Superradiance In Stars

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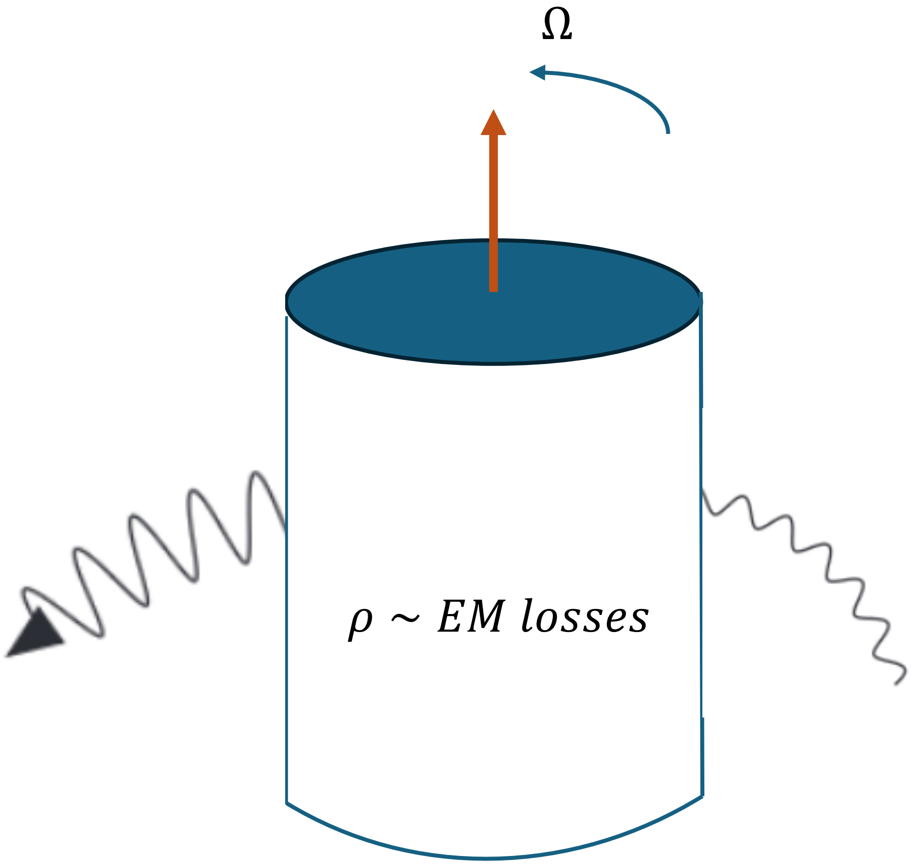
Black Holes and Fundamental Fields

Lisbon 1-5 July 2024

Superradiance is a Generic Phenomenon



$m\Omega > \omega$

Don't need horizons/ergoregions (black holes, fluid vortices):

Y. B. Zel'dovich Pis'ma Zh. Eksp. Teor. Fiz. 14 (1971) 270 [JETP Lett. 14, 180 (1971)].
Y. B. Zel'dovich Zh. Eksp. Teor. Fiz 62 (1972) 2076 [Sov.Phys. JETP 35, 1085 (1972)].

Neutron Stars are Incredible Objects!

Extremely Compact

$M_{NS} \simeq M_{\odot}, R \simeq 10km$

Spin 100s of times a second

Superradiance in stars:

JCAP 12 (2022) 008 Chadha-Day, Garbrecht, **JM**

Cardoso, Brito, Rosa Phys. Rev. D 91 (2015) 12, 124026

Strongest magnetic fields in Universe $B \simeq 10^{14}\ G$

Great axion labs in radio and X-ray (e.g. DM detection)

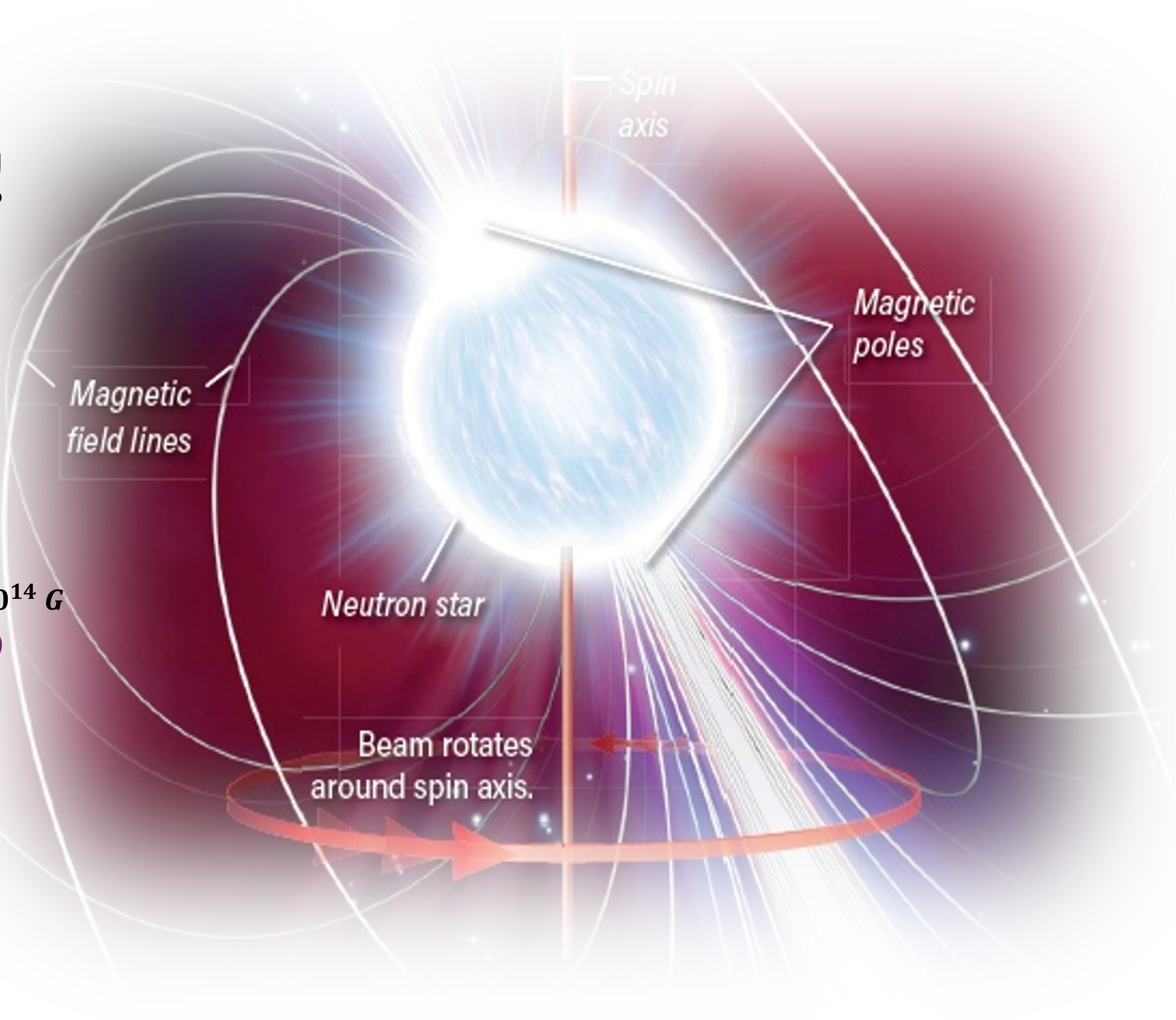
Accurate clocks (spin known very well)

Ultra-light dark matter tests

Khmelnitskya, Rubakovb JCAP 02 (2014) 019

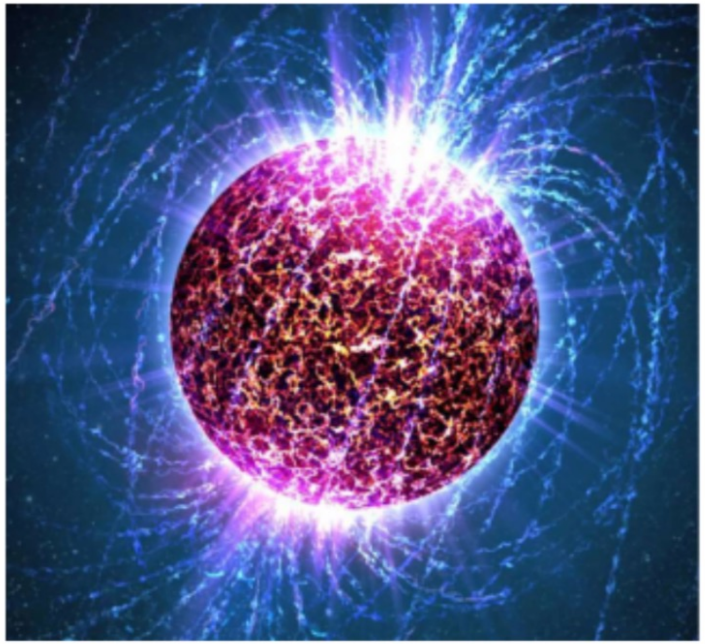
Very Dense (good dissipation)

Large scattering cross sections/short mean free path



Superradiance

Neutron Star



?

stellar matter

Black Hole

GR

Horizon

Superradiance in Stars

Superradiance in Stars: Non-equilibrium approach to damping of fields in stellar media

Chadha-Day, Garbrecht, **JM**

JCAP 12 (2022) 008

Superradiance in Stars

Cardoso, Brito, Rosa

Phys. Rev. D 91 (2015) 12, 124026

Superradiance in rotating stars and pulsar-timing constraints on dark photons

Cardoso, Pani, Yu

Phys. Rev. D 95, 124056 (2017)

Axion Superradiance in Rotating Neutron Stars

Day, **JM**

JCAP 10 (2019) 051

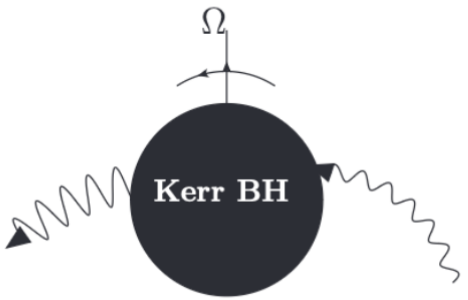
Particle Probes with Superradiance Pulsars

Kaplan, Rajendran, Riggins

[hep-ph1908.10440]

Superradiance

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + m_\phi^2\phi = 0 \qquad r_g = GM$$



$$Z_{lm}^{\mathrm{BH}} = \frac{|\phi_{\mathrm{out}}|^2}{|\phi_{\mathrm{in}}|^2} - 1 = r_g^2(m\Omega_{\mathrm{H}} - \omega)(\omega r_g)^{2l+1} \cdot \frac{1}{r_g}$$

<u>suppose for stars:</u> $\partial^2\phi + \gamma\dot{\phi} = 0$

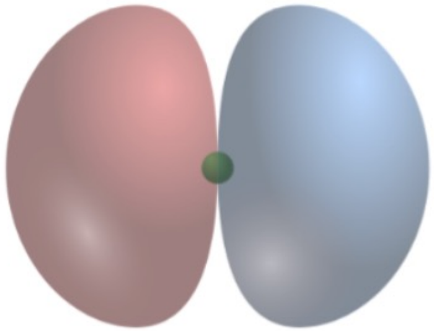
$$Z_{lm}^{\mathrm{Star}} = \frac{|\phi_{\mathrm{out}}|^2}{|\phi_{\mathrm{in}}|^2} - 1 = R^2\,(m\Omega - \omega)\,(\omega R)^{2l+1} \cdot \gamma$$

BH is damping membrane with effective damping $\sim 1/r_g$.

Cardoso, Brito, Rosa Phys. Rev. D 91 (2015) 12, 124026

Low Mass Fields are Bound $\implies$ Unstable

$$\phi = Y_{lm}(\theta, \phi)\psi_{lmn}(r)e^{\Gamma t}$$



$$\Gamma_{n\ell m} = r_g \left(m\Omega_{\mathrm{H}} - \omega\right)\left(GM\omega\right)^{4\ell+5} \cdot \frac{1}{r_g} \qquad \omega \simeq m_\phi.$$

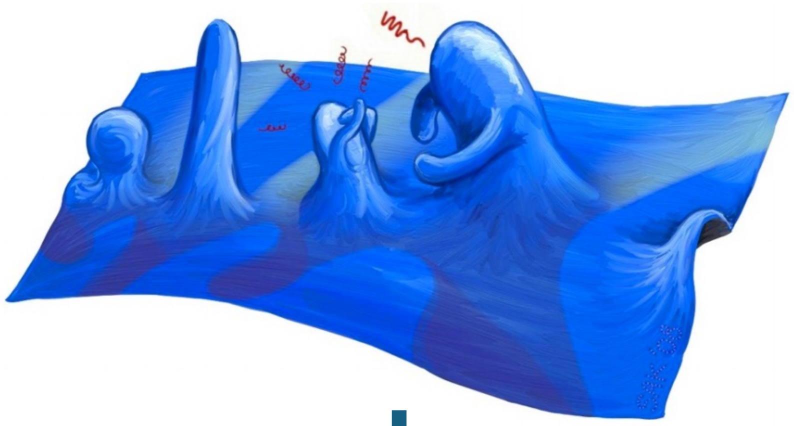
Expect stellar superradiance with

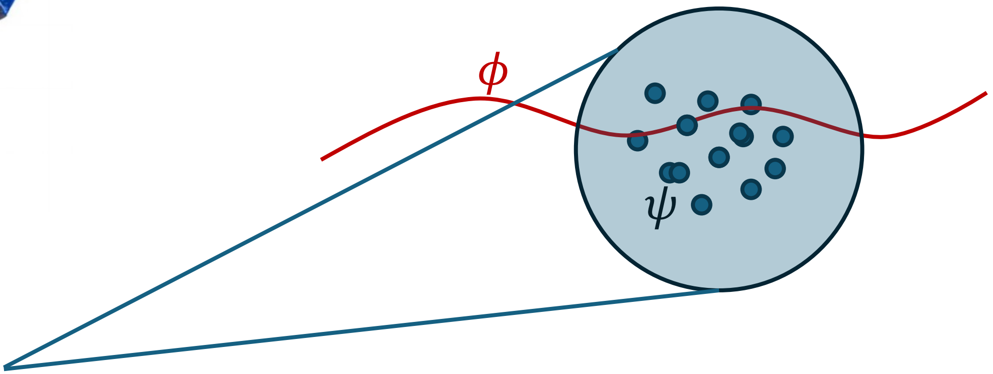
$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \gamma\dot{\phi} + m_\phi^2\phi = 0 \quad \implies \quad 1/r_g \to \gamma, \qquad r_g \to R$$

Path Integral

$$Z = \int d\phi \int d\psi \, d\bar{\psi} \, e^{S[\psi,\phi]}$$

Non-Eq QFT





$$\partial^2\phi(x) + m_\phi^2\phi(x) + \int dy \Pi_R(x,y)\phi(y) = 0$$

Dynamical Equation for ϕ

$$\partial^2_{x_1}\phi(x_1) + m^2_\phi\phi(x_1) + \int dx_2\Pi_R(x_1, x_2)\phi(x_2) = 0$$

Upon performing a Wigner transformation, defined by

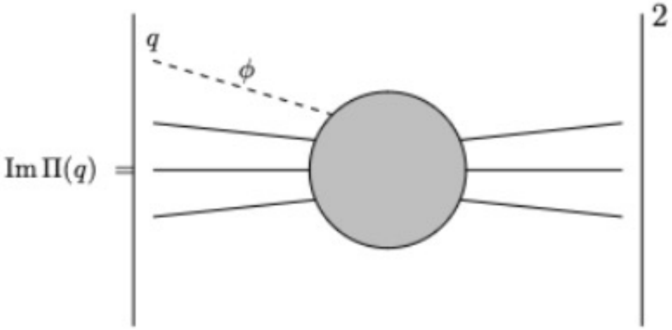
$$\Pi(q, x) = \int dy\Pi(x + y/2, x - y/2)e^{iy\cdot q}$$

$$\mathrm{x} = \frac{x_1 + x_2}{2}$$

$$y = x_1 - x_2$$

we can express this equation in Wigner space as

$$\partial^2\phi(x) + m^2_\phi\phi(x) + e^{i\partial_y\cdot\partial_q}\left[\Pi_R\left(q, \frac{x+y}{2}\right)\phi(y)\right]_{q\to 0, y\to x} = 0$$



$$-\nabla^2\phi(\omega, \mathbf{x}) + (m^2_\phi - \omega^2)\phi(\omega, \mathbf{x}) + i\mathrm{Im}[\Pi_R(\omega, \mathbf{x})]\phi(\omega, \mathbf{x}) \simeq 0$$

Flat Space + Static Star

$$Z_{lm}^{\mathrm{scat}} = \frac{|\phi_{\mathrm{out}}|^2}{|\phi_{\mathrm{in}}|^2} - 1 = -\frac{4R\,(\omega R)^{2l+2}}{(2l+1)!!(2l+3)!!} \cdot \frac{\mathrm{Im}\,\Pi}{\omega}$$

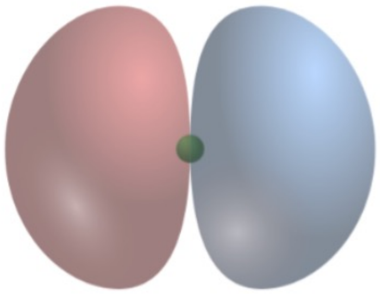
$$\downarrow$$

Add GR +Rotating Star

"A Modern approach to superradiance" Endlich, Penco JHEP 05 (2017) 052

$$\Gamma_{nlm} = (m_\phi R)^{(2l+3)}\,(m_\phi r_g)^{(2l+3)}\,\frac{\mathrm{Im}(\Pi)}{\omega}(\omega - m\Omega),$$

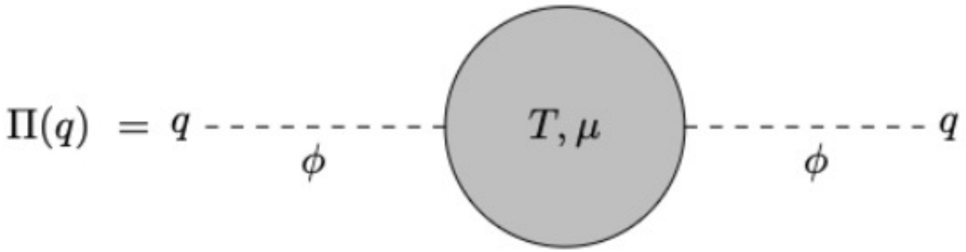
$$\phi = Y_{lm}(\theta,\phi)\psi_{lmn}(r)e^{i\Gamma t}$$

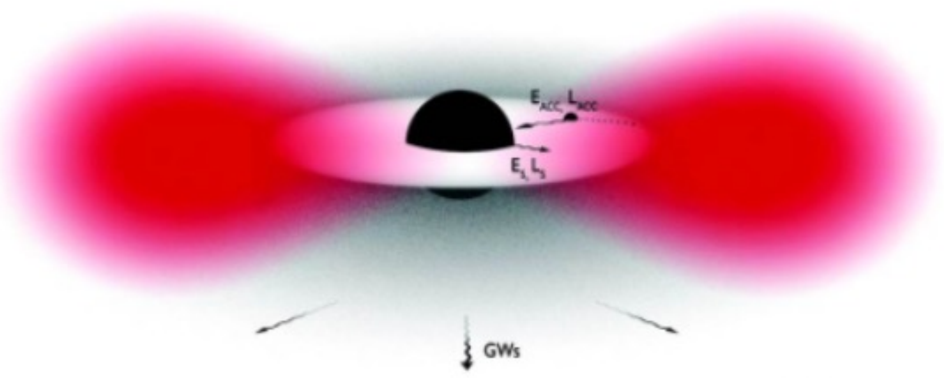


We now understand superradiance in stars!

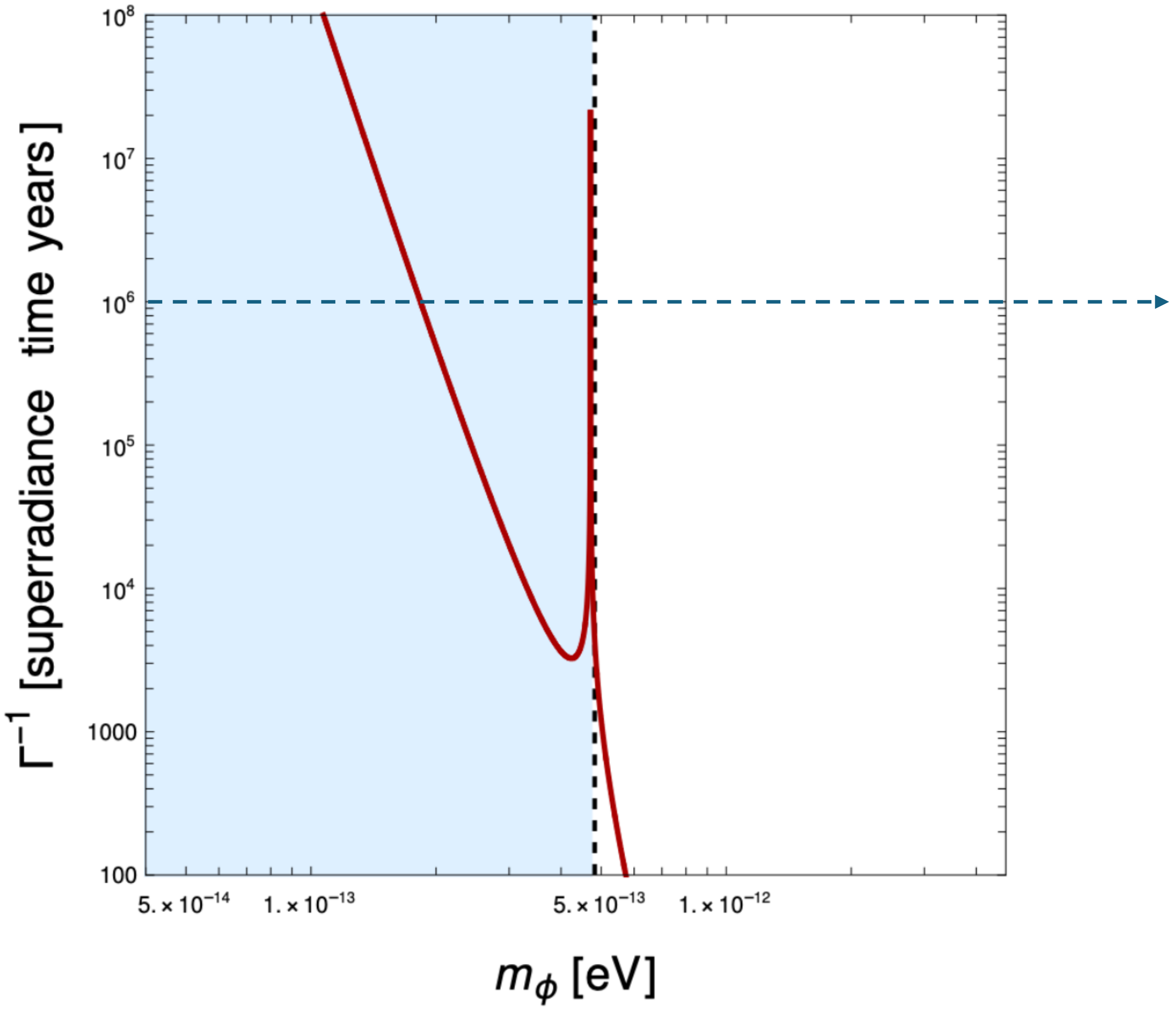
And we know how to calculate it:

$$-\nabla^2\phi(\omega,\mathbf{x}) + (m_\phi^2 - \omega^2)\phi(\omega,\mathbf{x}) + i\,\mathrm{Im}[\Pi_R(\omega,\mathbf{x})]\phi(\omega,\mathbf{x}) \simeq 0$$





Instability Rate (PSR J17482446ad)



Interesting NS timescales

$$\mathrm{Im}\Pi/\omega \sim cm^{-1}$$

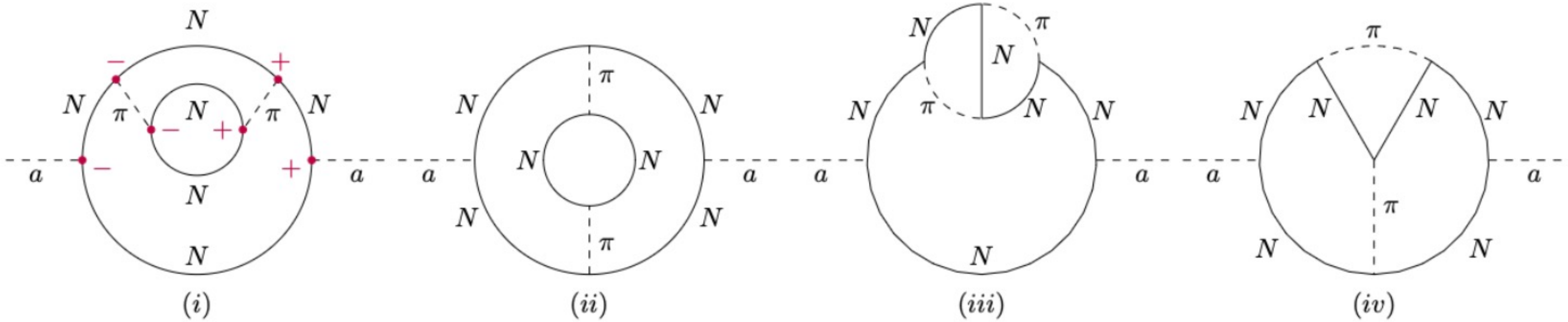
Inverse mean free path

Axions

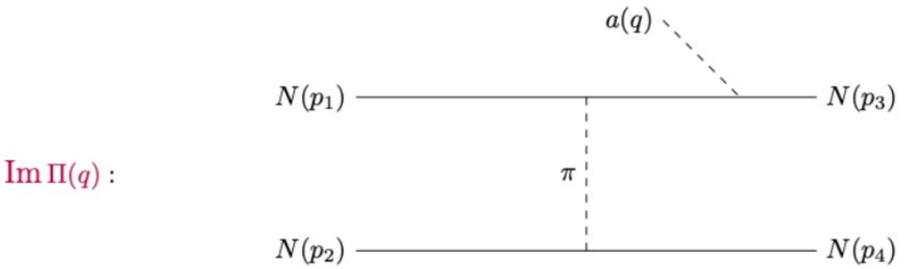
Axion nucleon interactions:

$$\mathcal{L}_{aNN} = G_{an}\partial_\mu \phi \bar{N}\gamma^\mu \gamma_5 N, \qquad \mathcal{L}_{\pi NN} = i(2m_n/m_\pi) f \pi_0 \bar{N}\gamma^5 N$$

$$-\nabla^2\phi(\omega,\mathbf{x}) + (m_\phi^2 - \omega^2)\phi(\omega,\mathbf{x}) + i\mathrm{Im}[\Pi_R(\omega,\mathbf{x})]\phi(\omega,\mathbf{x}) \simeq 0$$



Optical Theorem

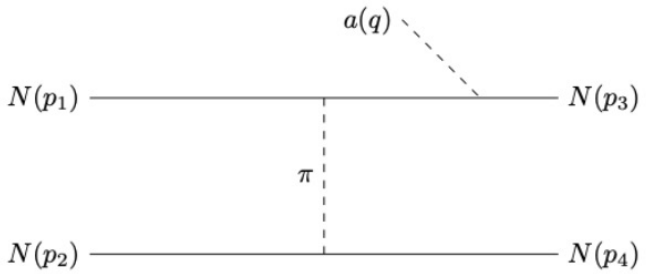


Axions

Superradiance rate for mode (l,m,n) :

$$\Gamma^a_{nlm} \simeq \frac{G_{an}^2 m_n^4}{m_\pi^4} p_F T^2 \cdot (m_\phi R)^{(2l+3)} (m_\phi r_g)^{(2l+3)} \cdot \frac{(\omega - m\Omega)}{\omega}$$

$\mathrm{Im}\,\Pi(q)$:

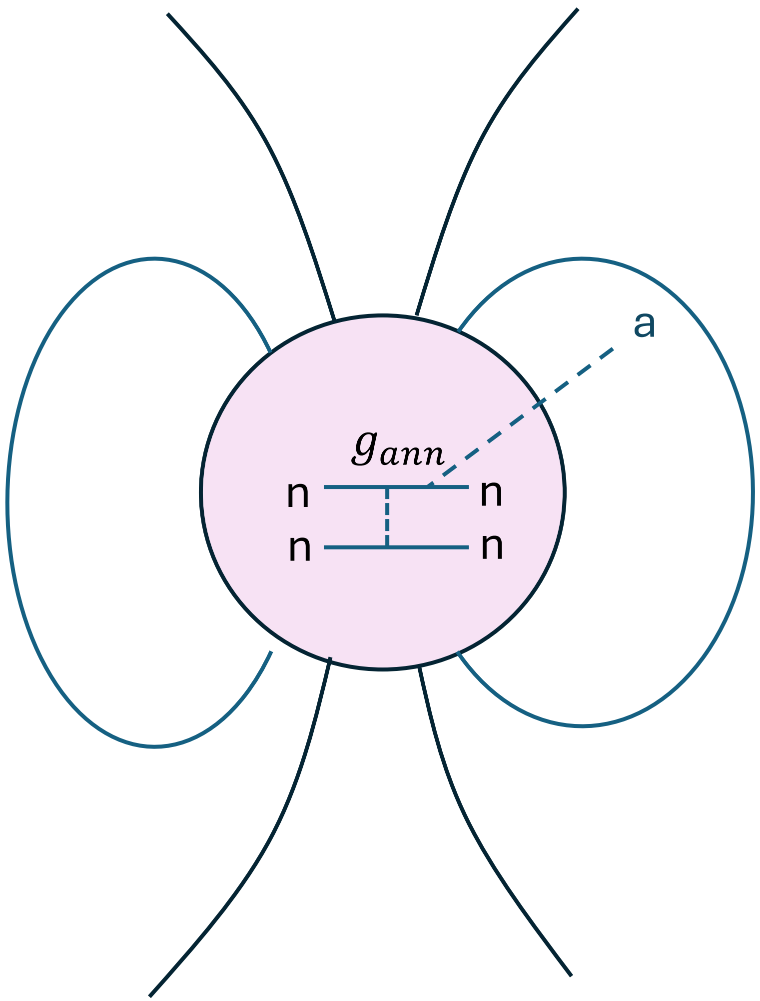


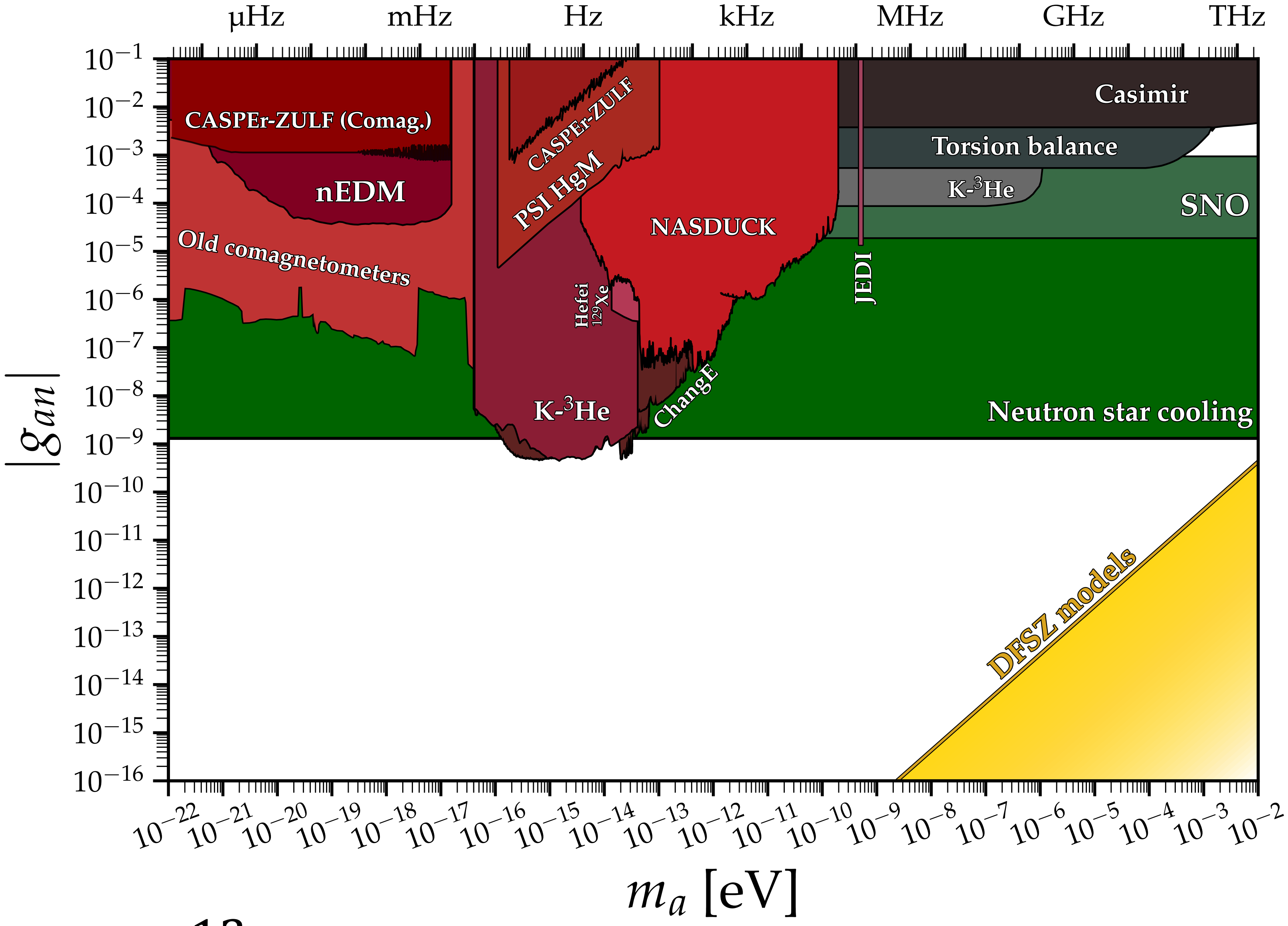
$$\mathrm{Im}\,\Pi = \prod_i \int \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} f_i \prod_j \int \frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j} (1 \pm f_j) \left|\mathcal{M}_{aNN\to NN}\right|^2$$

(calculate integral from axion mean free path: Harris (2020), Brinkmann + Turner (1988))

Axions

Stellar Cooling





$$\tau_{sup} \simeq 10^{13} \text{ yr}$$

What abouts (CP-even) Scalars ?

Pseudoscalar

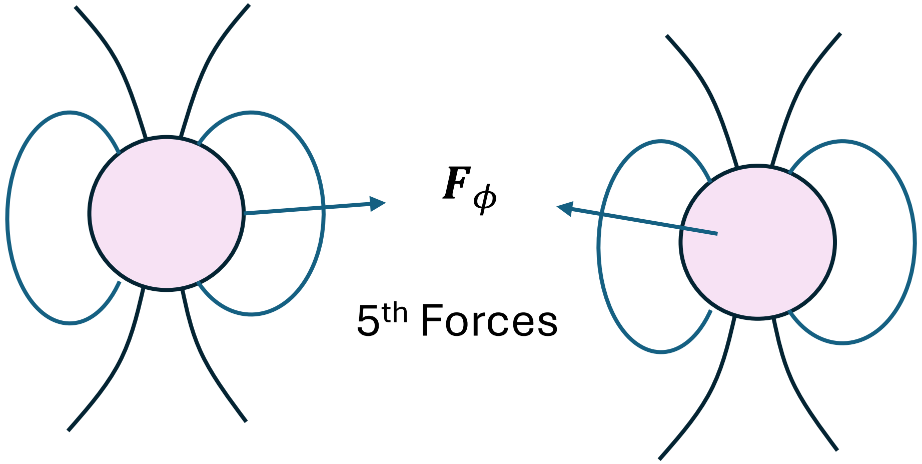
$$\mathcal{L}_{aNN} = G_{an}\partial_\mu\phi\bar{N}\gamma^\mu\gamma_5 N,$$

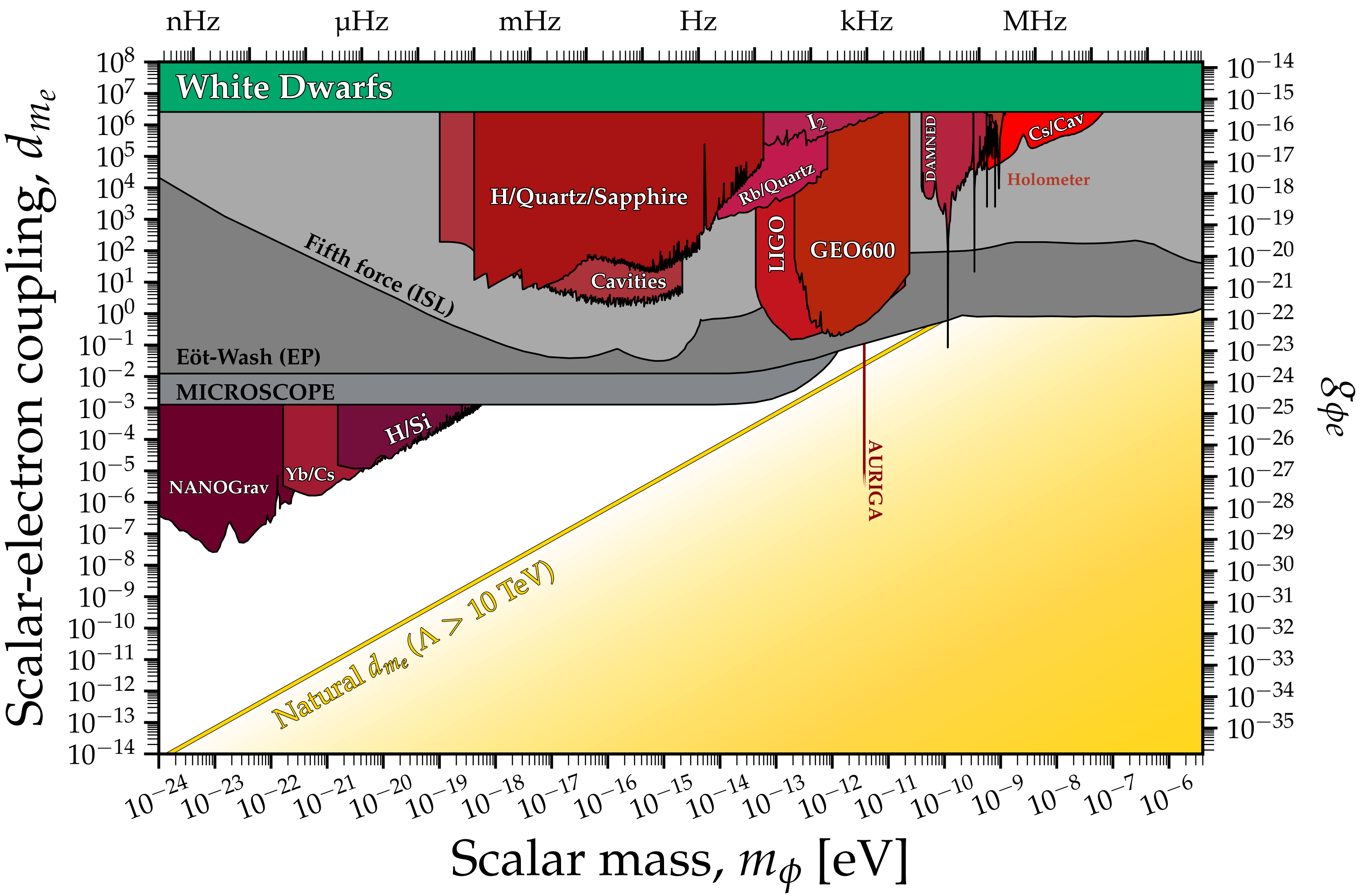
$g_{ann} = 10^{-9}$ $\quad \tau^a_{sup} \simeq 10^{13}$ yr

$$L_{\phi nn} = m_n (4\pi\, G)^{1/2} d_{m_e}\, \phi\, \bar{n}\, n = G_{\phi nn}\phi\, \bar{n}\, n$$

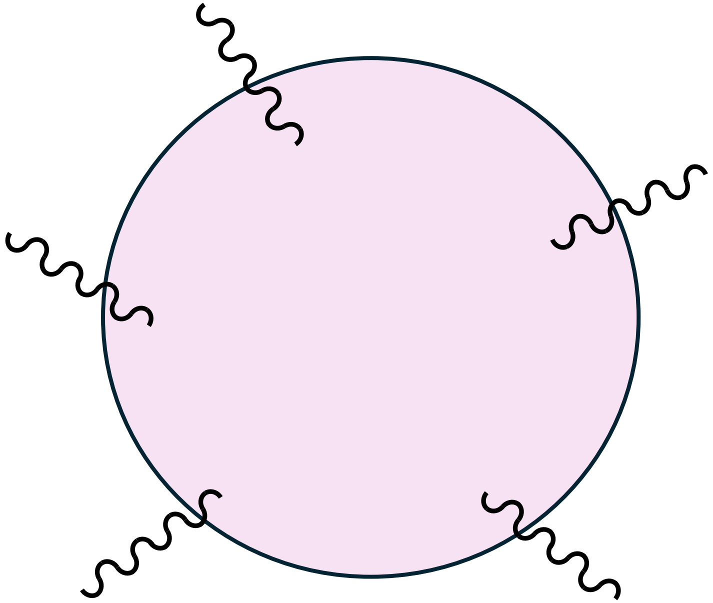
Scalar

$G_{\phi nn} = 10^{-24}$ $\quad \tau^\phi_{sup} \simeq 10^{43}$ yr





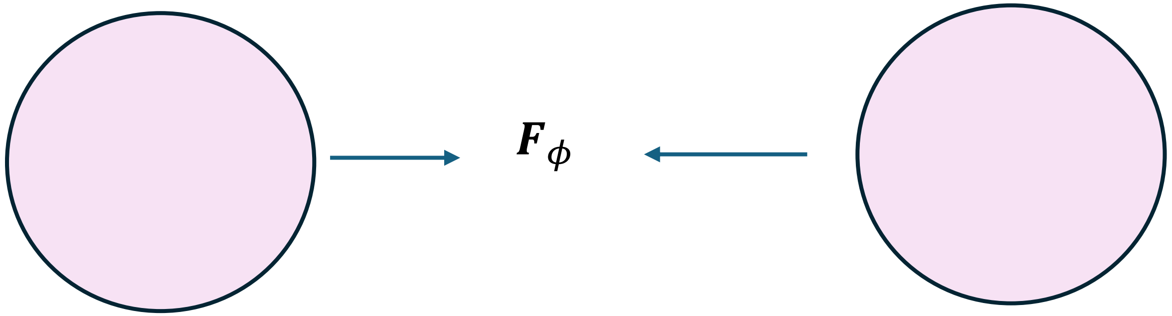
A "no-go" theorem for stellar superradiance ?



Stellar Cooling

$$Q \propto \int d\omega\, \omega^2 \; 1/\lambda_{MFP}(\omega)$$

(Schematically)



5th Forces

Backup: What about dark photons ?

Dent, Ferrer, Krauss (1201.2683)

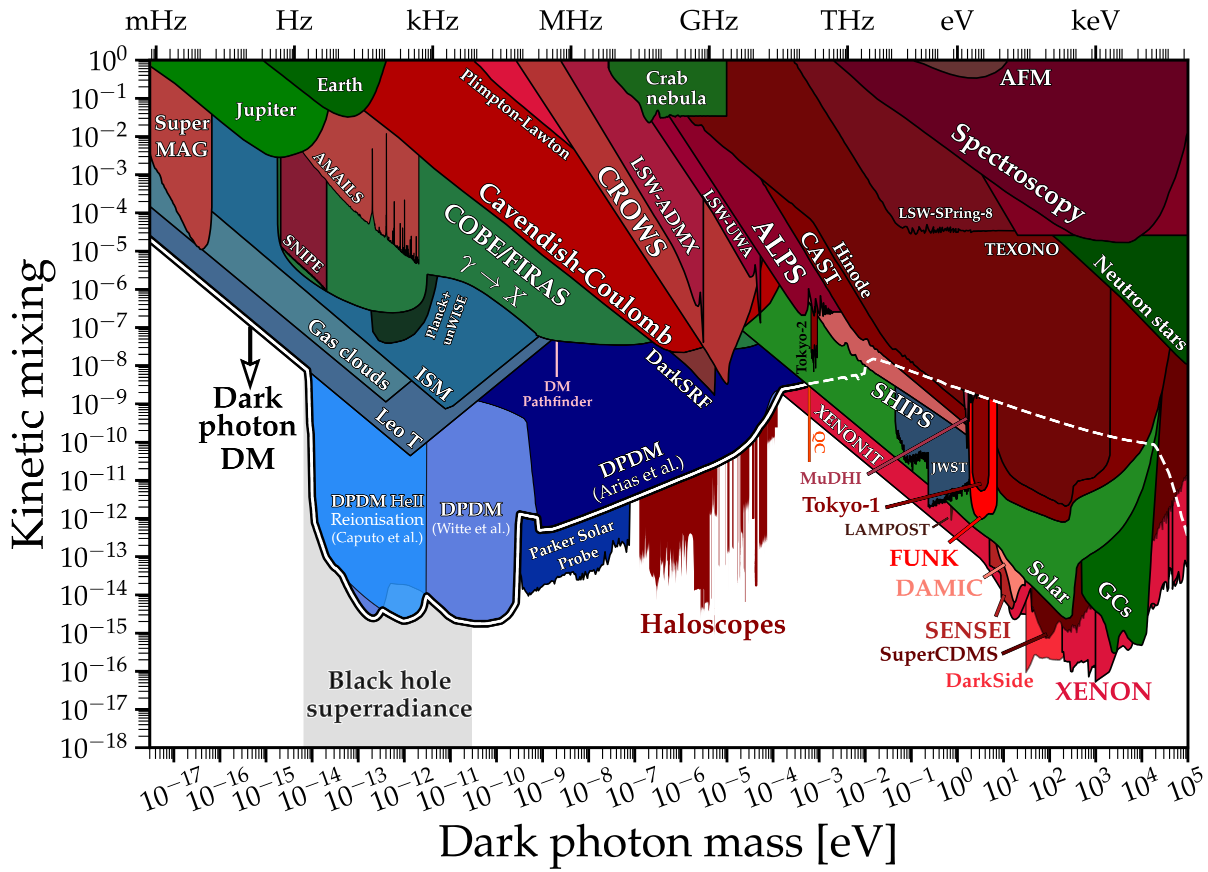
P + P -> P + P + A’

$$\sum_s |\mathcal{M}|^2_{p+p} = \frac{m_N^2}{m_\pi^4}\frac{64\mathbf{k}^2}{E_{A'}^2}\left(\frac{C_k|\mathbf{k}|^4}{(\mathbf{k}^2+m_\pi^2)(\mathbf{k}^2+m_\pi^2)} + \frac{C_l|\mathbf{l}|^4}{(\mathbf{l}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)} + \frac{C_{kl}(|\mathbf{k}|^2|\mathbf{l}|^2 - 2|\mathbf{k}\cdot\mathbf{l}|^2)}{(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)}\right)$$

$\sqrt{C_k} = \epsilon\, e$ $\quad$ k $\sim k_F \sim$100 MeV $\quad$ $m_A = 10^{-12}$ eV $\quad$ $\epsilon \sim 10^{-10}$

Some **VERY** rough estimates by comparing matrix elt for axions to dark photons:

$$\tau_{sup}^{A_\mu} \simeq 10^4 - 10^6 \text{ year (ish)}$$



More analysis needed: proper calculation + spin in rate

(I’m confused why we can evade 5th force/cooling what’s special about dark photons?)

See also Cardoso, Pani, Yu Phys. Rev. D 95, 124056 (2017)

Conclusions

1. Can compute stellar superradiance rates from first principles in terms of micro + macrophysics

2. Stellar cooling and firth forces limit rates
“no-go theorem” ?

3. However: stellar superradiance clearly worth pursuing (stars observed, spin/spindown known, EM counterparts)

4. Can these constraints be overcome ?

- Dark photons/tensor fields?

- Macroscopic perturbations of the whole star?