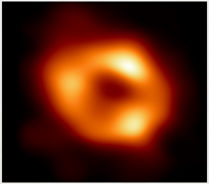
Beyond the Kerr paradigm: Black Holes and the fate of the Light ring Instability

Pedro V.P. Cunha

University of Aveiro
Department of Mathematics - CIDMA



Several work in collaboration with: C. Herdeiro, E. Radu, E. Berti, N. Sanchis-Gual

What is the nature of the Black Hole candidate in the *galactic center*?

What is the nature of the Black Hole candidate in the *galactic center*?

Is it described by the paradigmatic Kerr solution?

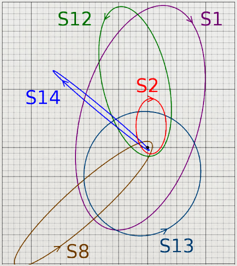
What is the nature of the Black Hole candidate in the *galactic center*?

Is it described by the paradigmatic Kerr solution?

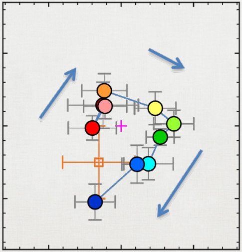
$\cdots$ or could models beyond Kerr *mimic* its phenomenology?

We have entered a precision era for strong gravity:

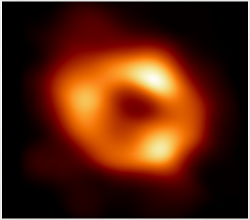
- motion of *S*-star orbits around Sgr A* Eisenhauer+, ApJ 628 246 (2005)
- access to the black hole (BH) shadow image of Sgr A*. EHT, ApJL 2022 ApJL 930 L12



S star orbits around SgrA*
Eisenhauer+, ApJ 628 246 (2005)



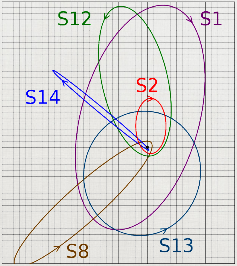
hot-spot orbiting Sgr A*
GRAVITY, A&A 618, L10 (2018)



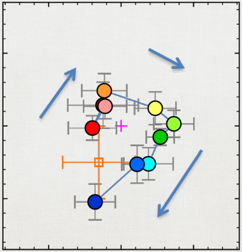
observed image Sgr A*
EHT, ApJL 2022 ApJL 930 L12

We have entered a precision era for strong gravity:

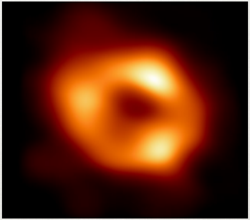
- motion of S-star orbits around Sgr A* Eisenhauer+, ApJ 628 246 (2005)
- access to the black hole (BH) shadow image of Sgr A*. EHT, ApJL 2022 ApJL 930 L12



S star orbits around SgrA*
Eisenhauer+, ApJ 628 246 (2005)



hot-spot orbiting Sgr A*
GRAVITY, A&A 618, L10 (2018)

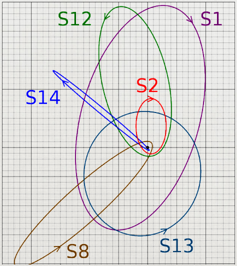


observed image Sgr A*
EHT, ApJL 2022 ApJL 930 L12

These observations can be used to test the true nature of Sgr A* (and BHs).

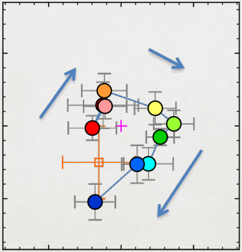
We have entered a precision era for strong gravity:

- motion of *S*-star orbits around Sgr A* Eisenhauer+, ApJ 628 246 (2005)
- access to the black hole (BH) shadow image of Sgr A*. EHT, ApJL 2022 ApJL 930 L12



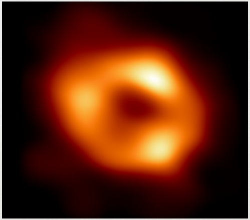
S star orbits around SgrA*

Eisenhauer+, ApJ 628 246 (2005)



hot-spot orbiting Sgr A*

GRAVITY, A&A 618, L10 (2018)



observed image Sgr A*

EHT, ApJL 2022 ApJL 930 L12

These observations can be used to test the true nature of Sgr A* (and BHs).

The shadow observation is connected to a special set of bound null orbits: *Light Rings* (**LRs**).

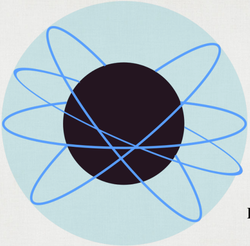
Plan of the talk

1. Why Light Rings (**LRs**) are relevant for observations.
2. LRs around horizonless compact objects
3. The fate of the Light Ring instability

Plan of the talk

1. Why Light Rings (**LRs**) are relevant for observations.
2. LRs around horizonless compact objects
3. The fate of the Light Ring instability

A Light Ring (**LR**) is a (spatially closed) circular null geodesic orbit.



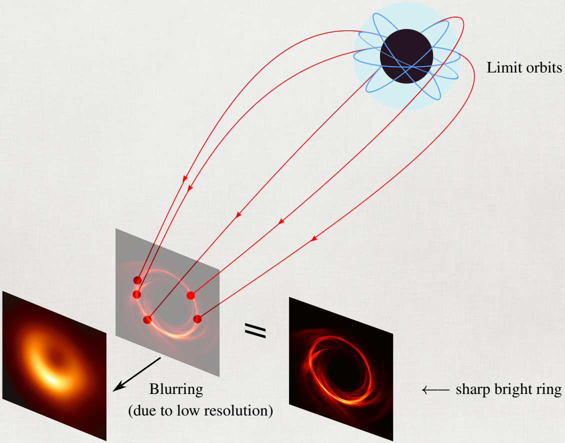
Photon Sphere as a collection of LRs

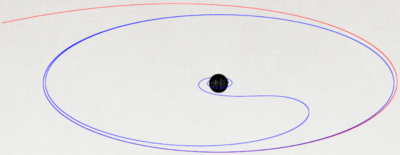
In spherically symmetry, the clustering of LRs forms a *Photon Sphere*.

LRs exist around Schwarzschild and Kerr BHs and very compact *horizonless* stars.

Why are Light Rings relevant to observations?

- Light Rings and similar limit orbits determine the BH shadow edge.
- It leaves a signature of a sharp bright ring in an *astrophysical* image.
- The Sgr A* image is consistent with the (*blurred*) image of a Kerr shadow.





Definition:

Ultra-Compact Objects (UCO) $\Longleftrightarrow$ **any** object with a LR (with or without an horizon).

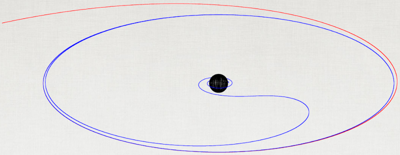
Motivation:

LRs are closely connected to direct astrophysical observables:

- Electromagnetic channel $\rightarrow$ BH shadow edge.
- GW channel $\rightarrow$ BH ringdown and Quasi-Normal modes.

Goebel, Astro. Jour. **172** (1972)

McWilliams, PRL **122** 191102 (2019)



Definition:

Ultra-Compact Objects (UCO) $\iff$ **any** object with a LR (with or without an horizon).

Motivation:

LRs are closely connected to direct astrophysical observables:

- Electromagnetic channel $\rightarrow$ BH shadow edge.
- GW channel $\rightarrow$ BH ringdown and Quasi-Normal modes.

Goebel, Astro. Jour. **172** (1972)

McWilliams, PRL **122** 191102 (2019)

Interest of UCOs:

Hypothetical exotic UCOs might *mimic* Kerr phenomenology because of LRs.

Plan of the talk

1. Why Light Rings (**LRs**) are relevant for observations.
2. LRs around horizonless compact objects
3. The fate of the Light Ring instability

A simple alternative to the Kerr paradigm are compact objects that have **no horizon**.

Example: Bosonic stars, which are horizonless solutions to scalar and Proca models:

Einstein-Klein-Gordon theory with a (complex) massive bosonic field

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G} - \nabla_\nu\phi\nabla^\nu\phi^* - \mu^2\phi^*\phi\right]. \qquad \text{(scalar)}$$

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{*\alpha\beta} - \frac{\mu^2}{2}\mathcal{A}_\alpha\mathcal{A}^{*\alpha}\right]. \qquad \text{(Proca)}$$

A simple alternative to the Kerr paradigm are compact objects that have **no horizon**.

Example: Bosonic stars, which are horizonless solutions to scalar and Proca models:

Einstein-Klein-Gordon theory with a (complex) massive bosonic field

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \nabla_\nu \phi \nabla^\nu \phi^* - \mu^2 \phi^* \phi \right]. \qquad \text{(scalar)}$$

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{4} \mathcal{F}_{\alpha\beta} \mathcal{F}^{*\alpha\beta} - \frac{\mu^2}{2} \mathcal{A}_\alpha \mathcal{A}^{*\alpha} \right]. \qquad \text{(Proca)}$$

This class of theories can lead to *viable* alternative Kerr objects:

- within a consistent and well motivated (effective field) theory of gravity.
- with a dynamical formation mechanism. Herdeiro, Radu PRL **119** 26 261101 (2017)
- can be (sufficiently) stable. Degollado+ 2018 PLB 781, 651; Sanchis-Gual+ PRL **123** 22 221101 (2019)

Simple example: Spherically-symmetric horizonless objects

The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + \left(\frac{E^2 - L^2\,H(r)^2}{g_{tt}}\right) = 0, \qquad H(r) = \frac{\sqrt{-g_{tt}}}{r}$$

E is photon's energy and L its angular momentum.

Conditions for a Light Ring: $H'(r) = 0$.

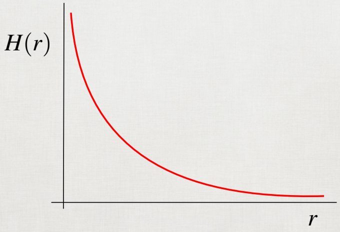
Simple example: Spherically-symmetric horizonless objects

The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + \left(\frac{E^2 - L^2 H(r)^2}{g_{tt}}\right) = 0, \qquad H(r) = \frac{\sqrt{-g_{tt}}}{r}$$

E is photon's energy and L its angular momentum.

Conditions for a Light Ring: $H'(r) = 0$.



Smooth deformation of metric fixing:

- asymptotic behavior (asymptotic flatness), *i.e.* $g_{tt} \to -1$.
- near origin behavior (smoothness), *i.e.* $g_{tt} \neq 0$.

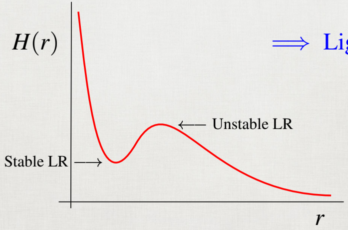
Simple example: Spherically-symmetric horizonless objects

The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + \left(\frac{E^2 - L^2\, H(r)^2}{g_{tt}}\right) = 0, \qquad H(r) = \frac{\sqrt{-g_{tt}}}{r}$$

E is photon's energy and L its angular momentum.

Conditions for a Light Ring: $H'(r) = 0$.

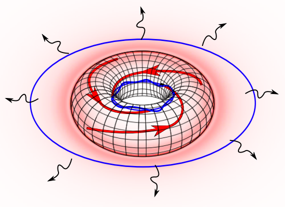


$\Longrightarrow$ Light Rings are created in pairs.

Smooth deformation of metric fixing:

- asymptotic behavior (asymptotic flatness), *i.e.* $g_{tt} \to -1$.
- near origin behavior (smoothness), *i.e.* $g_{tt} \neq 0$.

This is **not** a feature restricted to Spherical symmetry:



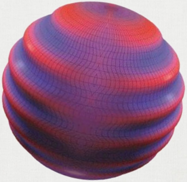
Phys. Rev. Lett. **119** (2017) no.25, 251102

P. Cunha, E. Berti and C. Herdeiro

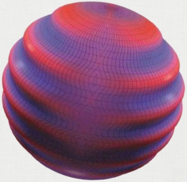
Theorem:

Horizonless UCOs must have at least *two* (non-degenerate) LRs, with one *stable*.

Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:

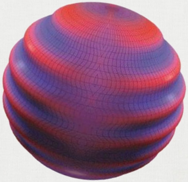


Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:



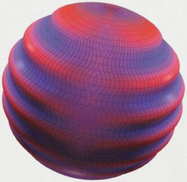
- stationarity and axial-symmetry.

Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:



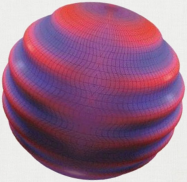
- stationarity and axial-symmetry.
- regularity at the origin.

Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:



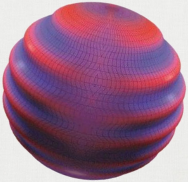
- stationarity and axial-symmetry.
- regularity at the origin.
- asymptotic flatness.

Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:



- stationarity and axial-symmetry.
- regularity at the origin.
- asymptotic flatness.
- metric is C^2-smooth (at least).

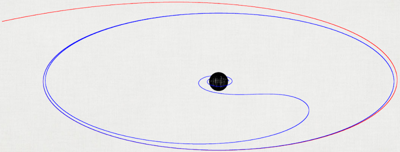
Consider a 3+1 spacetime **without any horizon** $(\mathcal{M}, g)$ and with the assumptions:



- stationarity and axial-symmetry.
- regularity at the origin.
- asymptotic flatness.
- metric is C^2-smooth (at least).
- circularity and causality.

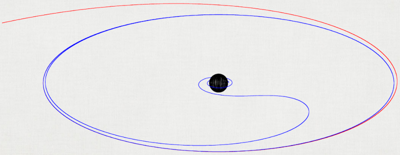
The null geodesic flow is determined by $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$.

This introduces a 2D potential $\boxed{U(r,\theta) \equiv g^{ab} p_a p_b}$ $\qquad a, b \in \{t, \varphi\}$.



The null geodesic flow is determined by $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$.

This introduces a 2D potential $\boxed{U(r,\theta) \equiv g^{ab} p_a p_b}$ $\quad a, b \in \{t, \varphi\}$.

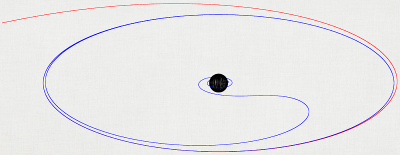


Since tangent vector field along LR path is a linear combination of (only) $\partial_t, \partial_\varphi$:

At a *Light Ring*: $\implies$ $\boxed{U = \nabla U = 0}$ PRL 124 (2020) 18, 181101

The null geodesic flow is determined by $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$.

This introduces a 2D potential $\boxed{U(r,\theta) \equiv g^{ab} p_a p_b}$ $\quad a, b \in \{t, \varphi\}$.



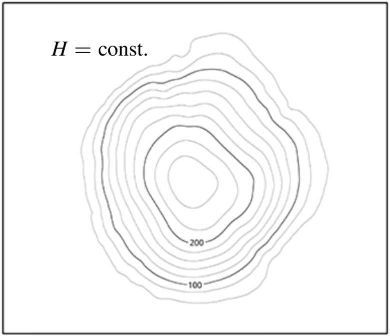
Since tangent vector field along LR path is a linear combination of (only) $\partial_t, \partial_\varphi$:

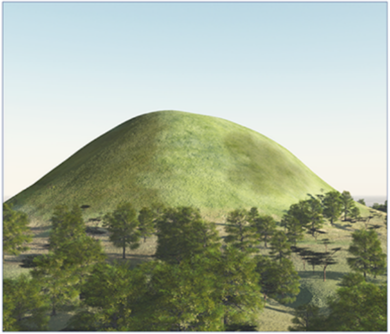
At a *Light Ring*: $\implies$ $\boxed{U = \nabla U = 0}$ PRL 124 (2020) 18, 181101

It is possible to factorize $U(r,\theta)$ into simpler 2D potentials $H_\pm(r,\theta)$:

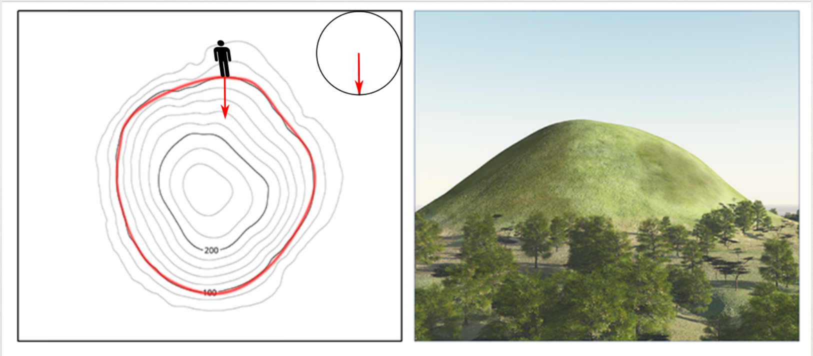
At a *Light Ring*: $\implies$ $\boxed{\nabla H_\pm(r,\theta) = 0}$ PRL 124 (2020) 18, 181101

The $\pm$ typically yields two different rotation directions.

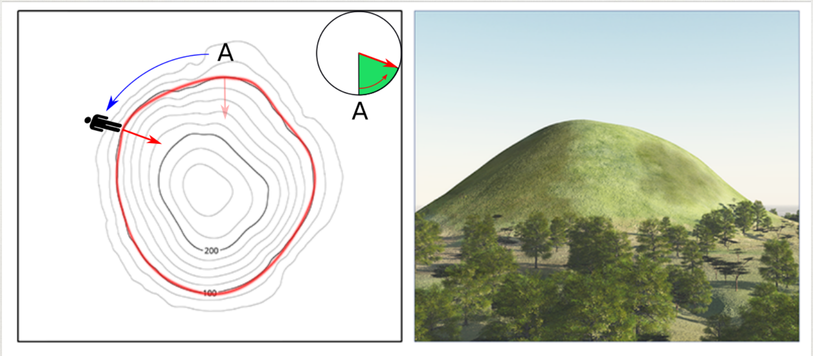

$H =$ const.
200
100



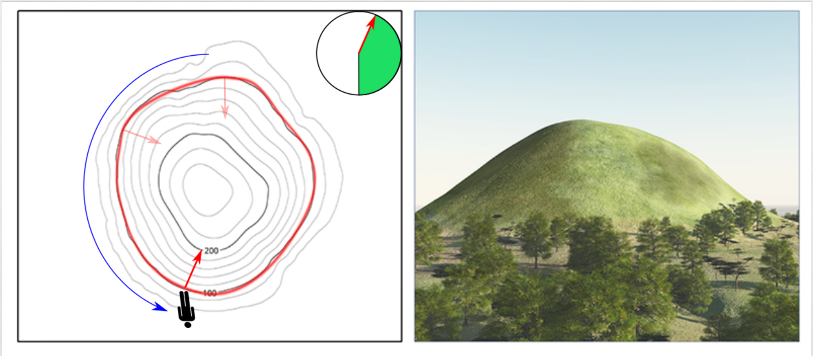
$\nearrow = \nabla H$



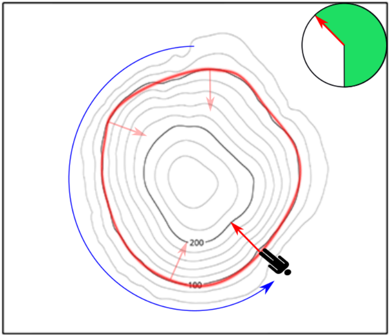
$\nearrow = \nabla H$

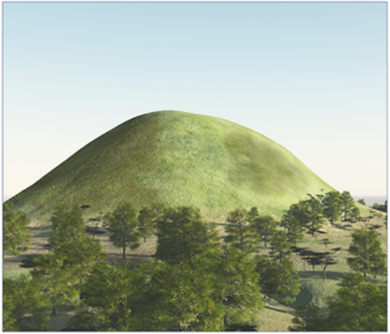


$\nearrow = \nabla H$

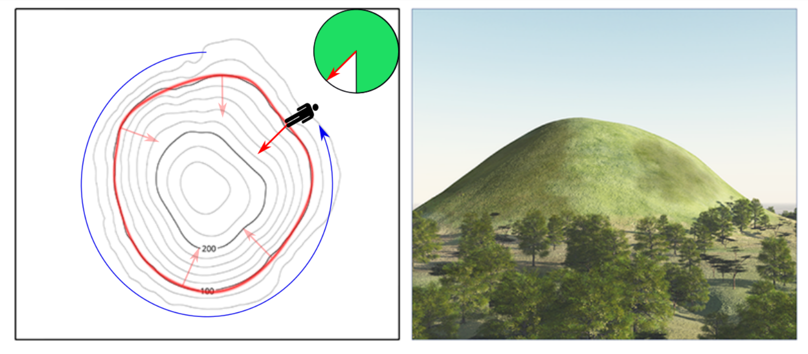


$\nearrow = \nabla H$

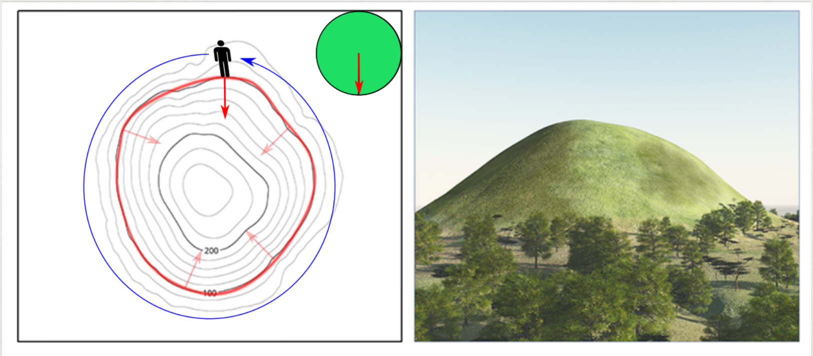




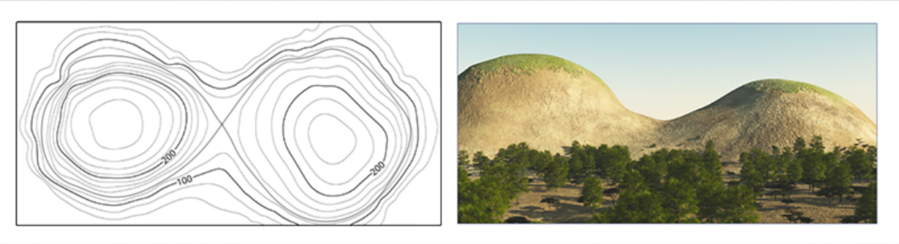
$\nearrow = \nabla H$



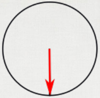
$\nearrow = \nabla H$

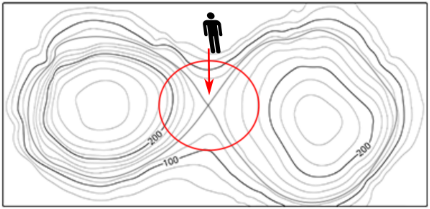


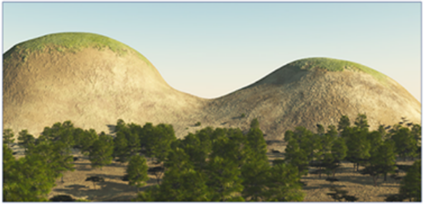
Rule 1 $\implies$ a Maximum (or Min.) leads to $(+\mathbf{1})$ full turns of vector field.


200
100
200

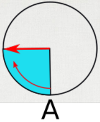
 $= \nabla H$

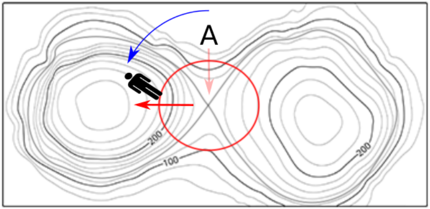


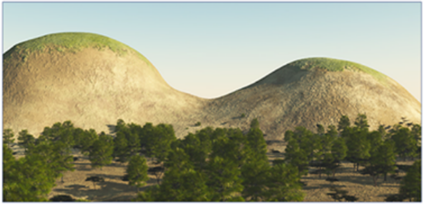




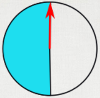
 $= \nabla H$

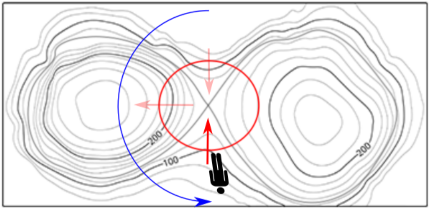


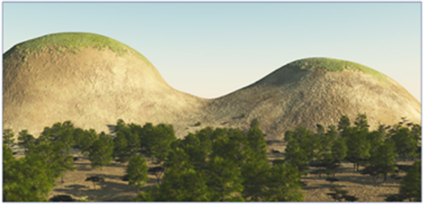




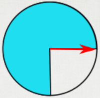
 $= \nabla H$

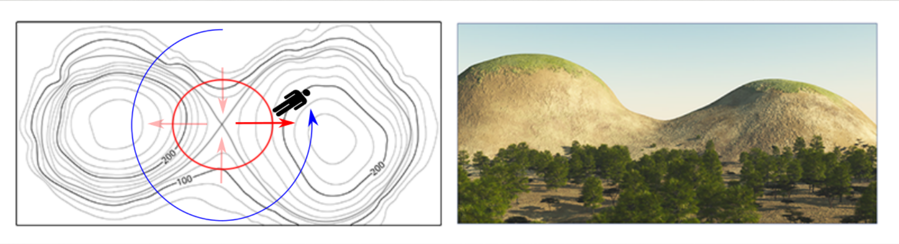




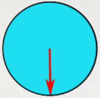


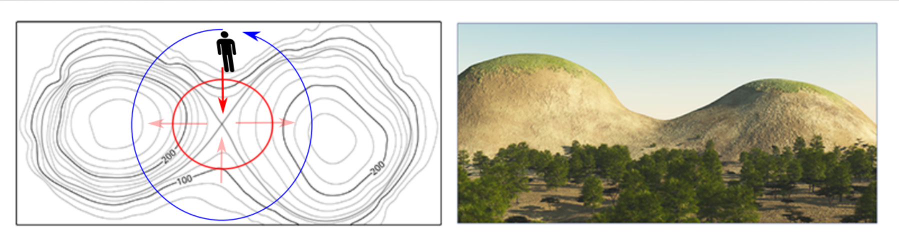
 $= \nabla H$





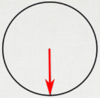
 $= \nabla H$

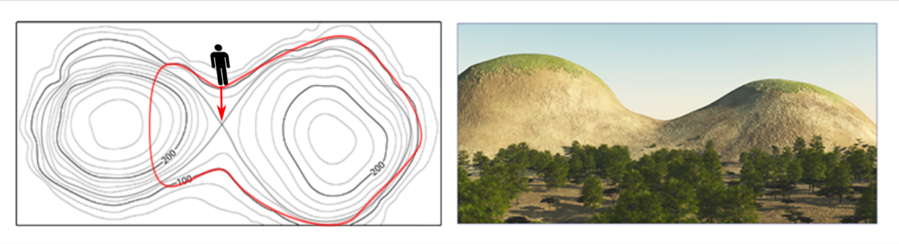




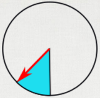
Rule 2 $\implies$ Saddle point leads to (-1) full turns of vector field (*i.e.* inverse sense).

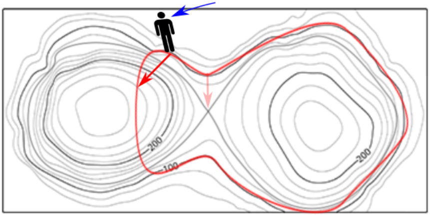
 $= \nabla H$

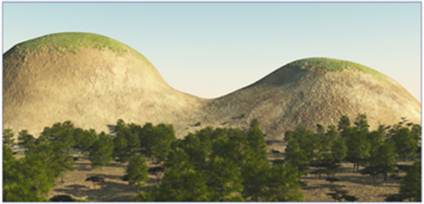




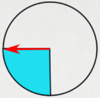
 $= \nabla H$

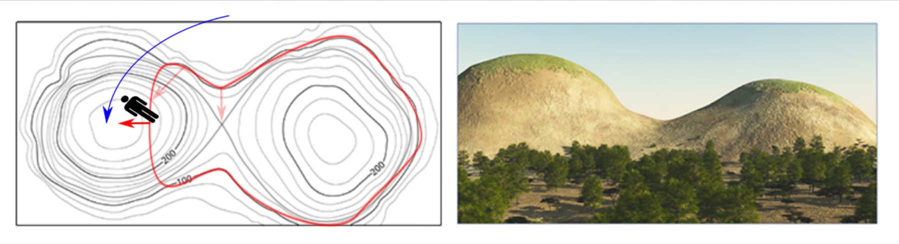




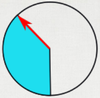


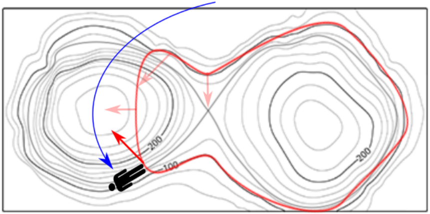
 $= \nabla H$

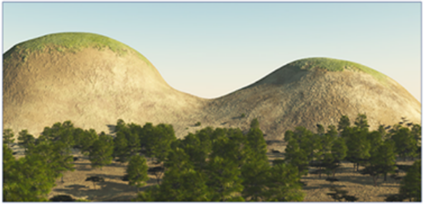




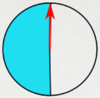
 $= \nabla H$

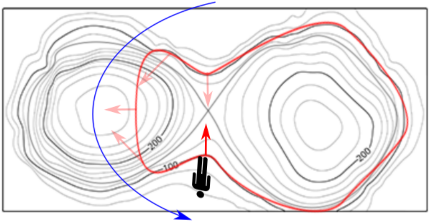


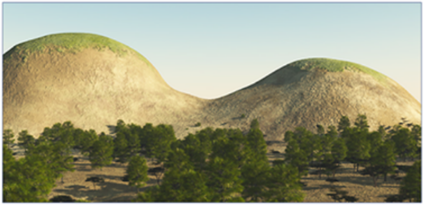




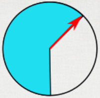
 $= \nabla H$

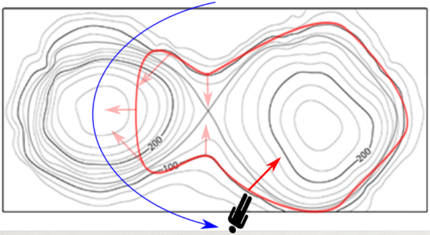


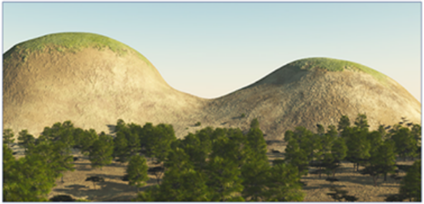




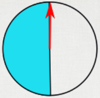
 $= \nabla H$

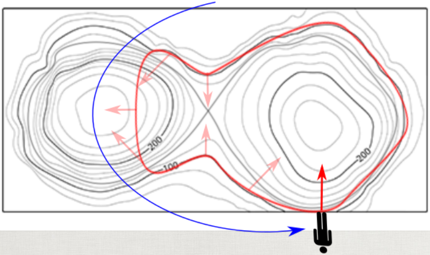


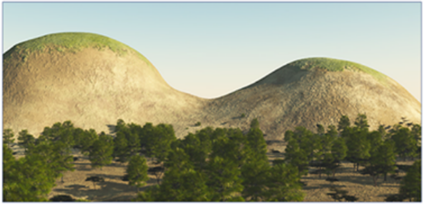




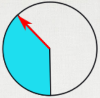
 $= \nabla H$

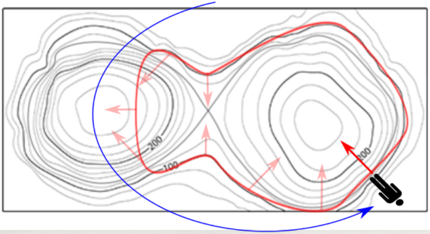


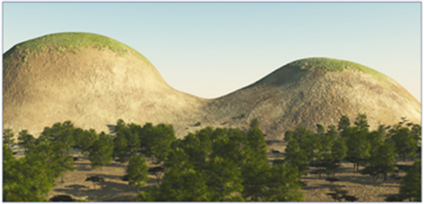




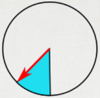
 $= \nabla H$

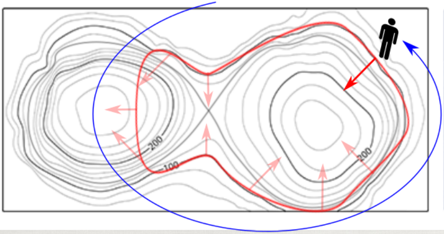


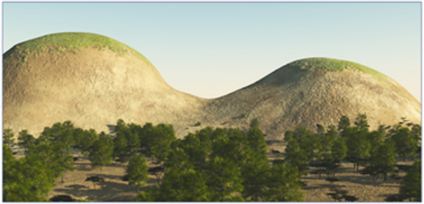




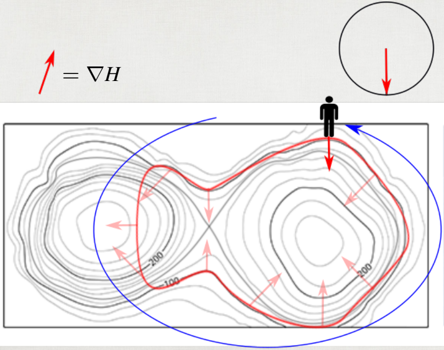
 $= \nabla H$

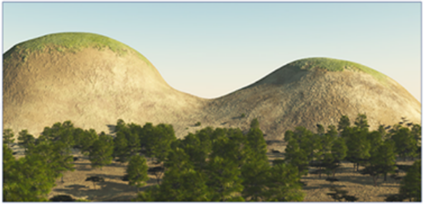




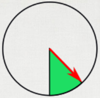


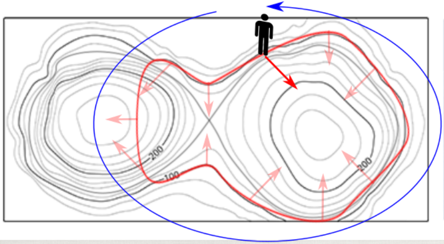
$= \nabla H$

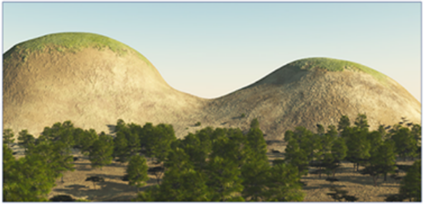




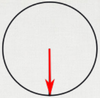
 $= \nabla H$

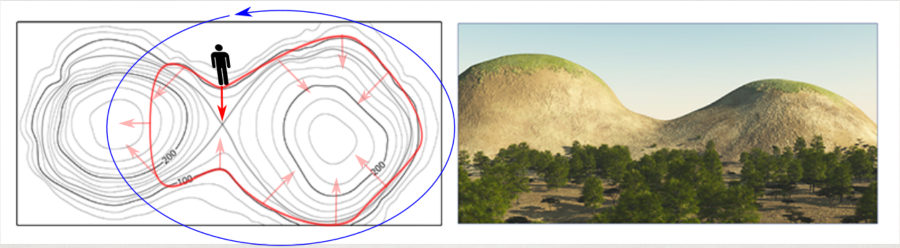




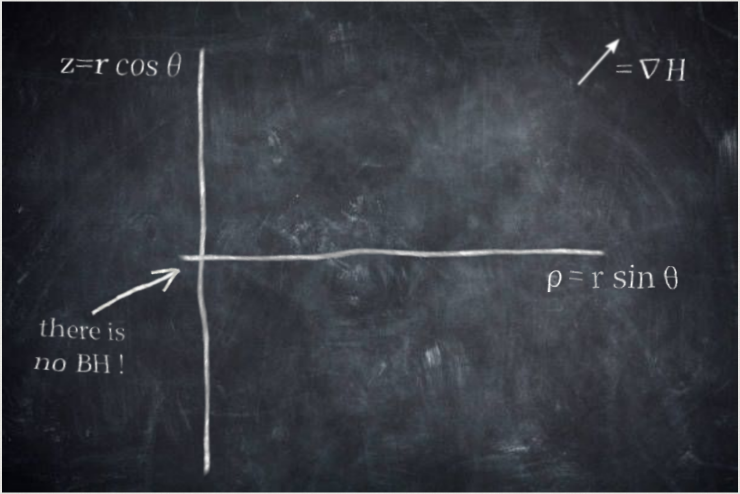


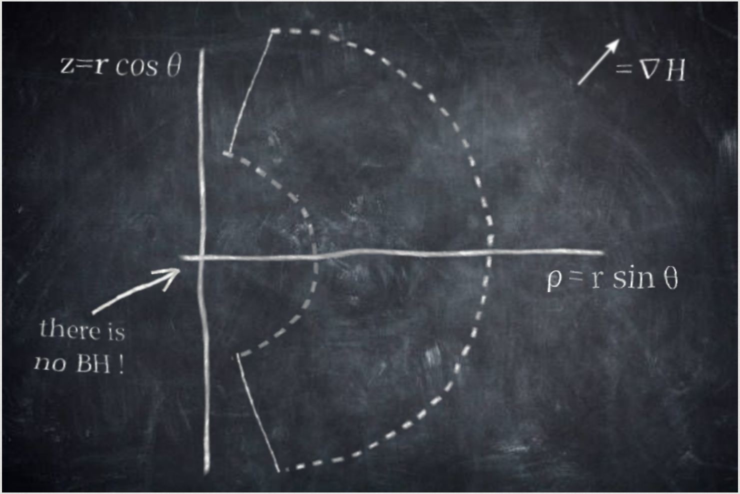
 $= \nabla H$

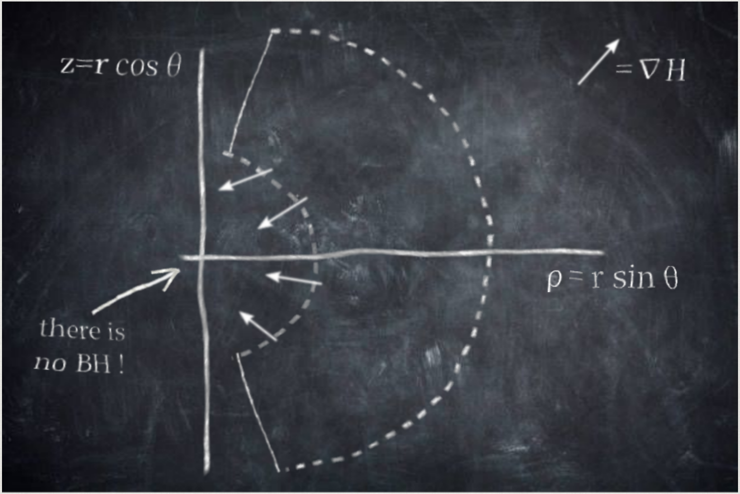




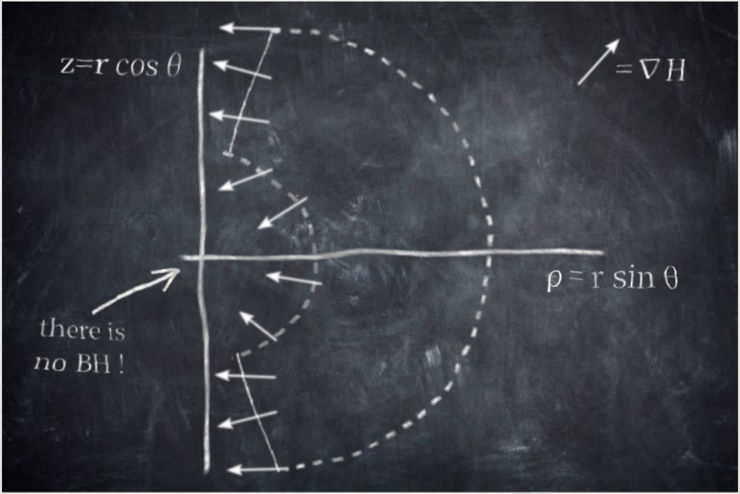
Rule 3 $\Longrightarrow$ number of full turns is additive, *e.g.* Saddle point (-1) + Max (+1) = 0.


z=r cos θ
= ∇H
ρ = r sin θ
there is
no BH !

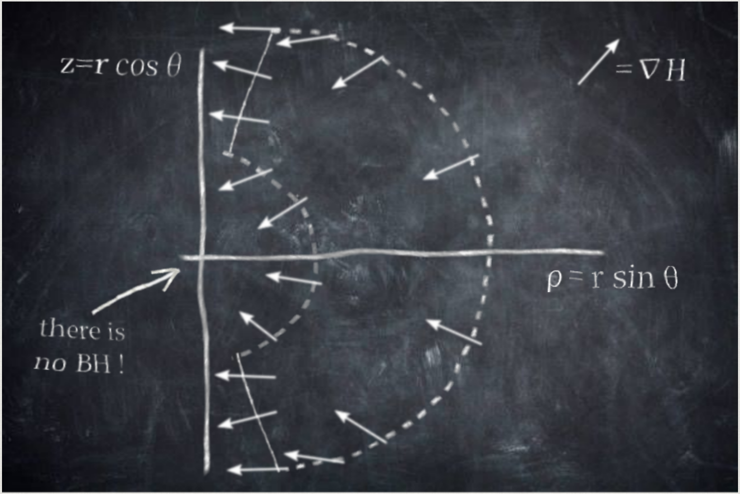

z=r cos θ
=∇H
ρ=r sin θ
there is
no BH !



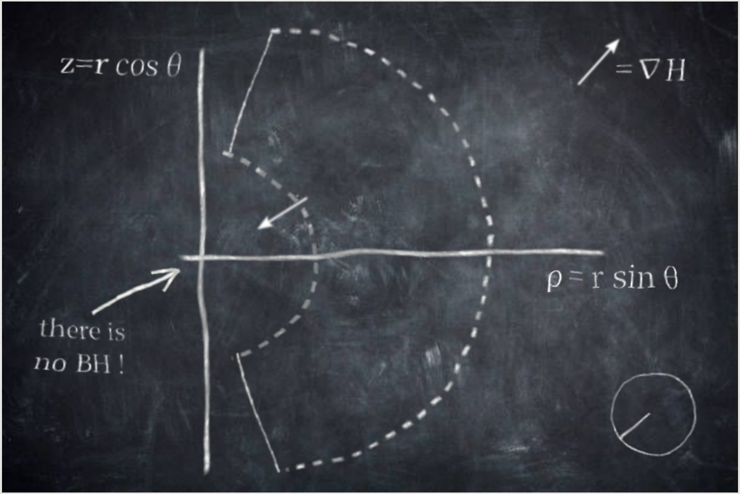
Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101



Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101



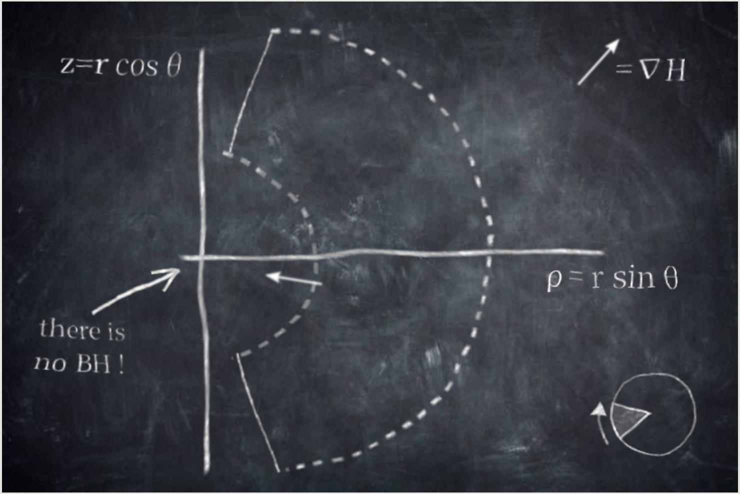
Asymptotic boundary $\implies$ *flat* spacetime.



Origin boundary $\implies$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\implies$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

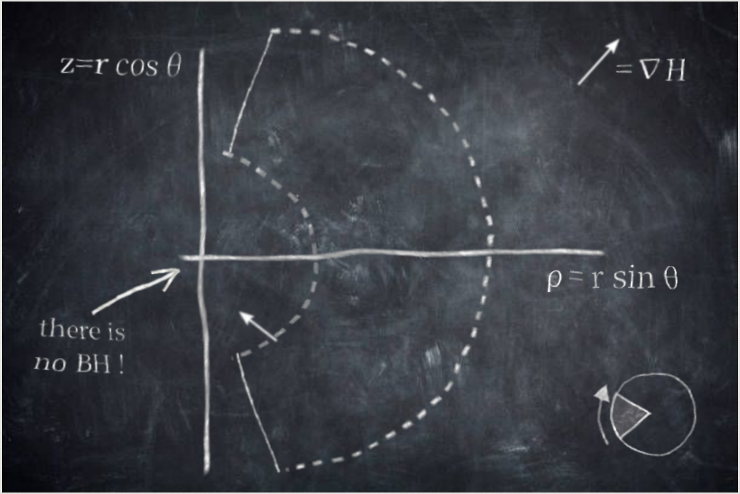
Asymptotic boundary $\implies$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

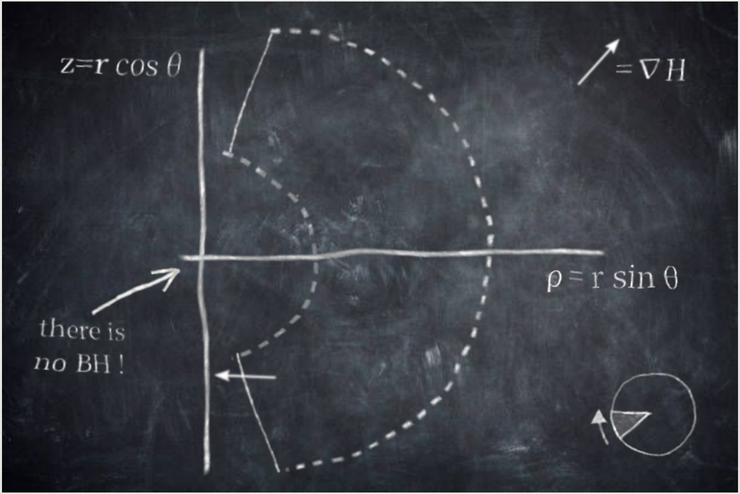
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

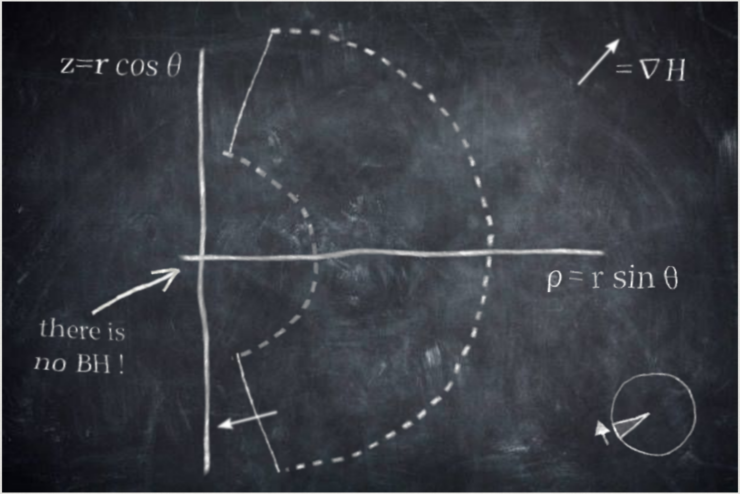
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

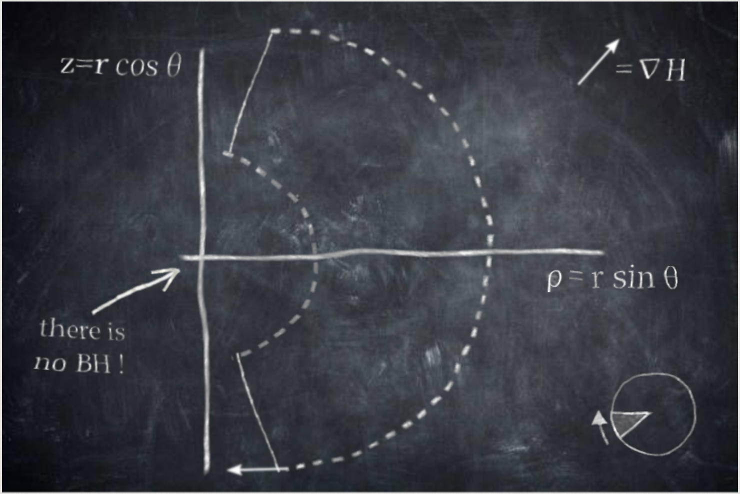
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

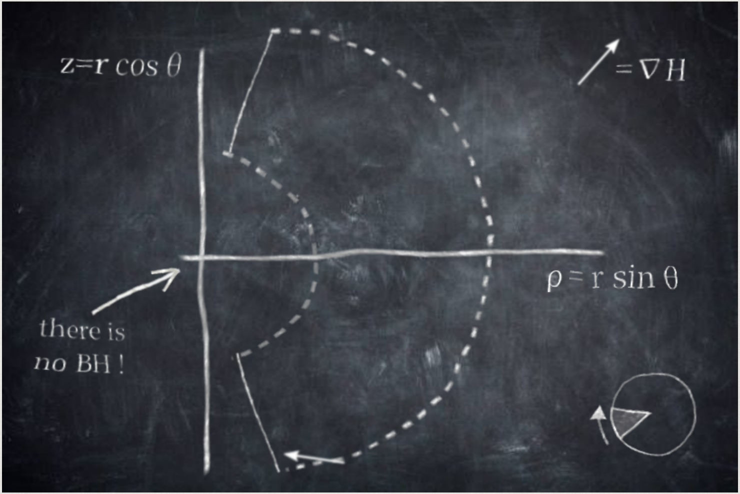
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

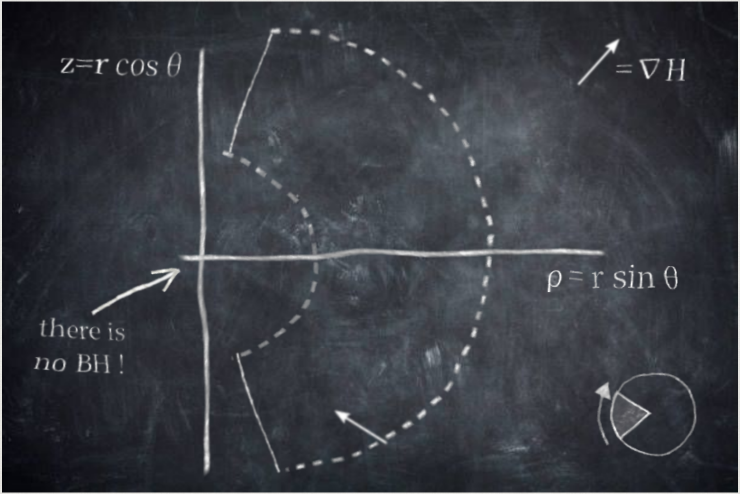
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

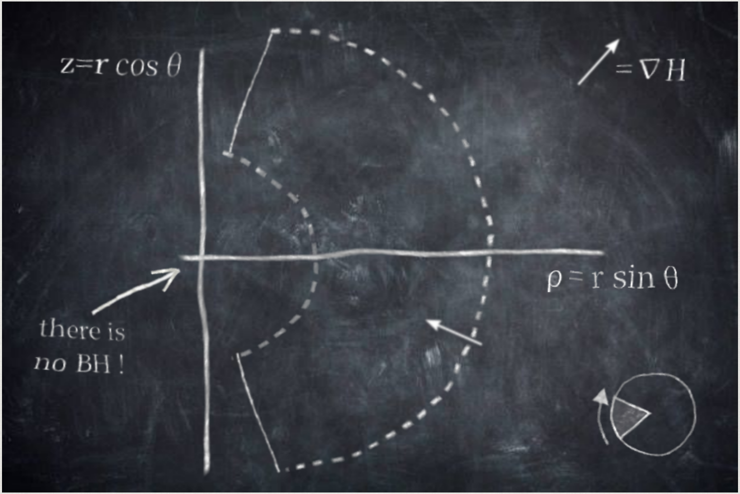
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

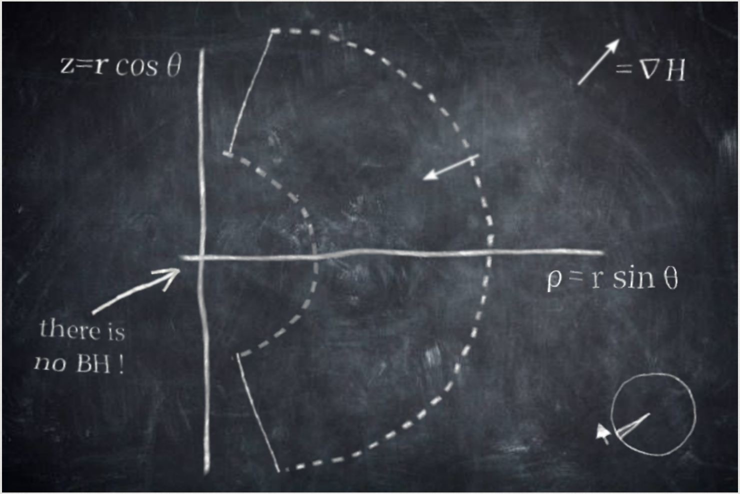
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

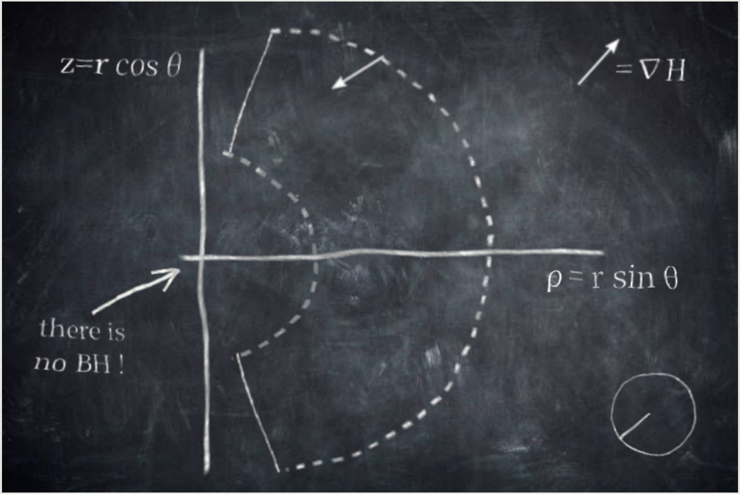
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

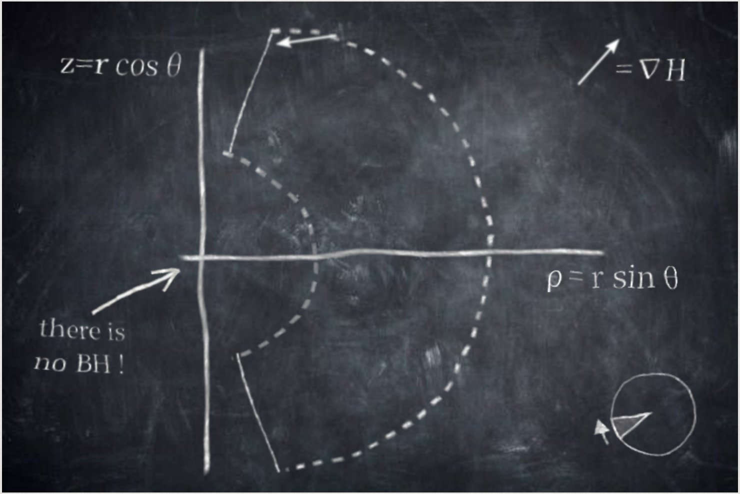
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

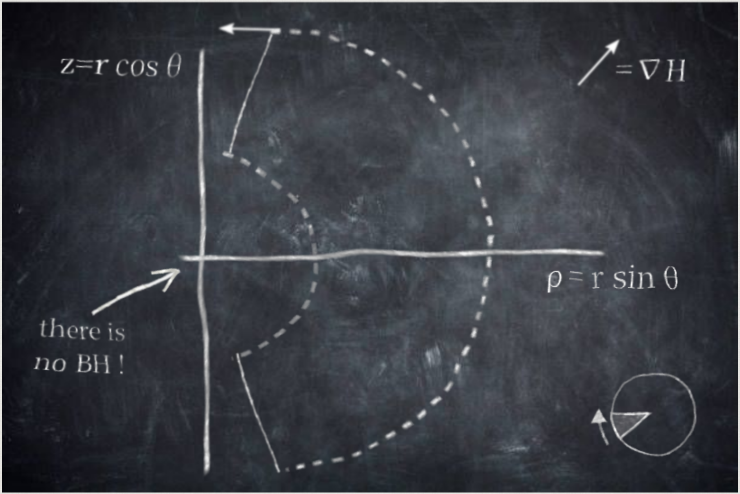
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

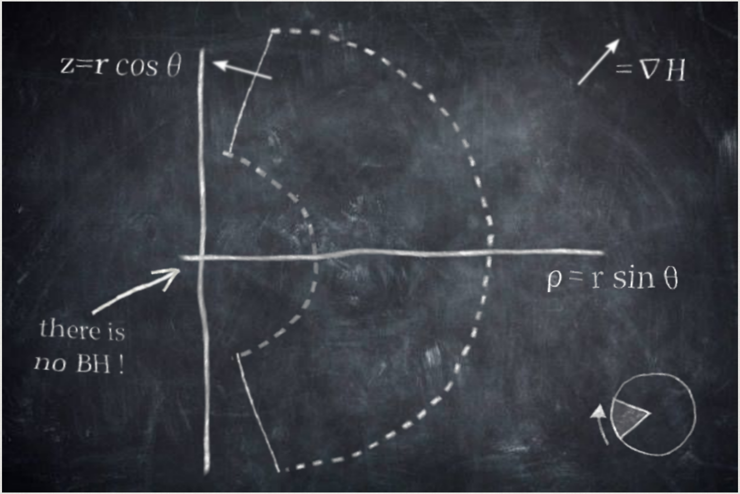
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

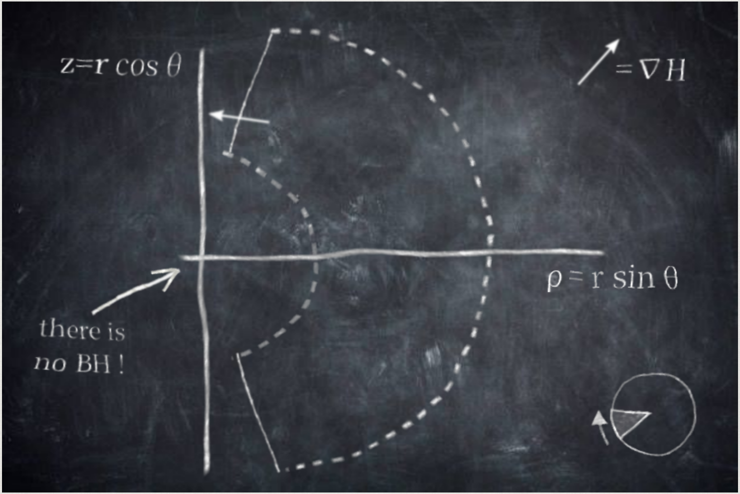
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

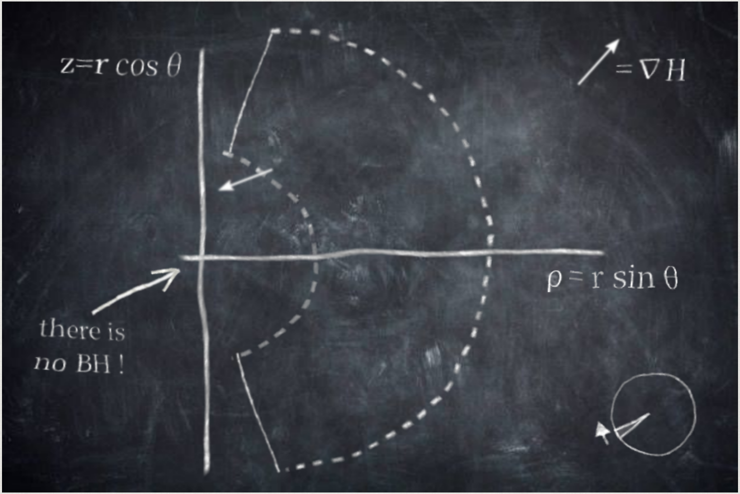
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

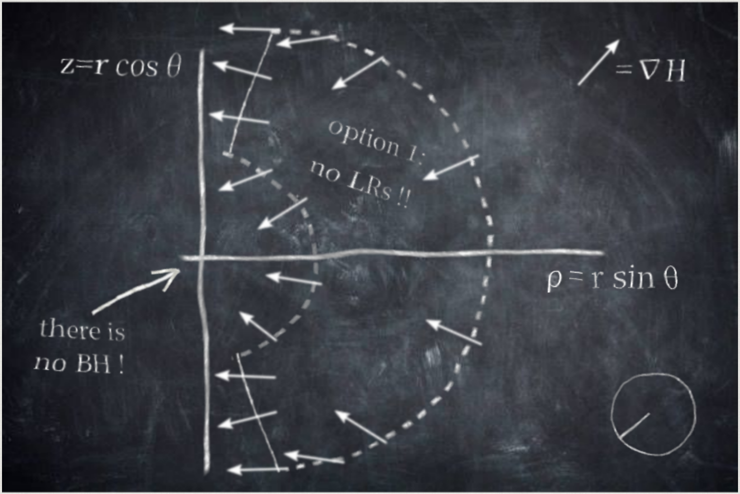
Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Origin boundary $\Longrightarrow$ *regular Ricci scalar* close to origin. PRL **124** (2020) 18, 181101

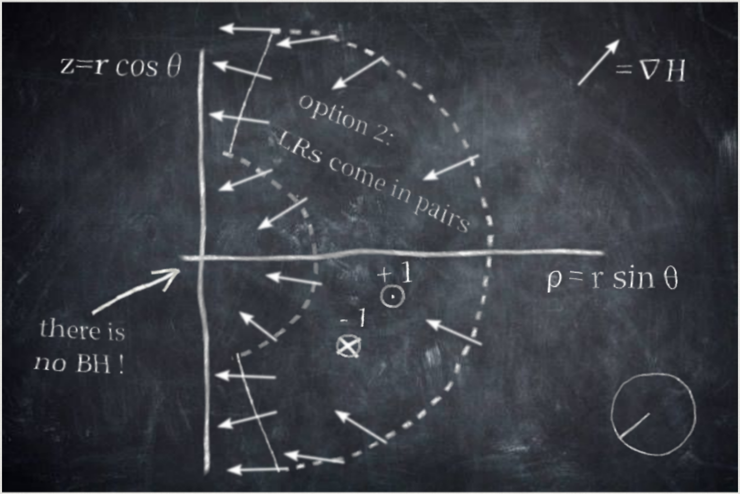
Axis boundary $\Longrightarrow$ *regular Ricci scalar* close to axis. PRL **124** (2020) 18, 181101

Asymptotic boundary $\Longrightarrow$ *flat* spacetime.



Conclusion: the total topological charge is **zero!**

Option 1: LRs **do not** exist inside the contour.



Conclusion: the total topological charge is **zero!**

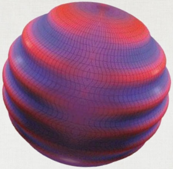
Option 1: LRs **do not** exist inside the contour.

Option 2: LRs exist but *appear in pairs* of charge $\{+1, -1\}$.

The *boundary conditions* affect the total topological charge:

The *boundary conditions* affect the total topological charge:

- Horizonless Star (asymptotically flat): $\sum_i w_i = 0$

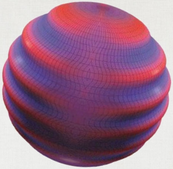


Phys.Rev.Lett. 124 (2020) 18, 181101

Phys.Rev.Lett. 119 (2017) 25, 251102

The *boundary conditions* affect the total topological charge:

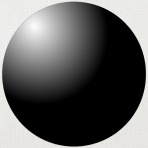
- Horizonless Star (asymptotically flat): $\sum_i w_i = 0$



Phys.Rev.Lett. 124 (2020) 18, 181101

Phys.Rev.Lett. 119 (2017) 25, 251102

- Black Hole (asymptotically flat): $\sum_i w_i = -1$



Phys.Rev.Lett. 124 (2020) 18, 181101

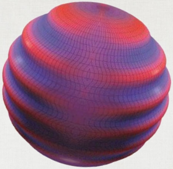
Phys.Rev.D 109 (2024) 6, 064050

The *boundary conditions* affect the total topological charge:

- Horizonless Star (asymptotically flat): $\sum_i w_i = 0$

Phys.Rev.Lett. 124 (2020) 18, 181101

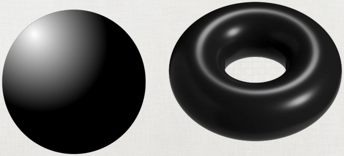
Phys.Rev.Lett. 119 (2017) 25, 251102



- Black Hole (asymptotically flat): $\sum_i w_i = -1$

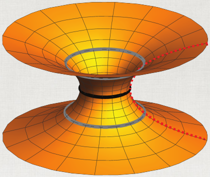
Phys.Rev.Lett. 124 (2020) 18, 181101

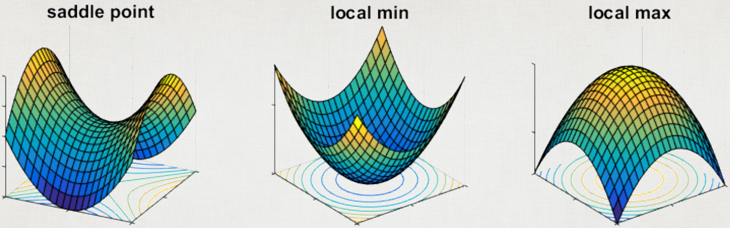
Phys.Rev.D 109 (2024) 6, 064050



- Wormhole (asymptotically flat): $\sum_i w_i = -1$

Phys.Rev.D 109 (2024) 12, 124065





$w = -1$ $\qquad$ $w = +1$ $\qquad$ $w = +1$

Different types of Light Rings:

- Saddle point of $U(r, \theta) \rightarrow (w = -1)$ “standard” LR (Kerr) .
- Local minimum of $U(r, \theta) \rightarrow (w = +1)$ *stable* LR (exotic)
- Local maximum of $U(r, \theta) \rightarrow (w = +1)$ **violates Null Energy Condition!**

Reasonable UCOs must have a stable LR $\implies$ spacetime instability?

Reasonable UCOs must have a stable LR $\Longrightarrow$ spacetime instability?

Paper by Keir: J. Keir, CQG 33 (2016) no.13, 135009 Benomio arXiv:1809.07795

- Scalar linear waves ϕ can be treated as a model for nonlinear perturbations:

$$\Box_g \phi + F(r)\,\phi = 0, \qquad \big(arbitrary\ F(r) > 0\big)$$

Reasonable UCOs must have a stable LR $\implies$ spacetime instability?

Paper by Keir: J. Keir, CQG 33 (2016) no.13, 135009 Benomio arXiv:1809.07795

- Scalar linear waves ϕ can be treated as a model for nonlinear perturbations:

$$\Box_g \phi + F(r)\,\phi = 0, \qquad \big(\textit{arbitrary } F(r) > 0\big)$$

They define integrated "energy" measure $\mathcal{E}[\phi](t)$ of wave ϕ across hypersurface Σ_t.

- In proving *non-linear stability*, one usually requires uniform fast decay:

$$\mathcal{E}[\phi](t) \lesssim \frac{1}{t^2}\,\mathcal{E}[\phi](0), \qquad (\textit{polynomial decay})$$

Reasonable UCOs must have a stable LR $\implies$ spacetime instability?

Paper by Keir: J. Keir, CQG 33 (2016) no.13, 135009 Benomio arXiv:1809.07795

- Scalar linear waves ϕ can be treated as a model for nonlinear perturbations:

$$\Box_g \phi + F(r)\,\phi = 0, \qquad (\textit{arbitrary } F(r) > 0)$$

They define integrated "energy" measure $\mathcal{E}[\phi](t)$ of wave ϕ across hypersurface Σ_t.

- In proving *non-linear stability*, one usually requires uniform fast decay:

$$\mathcal{E}[\phi](t) \lesssim \frac{1}{t^2}\,\mathcal{E}[\phi](0), \qquad (\textit{polynomial decay})$$

- If *stable* Light Ring exists, lower bound of uniform decay rate is slow:

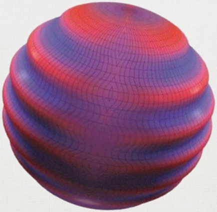
$$\mathcal{E}[\phi](t) \lesssim \frac{1}{[\log(2+t)]^2}\,\mathcal{E}[\phi](0), \qquad (\textit{logarithmic decay})$$

- This slow decay is highly suggestive that a *spacetime instability* exists.

Possible implications?

If one has an horizonless UCO that would mimic the BH observations:

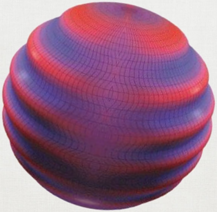
- If unstable Light Ring exists $\implies$ *stable* LR also exists (if NEC satisfied).
- The stable LR might trigger a *spacetime instability* $\implies$ not viable?



Possible implications?

If one has an horizonless UCO that would mimic the BH observations:

- If unstable Light Ring exists $\Longrightarrow$ *stable* LR also exists (if NEC satisfied).
- The stable LR might trigger a *spacetime instability* $\Longrightarrow$ not viable?



Questions:

- Has this hypothetical instability manifested itself in a concrete UCO model?
- What would be the timescale for such a putative instability?
- What would be the UCO endpoint state after the instability develops?

Plan of the talk

1. Why Light Rings (**LRs**) are relevant for observations.
2. LRs around horizonless compact objects
3. The fate of the Light Ring instability

The fate of the light-ring instability

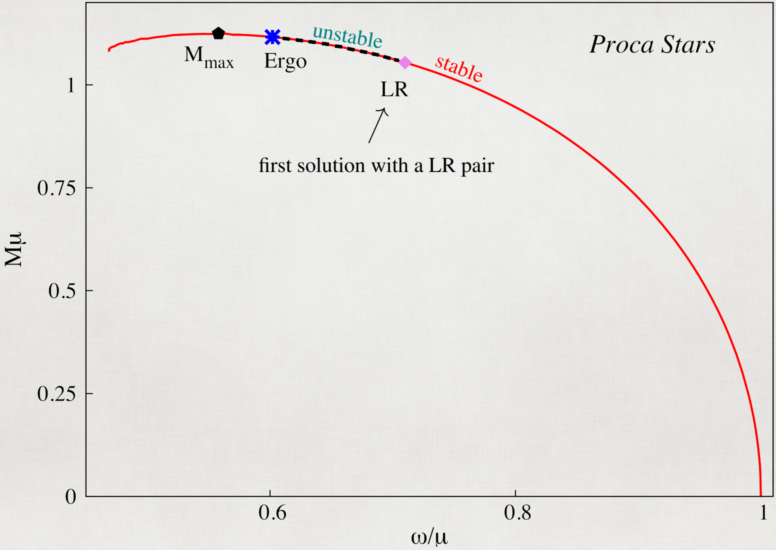
Pedro V. P. Cunha,[1] Carlos Herdeiro,[1] Eugen Radu,[1] and Nicolas Sanchis-Gual[2,1]

(Dated: July 2022)

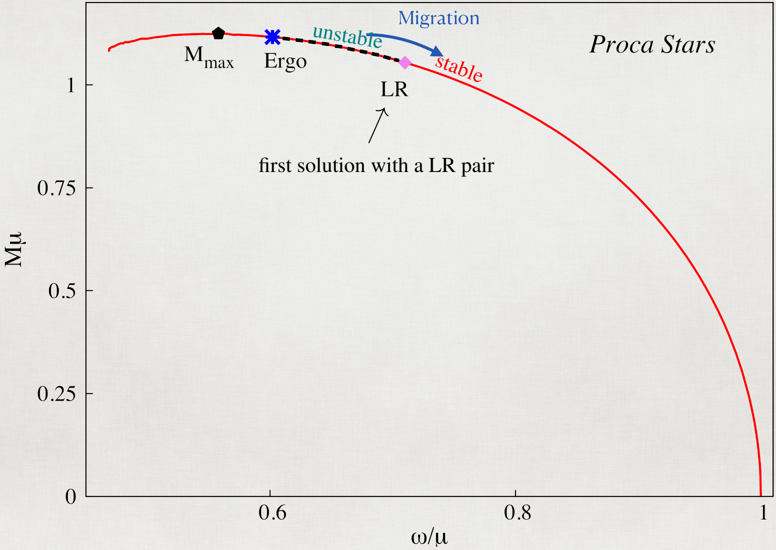
Ultracompact objects with light-rings (LRs) but without an event horizon could mimic black holes (BHs) in their strong gravity phenomenology. But are such objects dynamically viable? Stationary and axisymmetric ultracompact objects that can form from smooth, quasi-Minkowski initial data must have at least one *stable* LR, which has been argued to trigger a spacetime *instability*; but its development and fate have been unknown. Using fully non-linear numerical evolutions of ultracompact bosonic stars free of any other known instabilities and introducing a novel adiabatic effective potential technique, we confirm the LRs triggered instability, identifying two possible fates: migration to non-ultracompact configurations or collapse to BHs. In concrete examples we show that typical migration/collapse time scales are not larger than $\sim 10^3$ light-crossing times, unless the stable LR potential well is very shallow. Our results show that the LR instability is effective in destroying horizonless ultracompact objects that could be plausible BH imitators.

Phys. Rev. Lett. **130** (2023) 061401

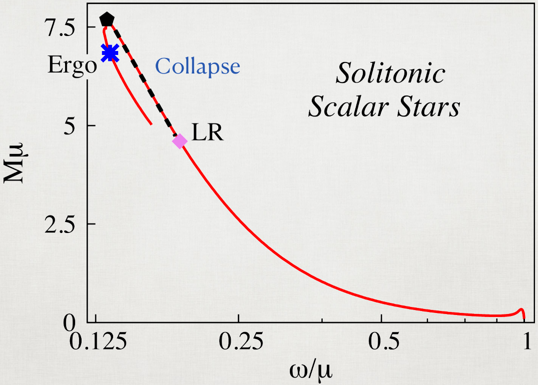
Solution diagram of Proca Stars



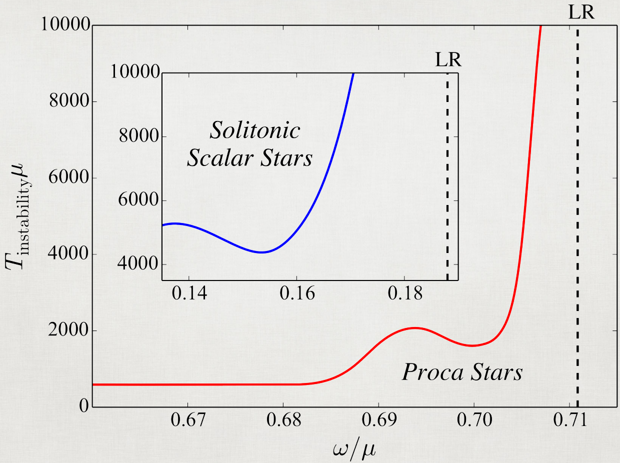
Solution diagram of Proca Stars



Solution diagram of Boson Stars with sextic self-interactions

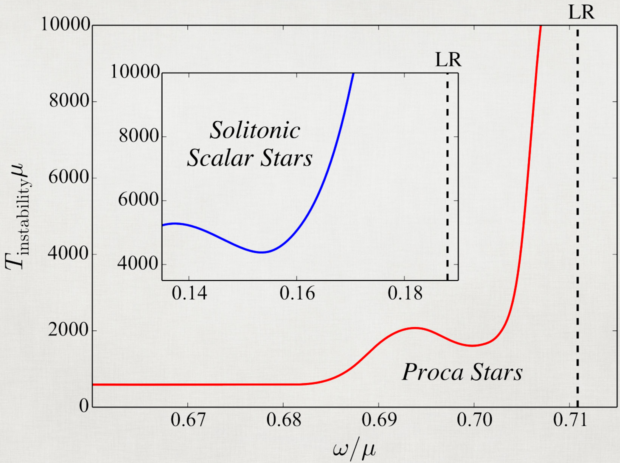


Time it takes for the instability to begin during the time evolution:



- The timescale **diverges** as we approach the solution where the LR disappears.

Time it takes for the instability to begin during the time evolution:



- The timescale **diverges** as we approach the solution where the LR disappears.
- For the mass of Sgr A*, an instability timescale $T\mu \sim 10^3$ is around $\sim$ 6 h.

To monitor the LRs in a dynamical spacetime we will introduce an *adiabatic effective potential* $V_{\pm}(r,t)$.

To monitor the LRs in a dynamical spacetime we will introduce an *adiabatic effective potential* $V_{\pm}(r,t)$.

This is well motivated if the dynamical evolution is:

To monitor the LRs in a dynamical spacetime we will introduce an *adiabatic effective potential* $V_{\pm}(r,t)$.

This is well motivated if the dynamical evolution is:

- **slow** enough in time.

To monitor the LRs in a dynamical spacetime we will introduce an *adiabatic effective potential* $V_{\pm}(r,t)$.

This is well motivated if the dynamical evolution is:

- **slow** enough in time.
- **violations** of axial-symmetry are mild.

The *adiabatic effective potential* $V_{\pm}(r,t)$ takes the explicit form:

The *adiabatic effective potential* $V_{\pm}(r,t)$ takes the explicit form:

$$V_{\pm}(r,t) = M_t \, \frac{\langle g_{t\varphi}\rangle \mp \sqrt{\langle g_{t\varphi}\rangle^2 - \langle g_{tt}\rangle \langle g_{\varphi\varphi}\rangle}}{\langle g_{\varphi\varphi}\rangle} \, .$$

The *adiabatic effective potential* $V_{\pm}(r,t)$ takes the explicit form:

$$V_{\pm}(r,t) = M_t \, \frac{\langle g_{t\varphi}\rangle \mp \sqrt{\langle g_{t\varphi}\rangle^2 - \langle g_{tt}\rangle \langle g_{\varphi\varphi}\rangle}}{\langle g_{\varphi\varphi}\rangle} \, .$$

Violations of axial-symmetry *diluted* by averaging over ϕ:

$$\text{lapse function:} \qquad N^2 \quad \longrightarrow \quad \langle N^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} N^2 \, d\varphi$$

The *adiabatic effective potential* $V_\pm(r,t)$ takes the explicit form:

$$V_\pm(r,t) = M_t \, \frac{\langle g_{t\varphi}\rangle \mp \sqrt{\langle g_{t\varphi}\rangle^2 - \langle g_{tt}\rangle \langle g_{\varphi\varphi}\rangle}}{\langle g_{\varphi\varphi}\rangle} \, .$$

Violations of axial-symmetry *diluted* by averaging over ϕ:

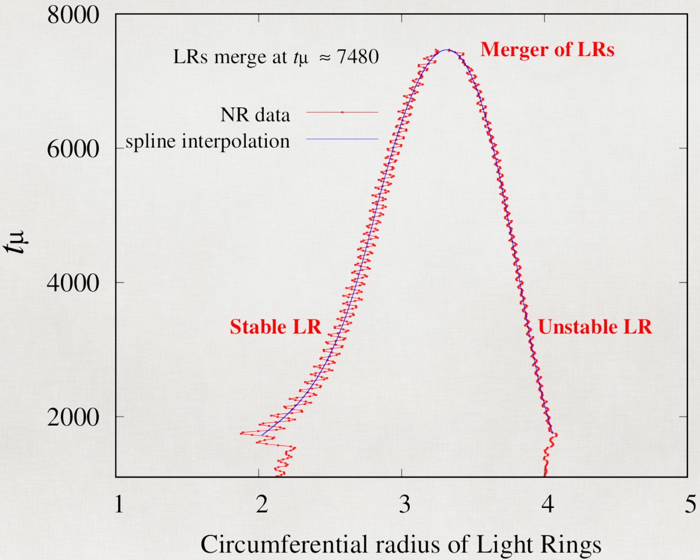
$$\text{lapse function:} \qquad N^2 \quad \longrightarrow \quad \langle N^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} N^2 \, d\varphi$$

We can also time-average to wash away star-oscillations around a *trend*:

$$V^*_\pm(r,t) = \frac{1}{2\,T(t)} \int_{t-2\,T(t)}^{t} V_\pm(r,\tau)\, d\tau,$$

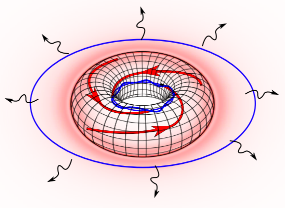
$\longrightarrow$ $T(t)$ is a measure of the oscillation period of the lapse $N(t)$.

Following the location of LRs during the evolution of PSs:



The road ahead and conclusions

- Testing the Kerr-paradigm is very challenging....
- One step is to **rule out** classes of inadequate models as BH alternatives.
- The numerical evidence presented *supports the inadequacy of a large class of UCOs*.



There might exist possible mechanisms to avoid this conclusion:

a) topological non-triviality, *e.g.* wormholes.

b) non-axisymmetry of the horizonless UCO.

c) non-circularity.

Acknowledgements

P.V.P.Cunha is supported by the postdoctoral fellowship FCT Individual CEEC 2020.

This work is supported by CIDMA through FCT, with references UIDB/04106/2020 and UIDP/04106/2020, and from the projects PTDC/FIS-OUT/28407/2017, CERN/FIS-PAR/0027/2019 and PTDC/FIS-AST/3041/2020.

This work has further been supported by the EU's Horizon 2020 (RISE) programme H2020-MSCA-RISE-2017 Grant No. FunFiCO-777740.

We would like to acknowledge networking support by the COST Action CA16104.

universidade de aveiro

CIDMA CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES CENTER FOR R&D IN MATHEMATICS AND APPLICATIONS

FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR