Compact binaries in astrophysical environments

Caio F. B. Macedo

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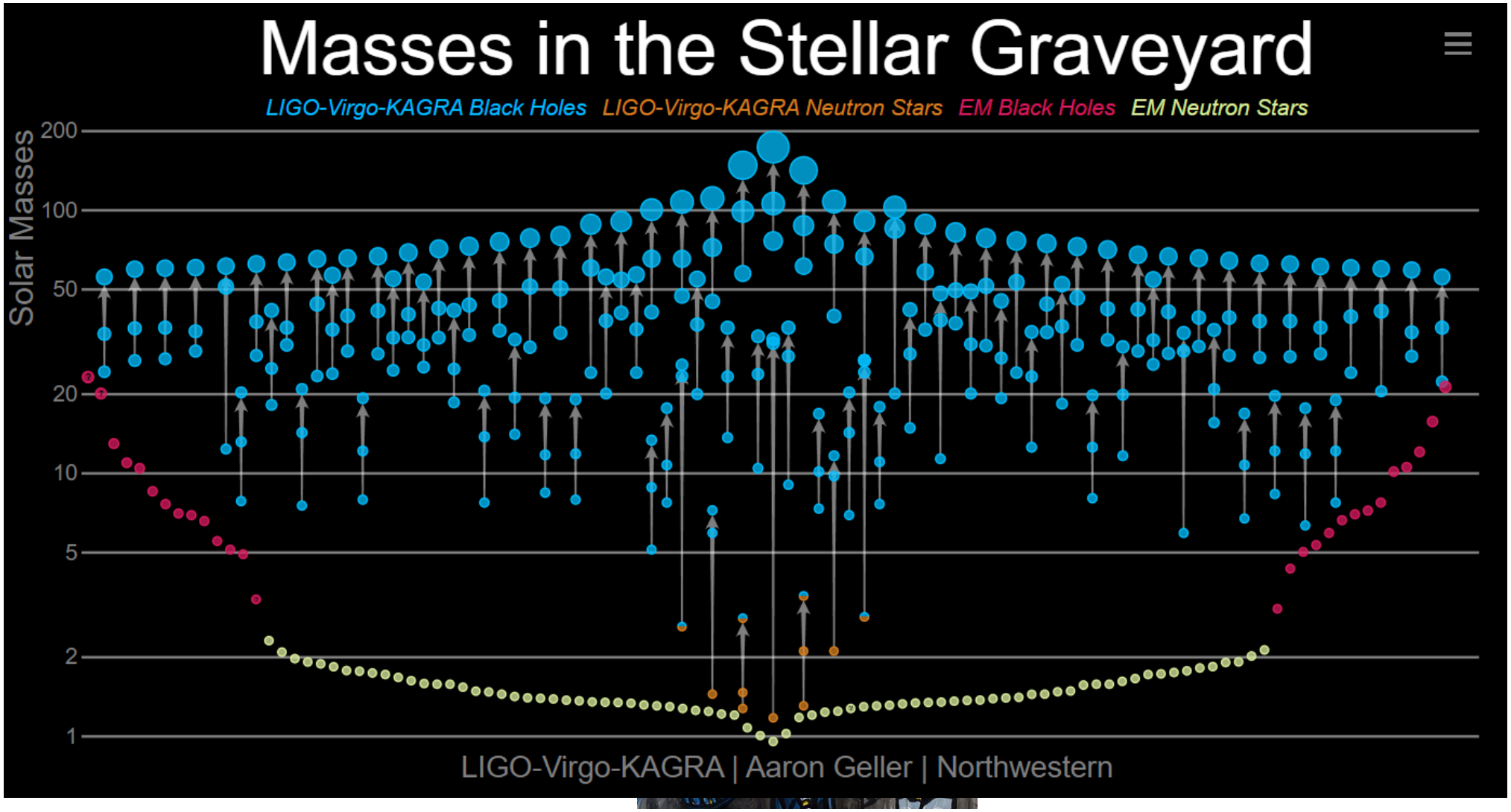
NEW HORIZONS FOR PSI

Outline

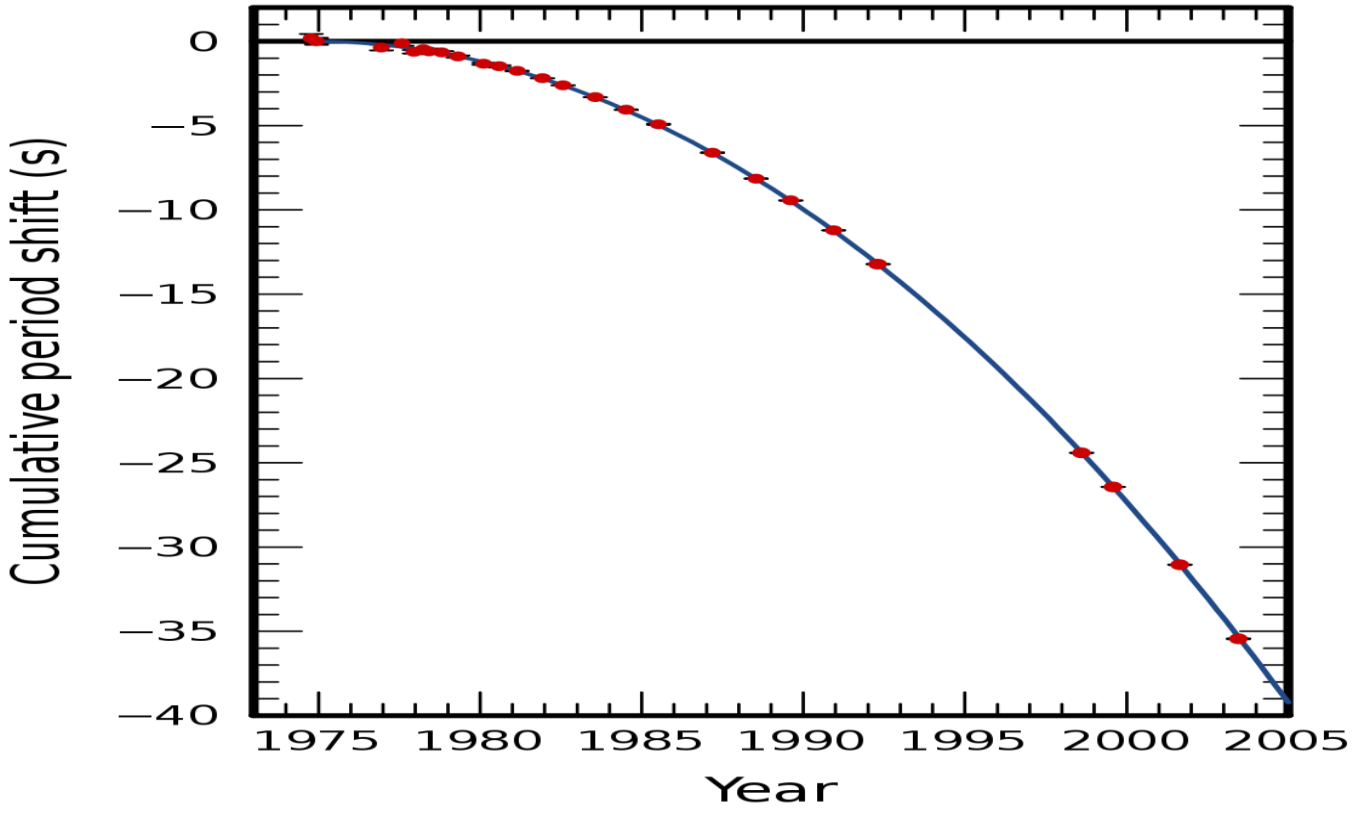
- General motivations
- Physical setups
- Binaries in environments
- Final remarks

General motivations

Main targets for GW detectors



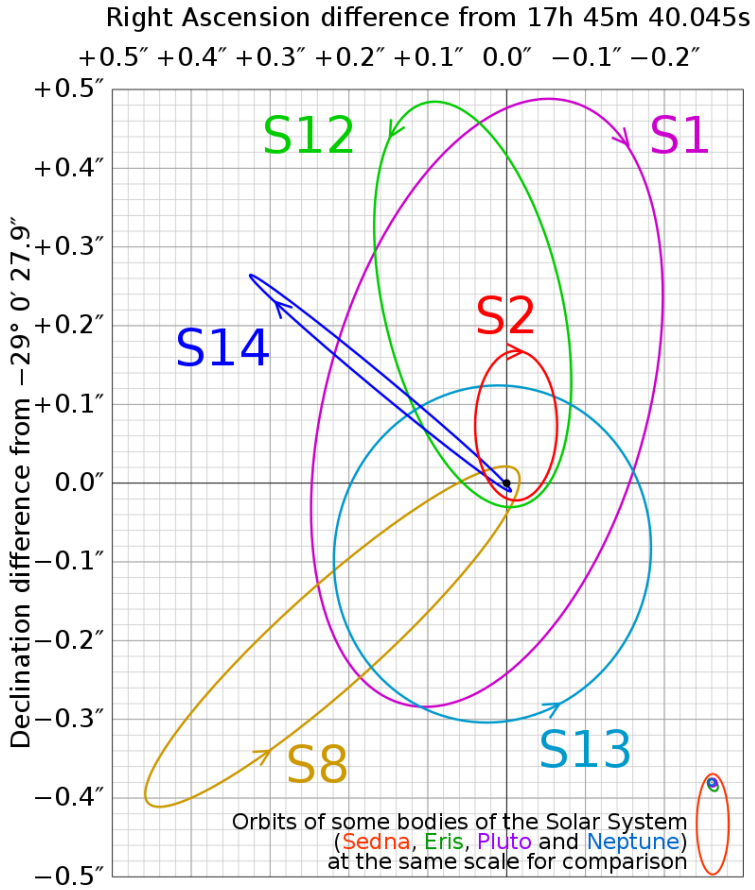
Time passes, and shift tells



$$\frac{\dot{T}}{T} = \frac{3}{2}\frac{Energy\ Lost}{Orbital\ energy}$$

Accretion disks, plasmas and dark matter

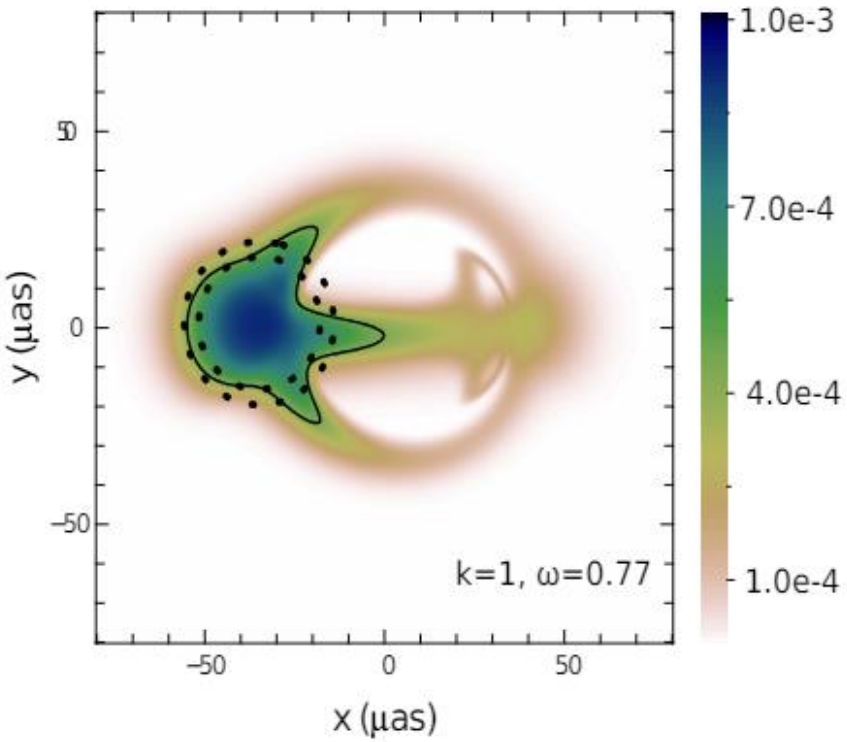
Barausse *et. al* PRD **89**, 104059 (2014), Wayne Hu, Rennan Barkana, and Andrei Gruzinov Phys. Rev. Lett. 85, 1158 (2000)



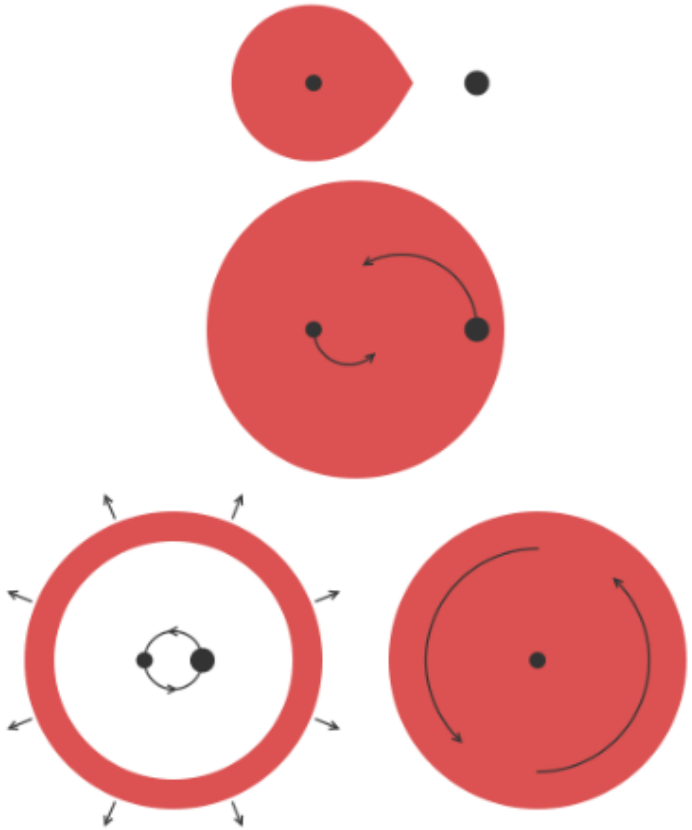
Environmental forces play a role

GRAVITY Collaboration

Boson stars and common envelopes



Vicent et al. Class.Quant.Grav. 33 (2016) 10, 105015



Ivanova et al., AARev., 21, 59 (2013)

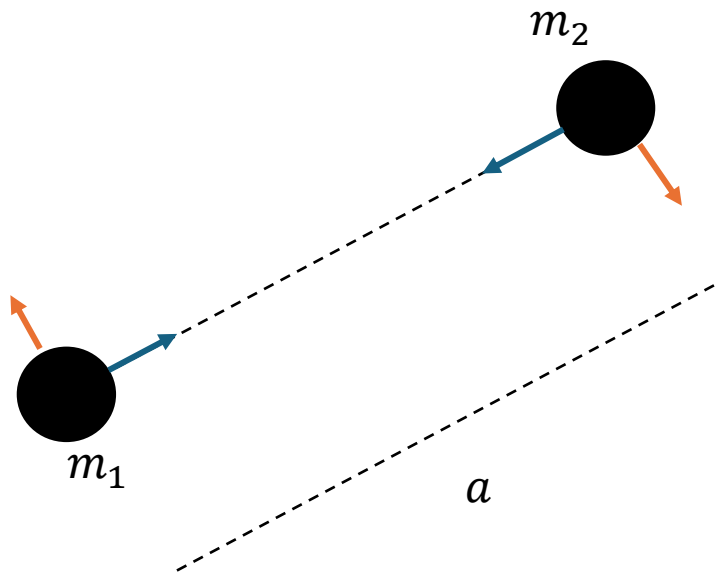
Physical Setups

Physical setups

Newtonian elements

$$\ddot{\mathbf{r}} = f_1\dot{\mathbf{r}} + f_2\dot{\mathbf{R}} + f_3\mathbf{r}$$
$$\ddot{\mathbf{R}} = f_4\dot{\mathbf{r}} + f_5\dot{\mathbf{R}} + f_6\mathbf{r}$$

$$\mathbf{F}_{\mathrm{d},i} = -G^2 m_i^2 \rho I_{\mathrm{d}}(v_i)\dot{\mathbf{r}}_i$$

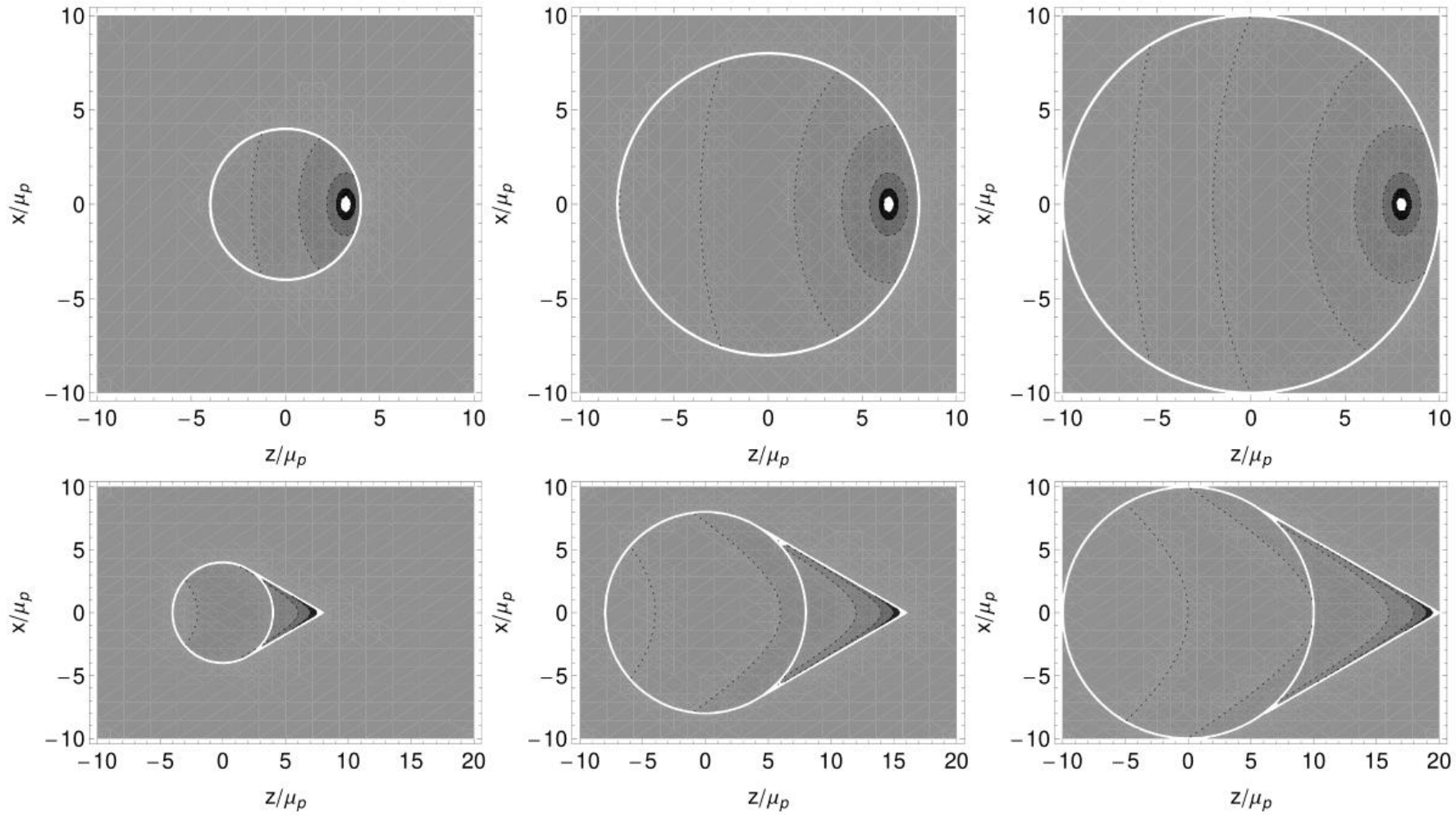


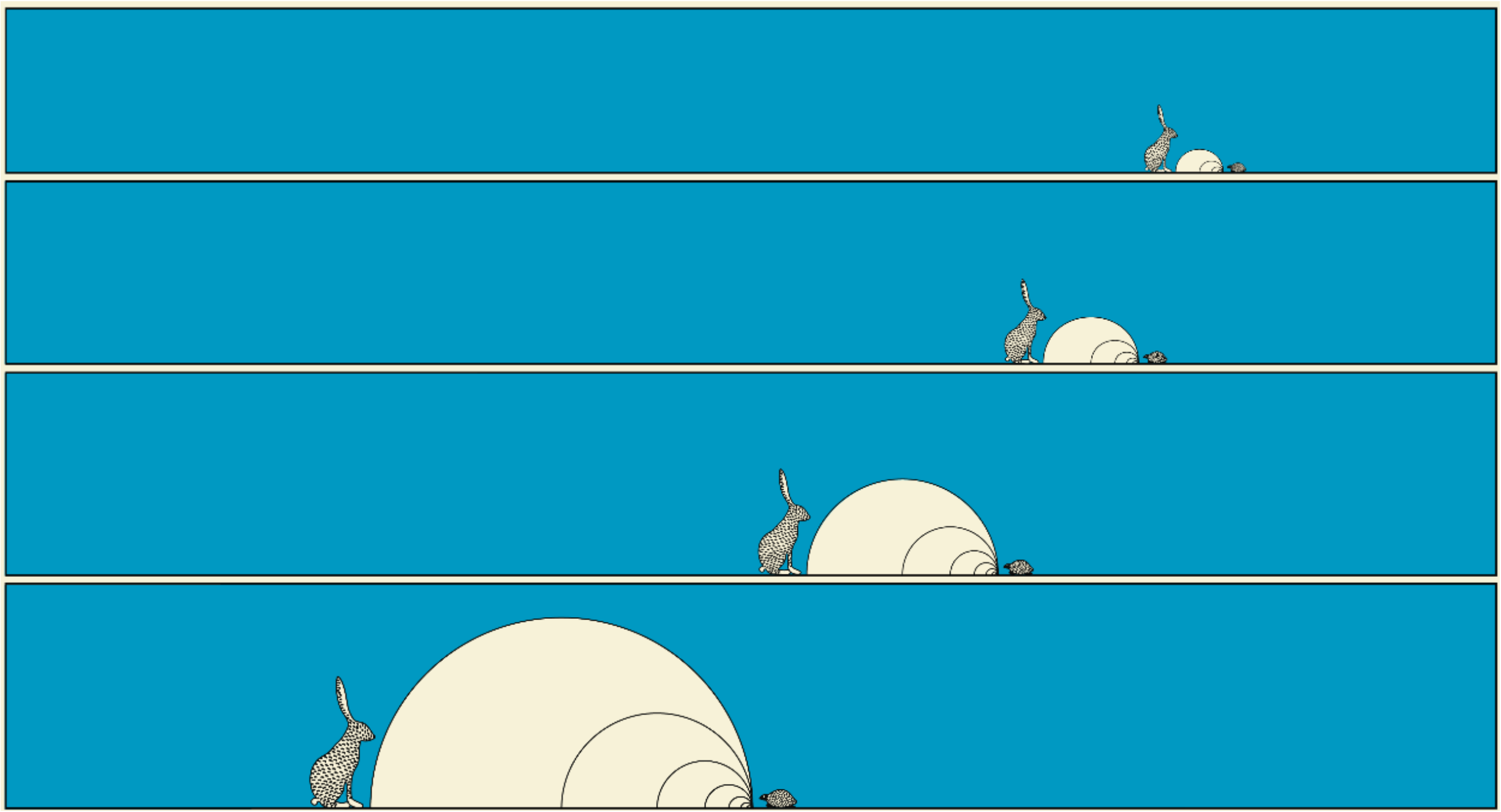
$$f_1 = -\frac{G^2 M q \rho (I_{\mathrm{a}1} + I_{\mathrm{a}2} + I_{\mathrm{d}1} + I_{\mathrm{d}2})}{(q+1)^2},$$
$$f_2 = \frac{G^2 M \rho [I_{\mathrm{a}1} + I_{\mathrm{d}1} - q(I_{\mathrm{a}2} + I_{\mathrm{d}2})]}{q+1},$$
$$f_3 = GM\left\{\frac{G^3 M q \rho^2 (I_{\mathrm{a}1} - qI_{\mathrm{a}2})[I_{\mathrm{a}1} + I_{\mathrm{d}1} - q(I_{\mathrm{a}2} + I_{\mathrm{d}2})]}{(q+1)^4} - \frac{1}{r^3}\right\},$$
$$f_4 = \frac{G^2 M q \rho [q(I_{\mathrm{a}2} - I_{\mathrm{d}2}) - I_{\mathrm{a}1} + I_{\mathrm{d}1}]}{(q+1)^3},$$
$$f_5 = -\frac{G^2 M \rho \left[q^2(I_{\mathrm{a}2} + I_{\mathrm{d}2}) + I_{\mathrm{a}1} + I_{\mathrm{d}1}\right]}{(q+1)^2},$$
$$f_6 = -\frac{G^4 M^2 q \rho^2 (I_{\mathrm{a}1} - qI_{\mathrm{a}2})\left[q^2(I_{\mathrm{a}2} + I_{\mathrm{d}2}) + 2q(I_{\mathrm{a}1} + I_{\mathrm{a}2}) + I_{\mathrm{a}1} + I_{\mathrm{d}1}\right]}{(q+1)^5}.$$

Physical setups

Newtonian elements: Dynamical friction

(Chandrasekhar, APJ (1943)), (Ostriker, astro-ph/9810324 (1998))



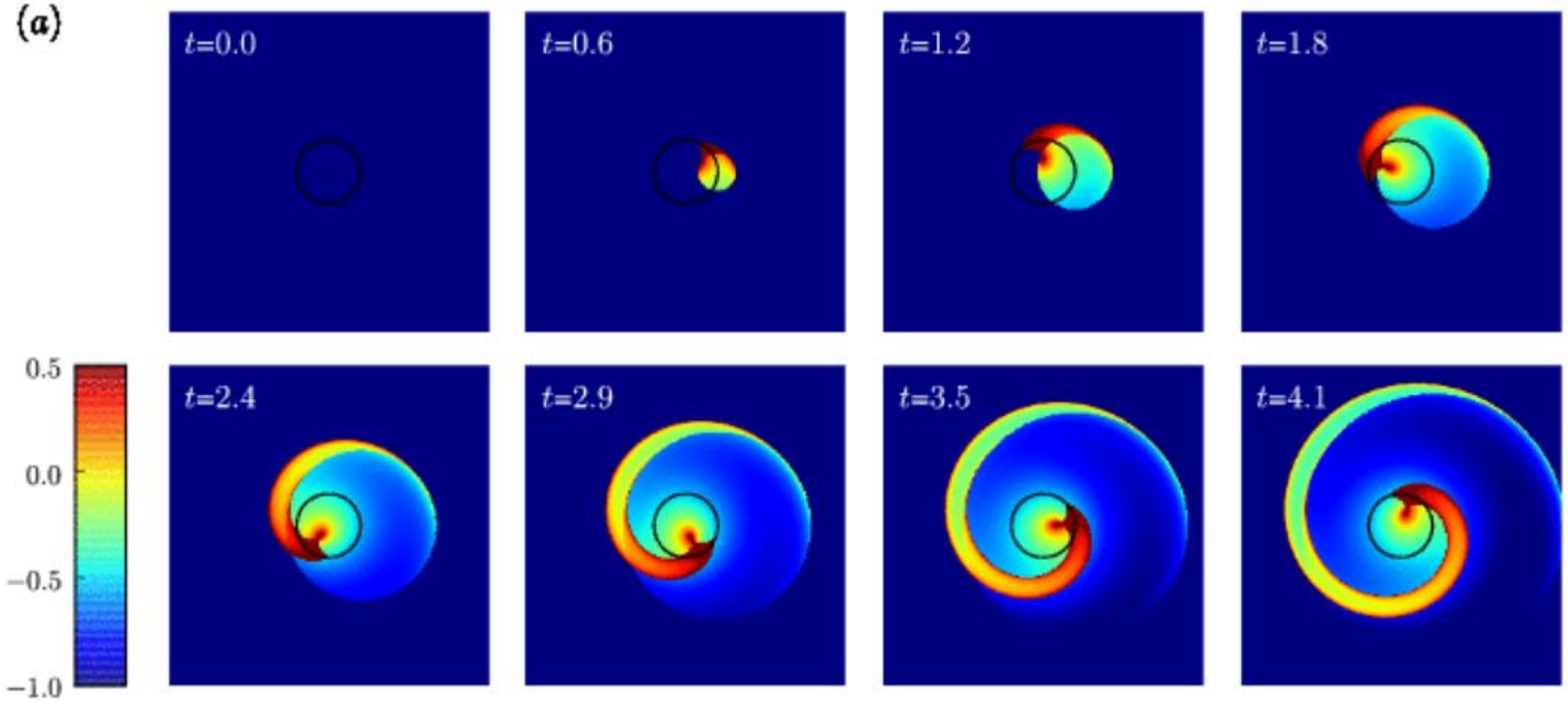


Physical setups

Newtonian elements: Dynamical friction in other scenarios

Kim&Kim, 0705.0084 (2007); Kim et al. 0804.2010 (2018); Vicente et. al 1905.06353 (2019)

"These are my principles. If you don't like them, I have others".



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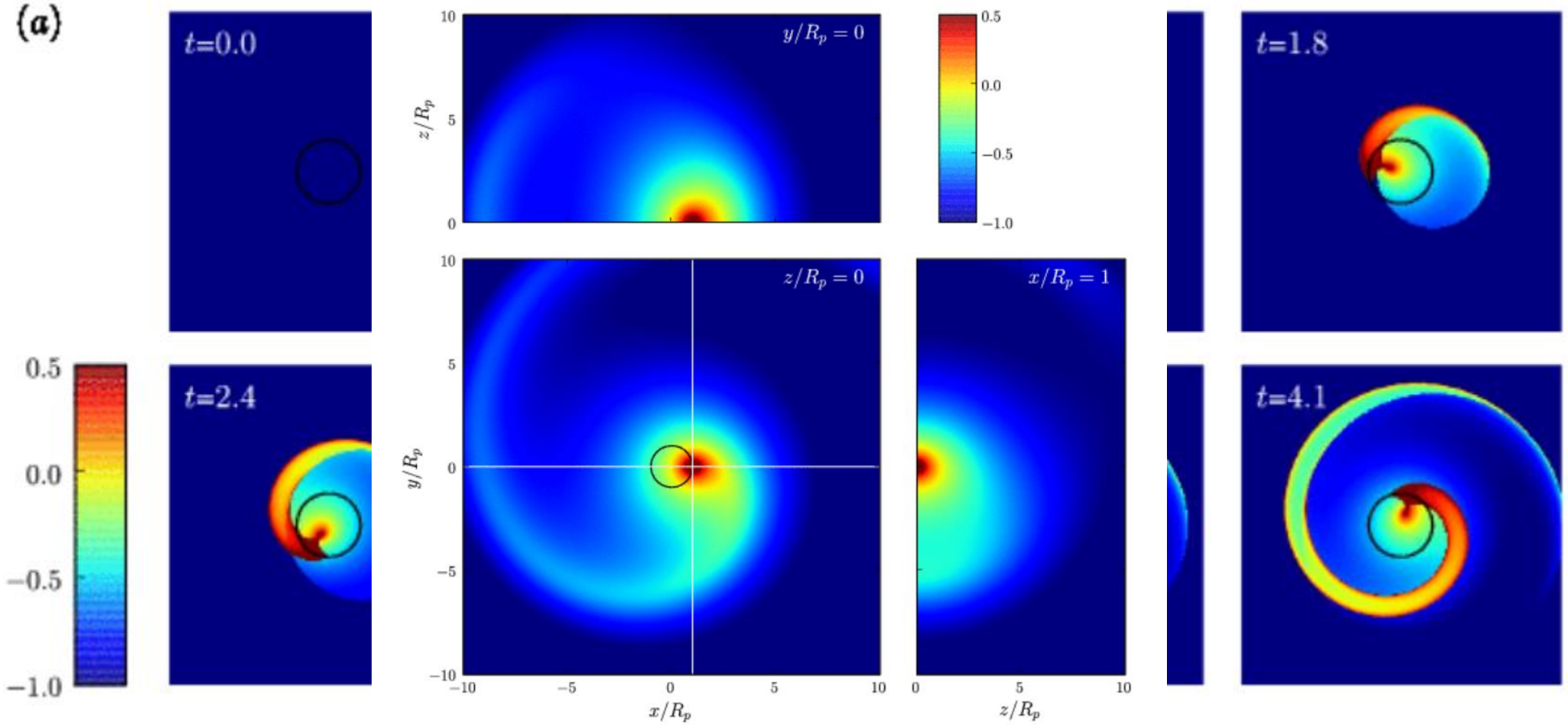

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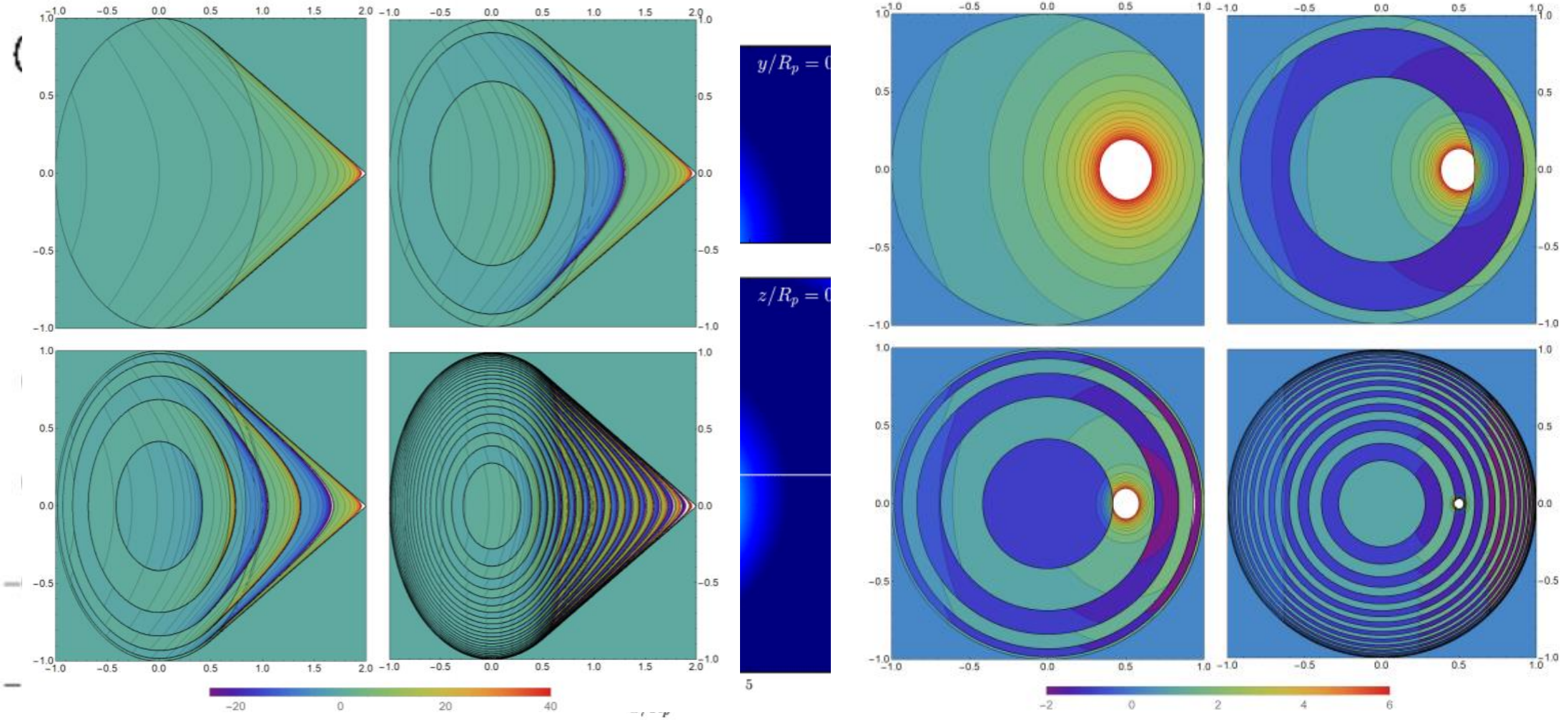


Physical setups

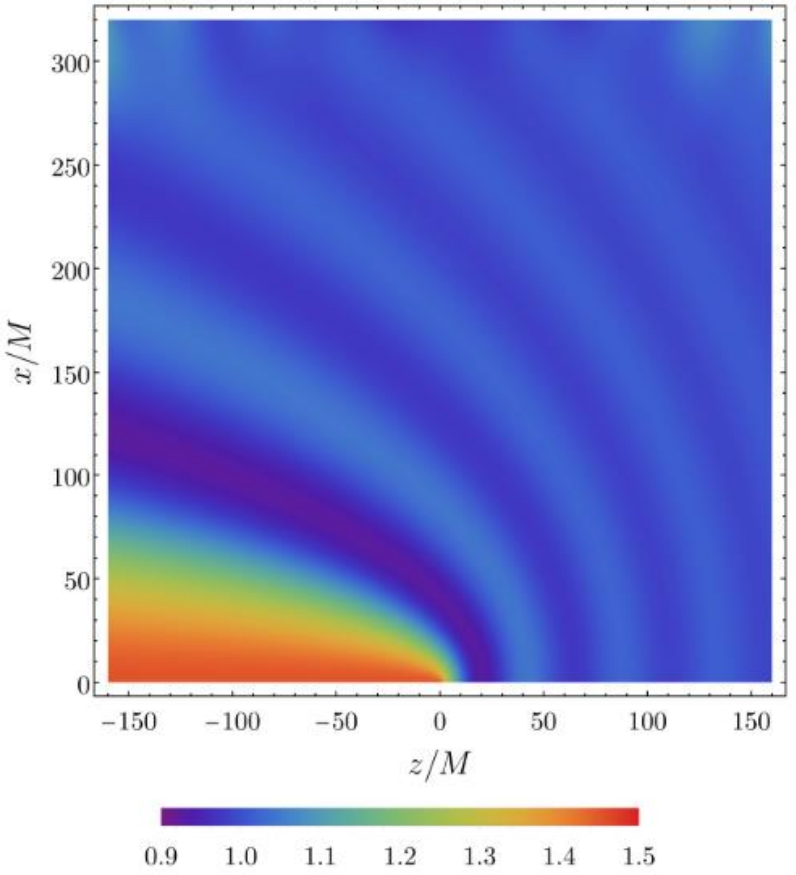
GR elements: Dynamical friction and scattering amplitudes

Traykova *et al. Phys. Rev. D 104, 103014 (2021), Vicente and Cardoso. Phys.Rev.D* 105 8, 083008 (2022), *Traykova et al.,Phys.Rev.D 108 12, L121502 (2023)*

$$\dot{E}_{\rm BH} = \frac{\pi\hbar\omega n}{\mu k_\infty}\sum_{\ell,m}(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}(\mathrm{Ps}_\ell^m)^2\left(1-\left|\frac{R}{I}\right|^2\right).$$

$$P_S^i(t') = \int_{S_{t'}} dV_3 T^{\alpha i} N_\alpha.$$

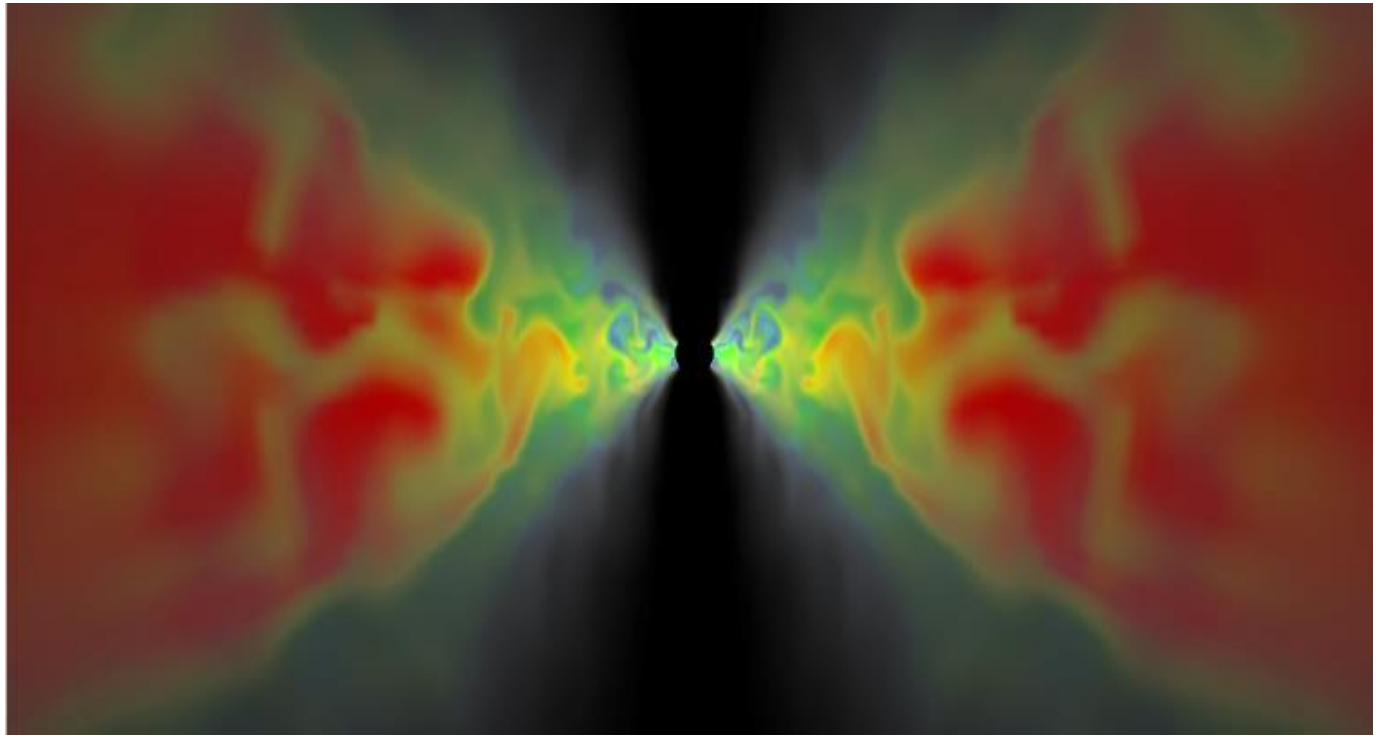
$$\boldsymbol{F}' = -\frac{4\pi M^2\rho\boldsymbol{v}}{v^3}\log\left(\sqrt{1+\frac{b_{\rm max}^2}{(M/v^2)^2}}\right)$$

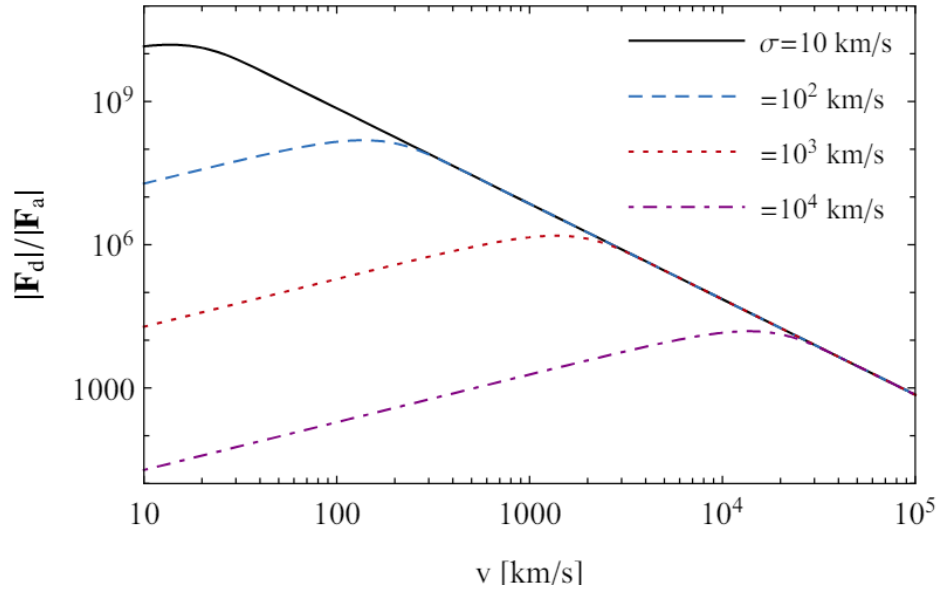


Note that the Chandrasekhar case is recovered

Physical setups

Newtonian(-ish) elements: Accretion





$$m_i\ddot{\mathbf{r}}_i + \dot{m}_i\dot{\mathbf{r}}_i = \pm\frac{Gm_1m_2}{r^3}\mathbf{r} + \mathbf{F}_i$$

E.g. Bondi-Hoyle-Littleton

$$\dot{m}_i = 4\pi G^2\rho\frac{m_i^2}{(v_i^2 + c_s^2)^{3/2}}$$

Physical setups

(Post-)Newtonian elements: quadru and octupole emissions

For ***gravitational waves***, we have

$$\langle \dot{E} \rangle = -\frac{32}{5}\frac{G^4 m_1^2 m_2^2 M}{a^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$\langle \dot{L} \rangle = -\frac{32}{5}\frac{G^{7/2} m_1^2 m_2^2 M^{1/2}}{a^{7/2}(1-e^2)^2}\left(1+\frac{7}{8}e^2\right).$$

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3 \mu\, M^2}{c^5 a^3 (1-e^2)^{\frac{7}{2}}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right),$$

$$\frac{de}{dt} = -\frac{304}{15}\frac{G^3 \mu\, M^2}{c^5 a^4 (1-e^2)^{\frac{5}{2}}}\left(1+\frac{121}{304}e^2\right).$$

Emission of gravitational waves ***circularizes*** orbits.

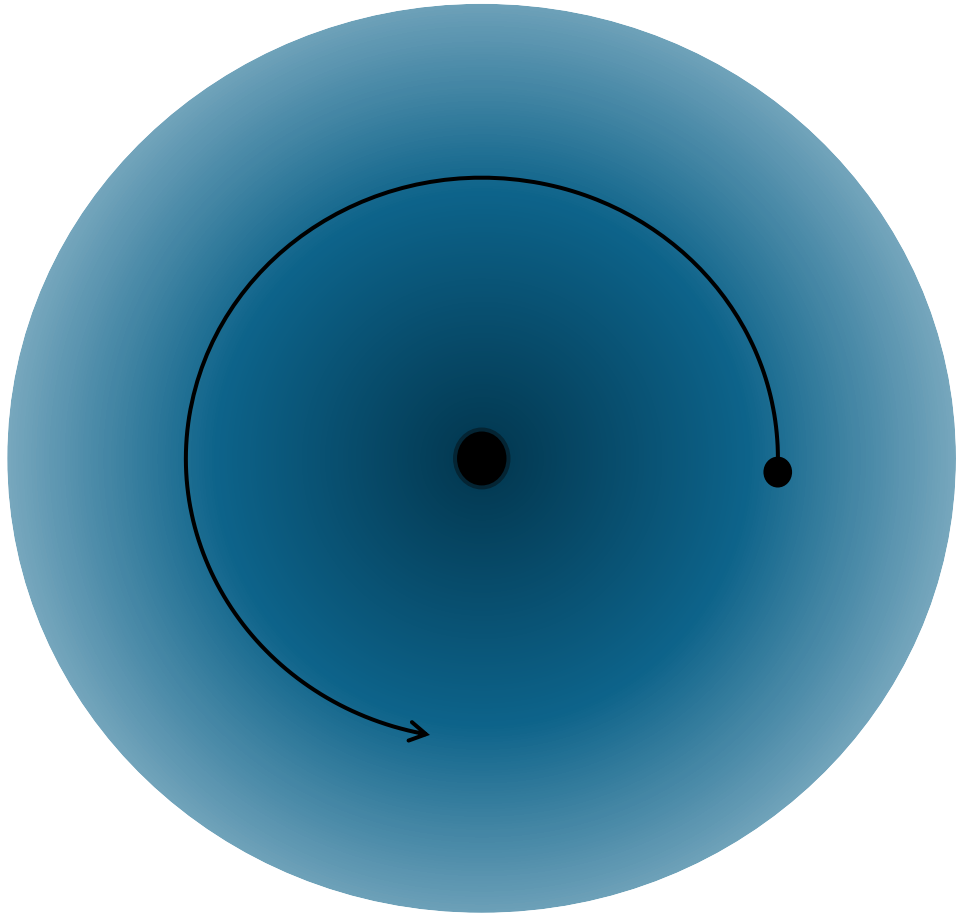
Physical setups

Radiation: scalar fields, black holes and particles

Macedo et al. 2013, Duque et al. 2023

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \qquad \Box_g \Phi = \frac{\partial V}{\partial \Phi^*}$$

$$g_{\mu\nu} \approx \widehat{g}_{\mu\nu} + q\,\delta g_{\mu\nu}, \qquad \Phi \approx \widehat{\Phi} + q\,\delta\Phi$$

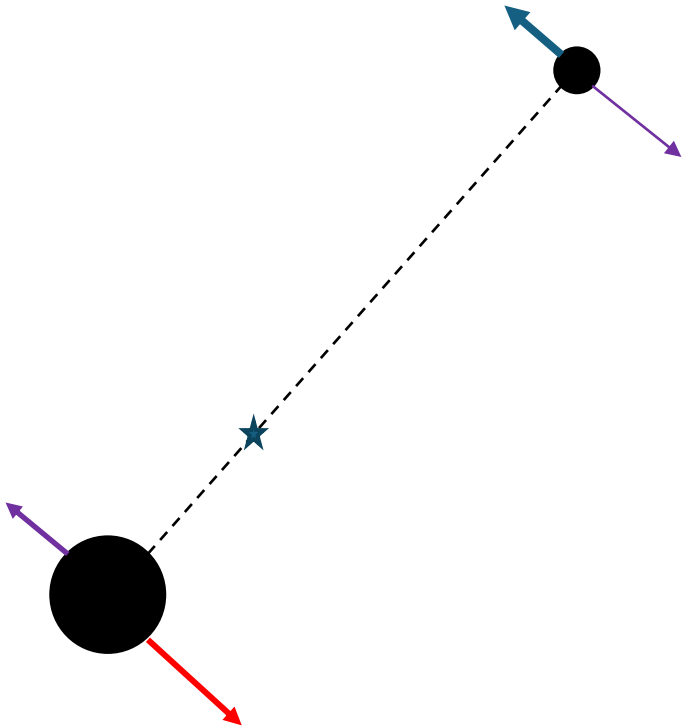


Binaries in environments

Let us first investigate the *asymmetric case*.

Accelerating the CM

Cardoso&CFBM, 2008.01091 (2020)



$$\ddot{\mathbf{R}} = -\frac{G^2 M\rho}{(q+1)^2}(I_1 + q^2 I_2)\dot{\mathbf{R}} + \frac{G^2 M\rho q}{(q+1)^3}(I_1 - qI_2)\dot{\mathbf{r}},$$

$$\ddot{\mathbf{r}} = -\frac{G^2 M\rho q}{(q+1)^2}(I_1 + I_2)\dot{\mathbf{r}} + \frac{G^2 M\rho}{q+1}(I_1 - qI_2)\dot{\mathbf{R}} - \frac{GM}{r^3}\mathbf{r}\,.$$

Given the initial conditions, we have the key expressions:

CM "acceleration": $\alpha \equiv \dfrac{G^2 M^2 \rho}{(q+1)\sigma^2}\,,$

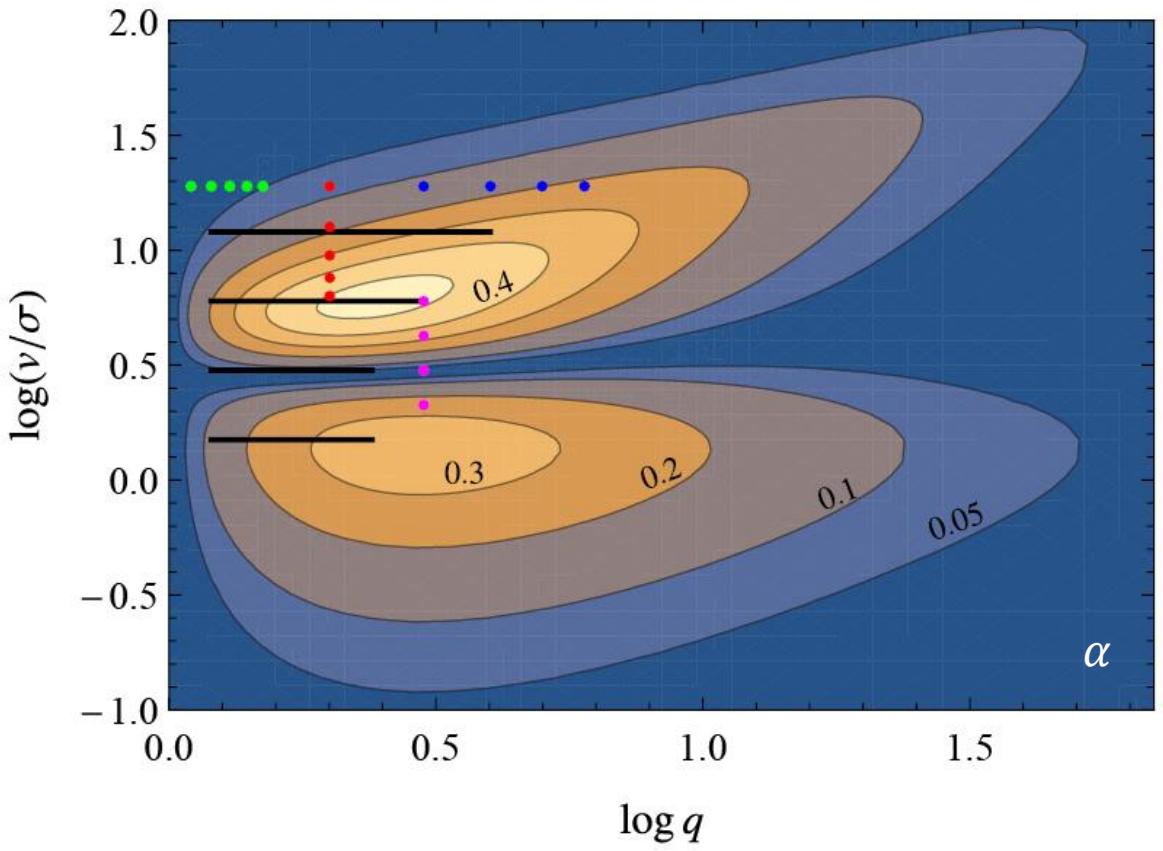
CM/drag: $\eta_1 = \dfrac{1}{q+1}\dfrac{(I_1 - qI_2)}{(I_1 + I_2)}\,.$

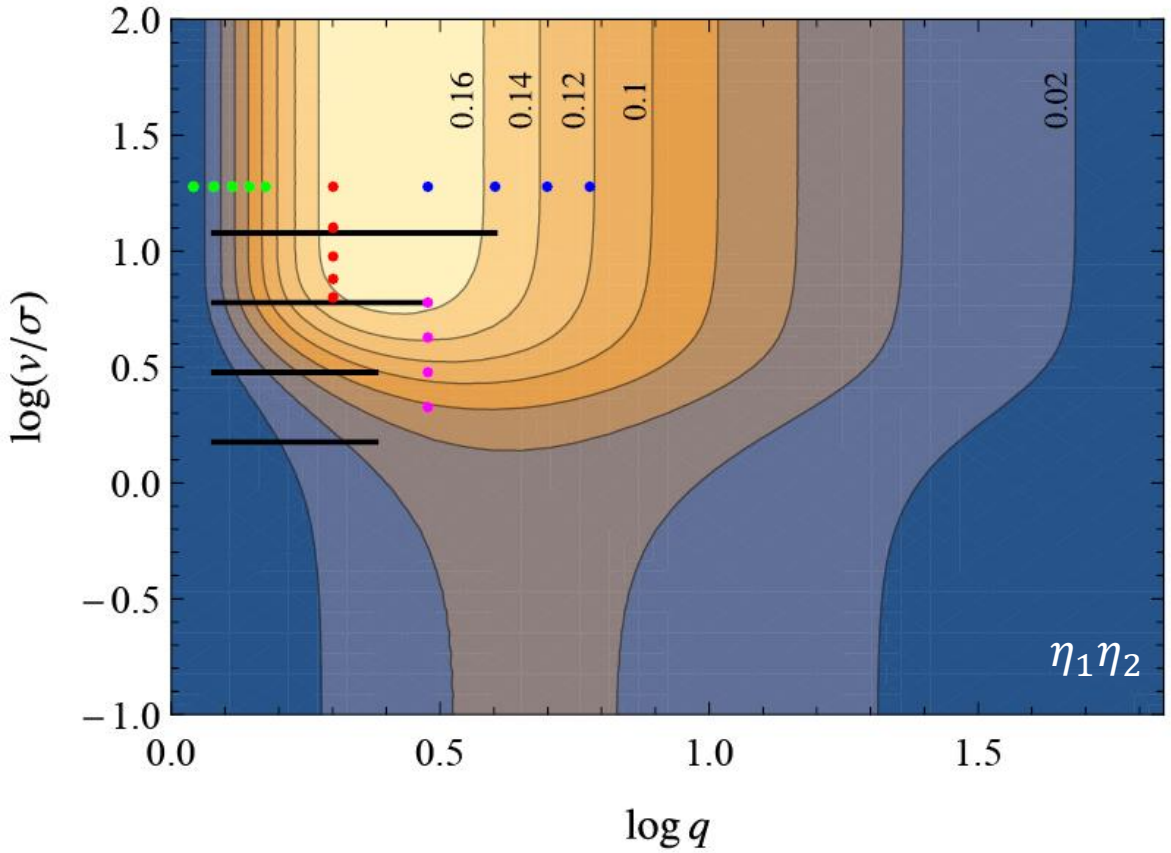
CM boost/drag: $\eta_2 = \dfrac{q}{q+1}\dfrac{(I_1 - qI_2)}{(I_1 + q^2 I_2)}\,.$

Let us first look into the *asymmetric case*.

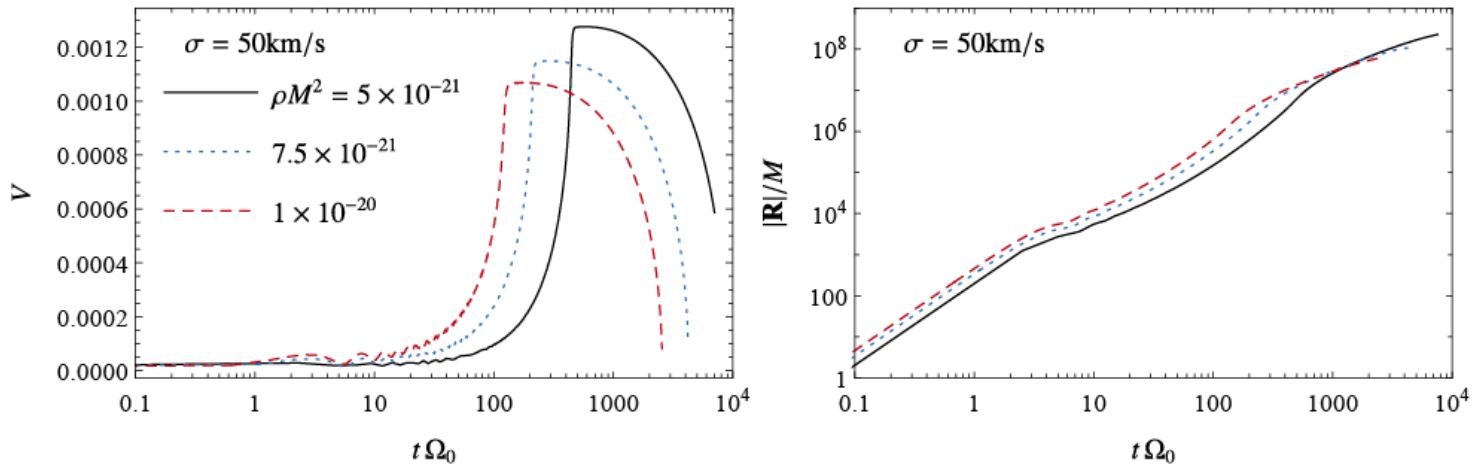
Accelerating the CM

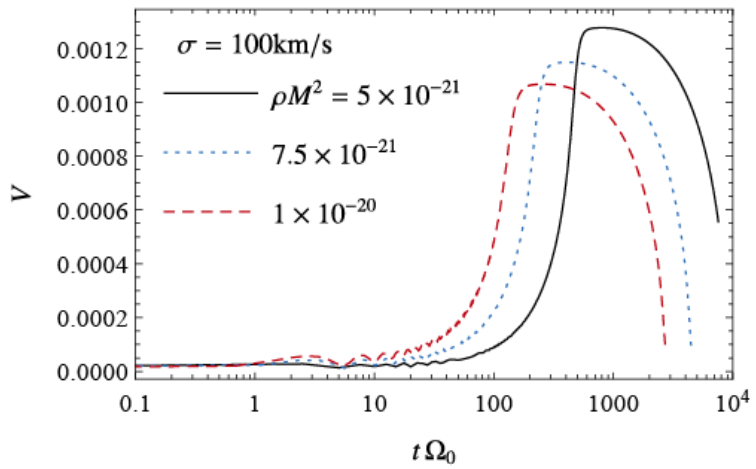
Cardoso&CFBM, 2008.01091 (2020)

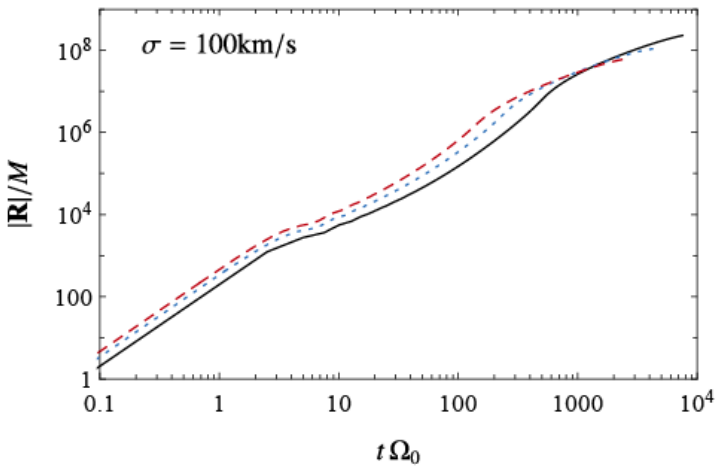




Boost in the center of mass







$V \sim 350\text{km/s}$

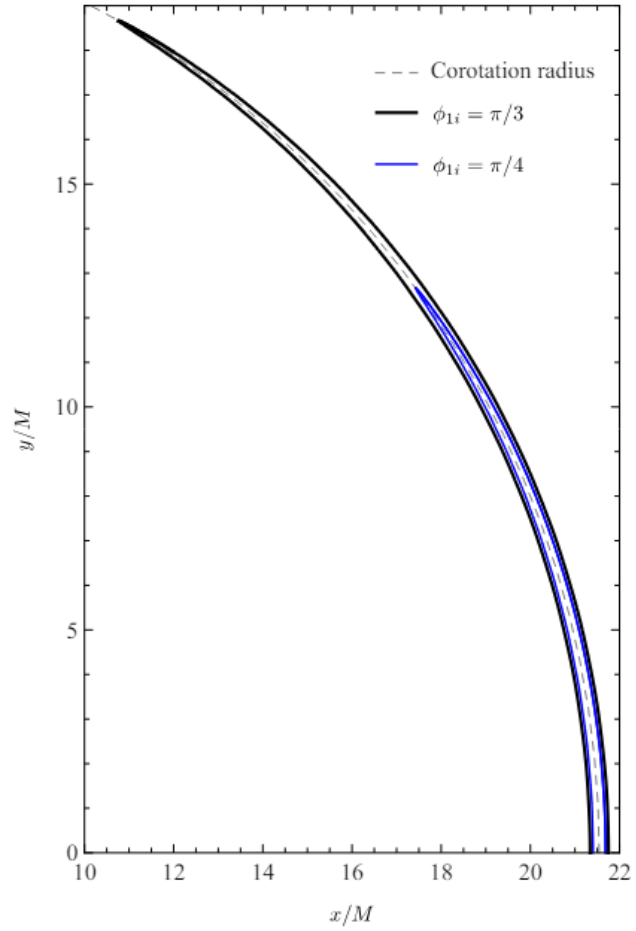
Binaries could escape galaxies

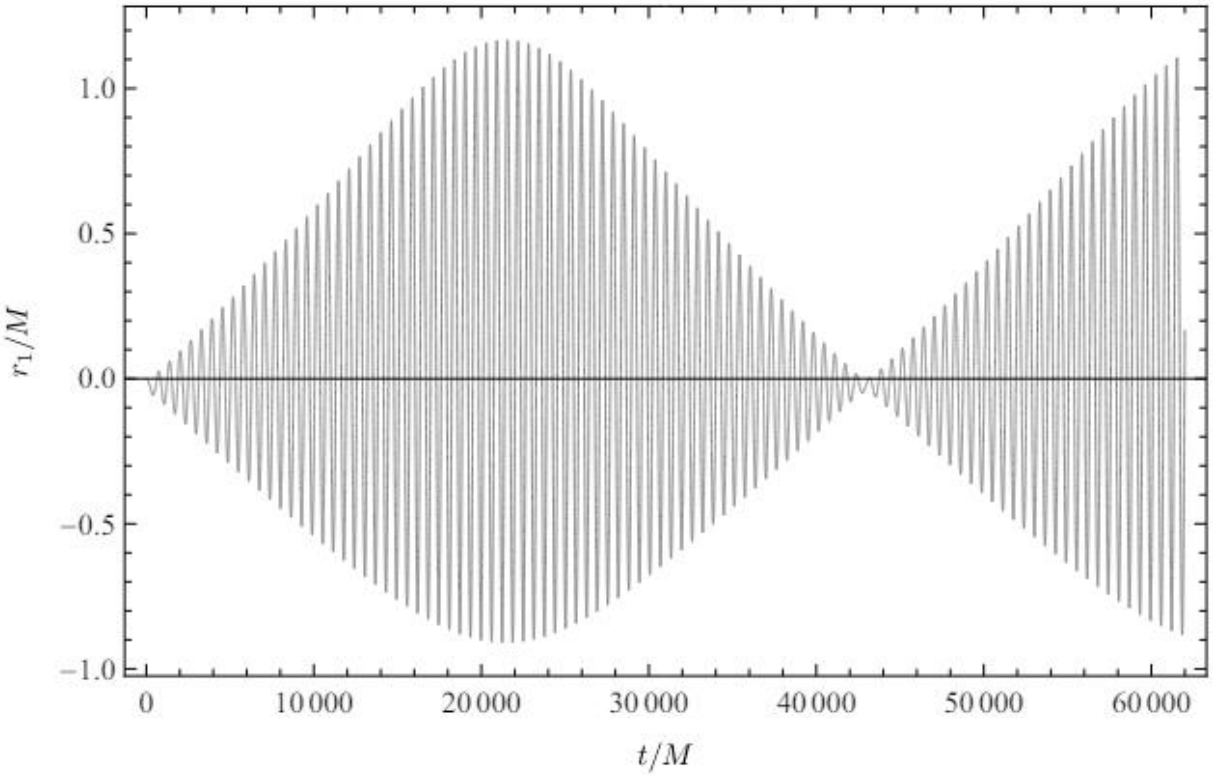
$$T_{\max} = 6\left(\frac{M_\odot}{M}\right)^3\left(\frac{10^{-6}\rho_{\text{water}}}{\rho}\right)^2 \text{years}$$

Orbital dynamics

Ferreira et al. 2017

A time dependent background dynamics introduces richer orbital dynamics





Eccentricity evolution and dynamical friction

Cardoso, CFBM, Vicente 2010.1515 (2021) Roedig&Sesana 1111.3742 (2012)

Considering dominant terms

$$\langle \dot{a} \rangle = -k\rho\sqrt{\frac{G\,a^5}{M}}\left(1+\frac{3e^2}{4}+\mathcal{O}(e^4)\right),$$
$$\langle \dot{e} \rangle = \frac{3}{2}k\rho\sqrt{\frac{G\,a^3}{M}}e\left(1+\frac{3e^2}{8}+\mathcal{O}(e^4)\right).$$

$$\frac{da}{de} = -\frac{2a}{3e}\left(1+\frac{3}{8}e^2+\mathcal{O}(e^3)\right)$$

Therefore, the environment favors eccentric motion. Considering both GW and the environment

$$\frac{da}{de} = \frac{6a\left(5c^5k\rho\sqrt{GMa^{11}}+32G^3M^4\right)}{e\left(304G^3M^4-45c^5k\rho\sqrt{GMa^{11}}\right)}$$

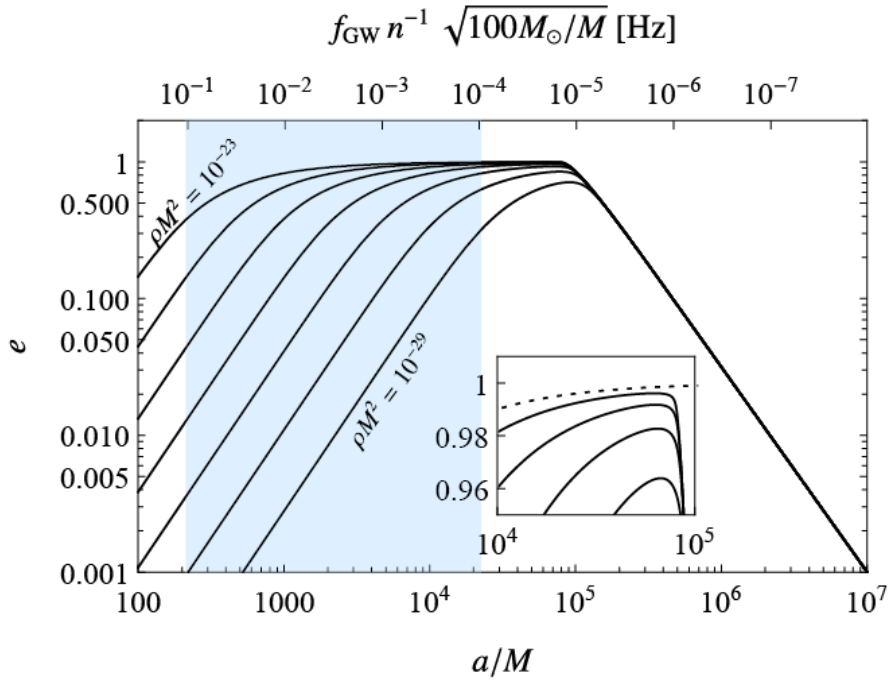
→ ***Critical distance***

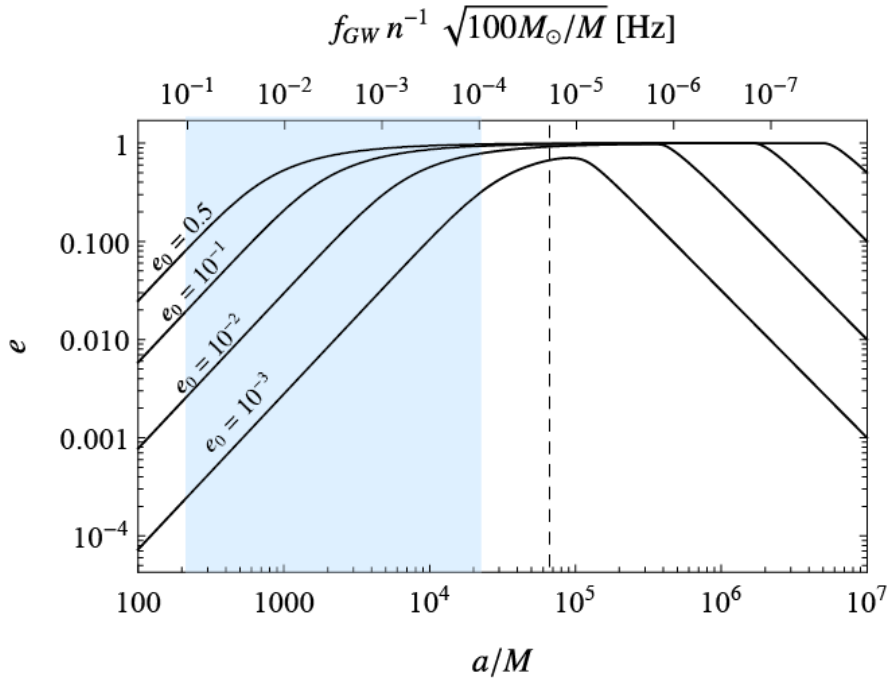
Eccentricity evolution and dynamical friction

Cardoso, CFBM, Vicente 2010.1515 (2021)) (Roedig&Sesana 1111.3742 (2012))

$$\frac{a_c}{\left(\frac{100GM_\odot}{c^2}\right)} = 3\times 10^4\, k^{-2/11}\left(\frac{M}{100M_\odot}\right)^{7/11}\left(\frac{\rho_{10}}{\rho}\right)^{2/11} \qquad \rho_{10} = 10^{-10}\mathrm{g\,cm^{-3}}$$

What about the evolution?





$$\frac{G^3}{c^6}\rho M^2 = 1.6\times 10^{-24}\frac{\rho}{\rho_{10}}\left(\frac{M}{100M_\odot}\right)^2$$

Eccentricity evolution and dynamical friction

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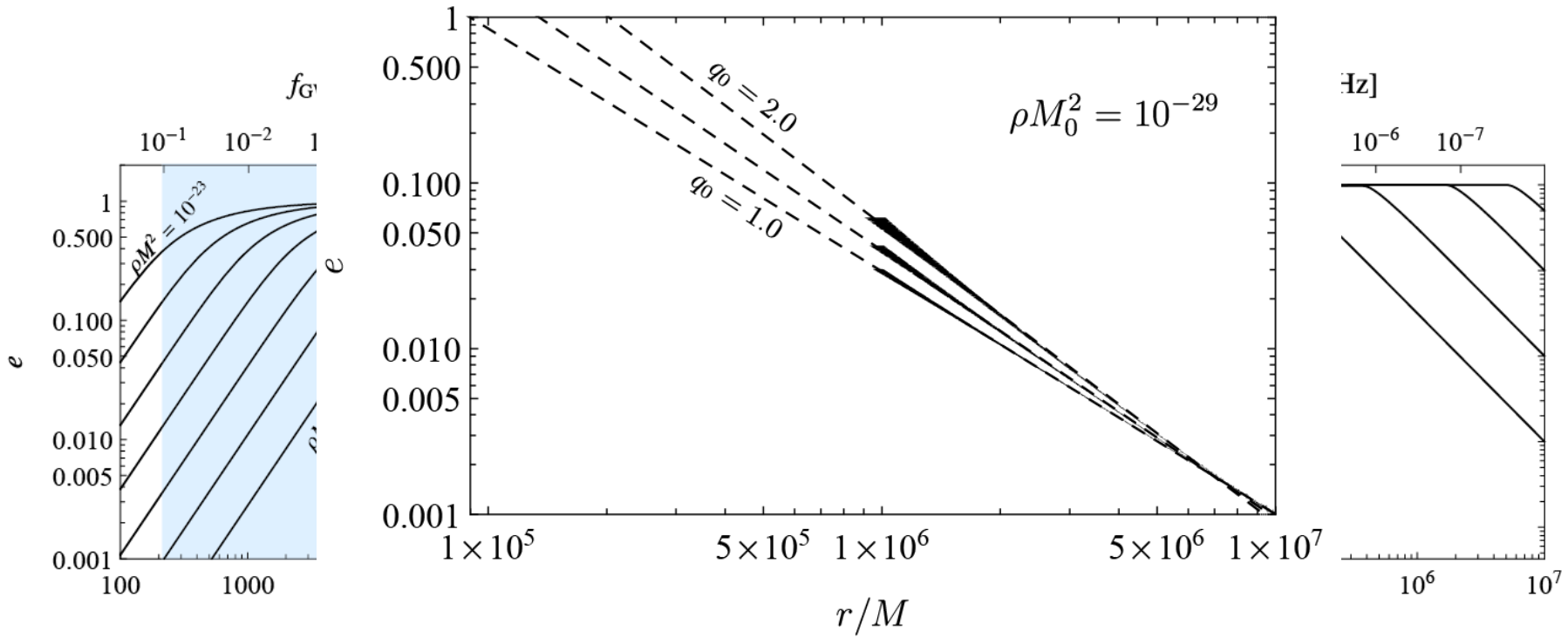

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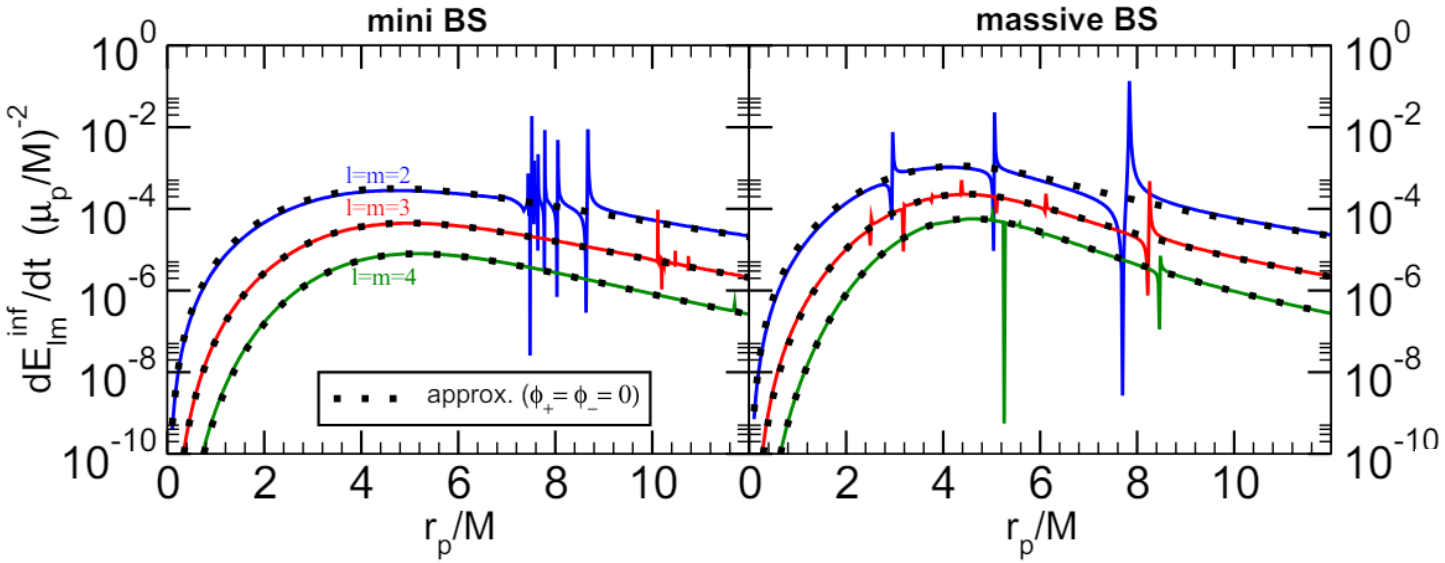
Mode excitation and dephasing

Macedo et al. Phys.Rev.D 88 6, 064046 (2013) & Astrophys.J. 774 (2013).



$$\dot{E} = \dot{E}_0 + h(t)\mathcal{H}\left[\delta t_{\rm res}^2 - (t - t_{\rm res})^2\right]$$

$$|\delta\phi_{\rm GW}| \sim \frac{5h}{16\nu^2}\frac{M\Delta\Omega_{\rm res}}{(M\Omega_{\rm res})^{10/3}}\frac{T_{\rm obs}}{M}$$

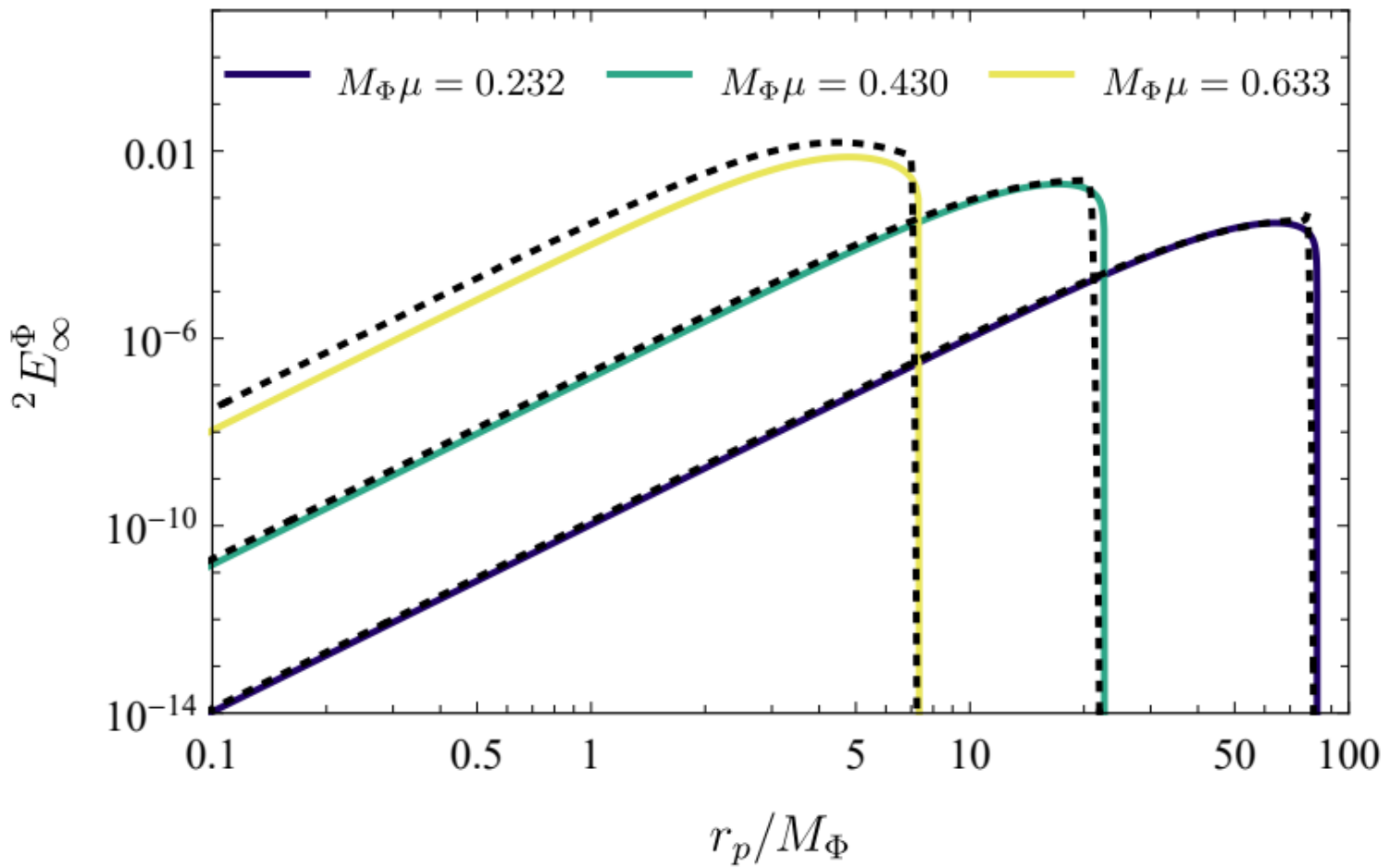
$$|\delta\phi_{\rm GW}| \approx 8.6\times 10^3 {\rm rads}\left[\frac{10^5 M_\odot}{M}\right]\left[\frac{T_{\rm obs}}{1{\rm yr}}\right]$$

$$\times\left[\frac{h/\nu^2}{10^{-2}}\right]\left[\frac{\sigma_I M}{10^{-5}}\right]\left[\frac{0.1}{\sigma_R M}\right]^{10/3}$$

DM fluxes

2312.06767

$$\delta\phi \sim -\frac{8\pi^{3/2}\phi_0(r_p)\left(m^{\frac{m}{2}-1}Y_m^m\left(\frac{\pi}{2},0\right)\left(r_p^3\Omega^3\right)^{m/3}\right)}{2^{\frac{m}{2}+2}\Omega^{\frac{m}{2}+1}\Gamma\left(m+\frac{3}{2}\right)}$$



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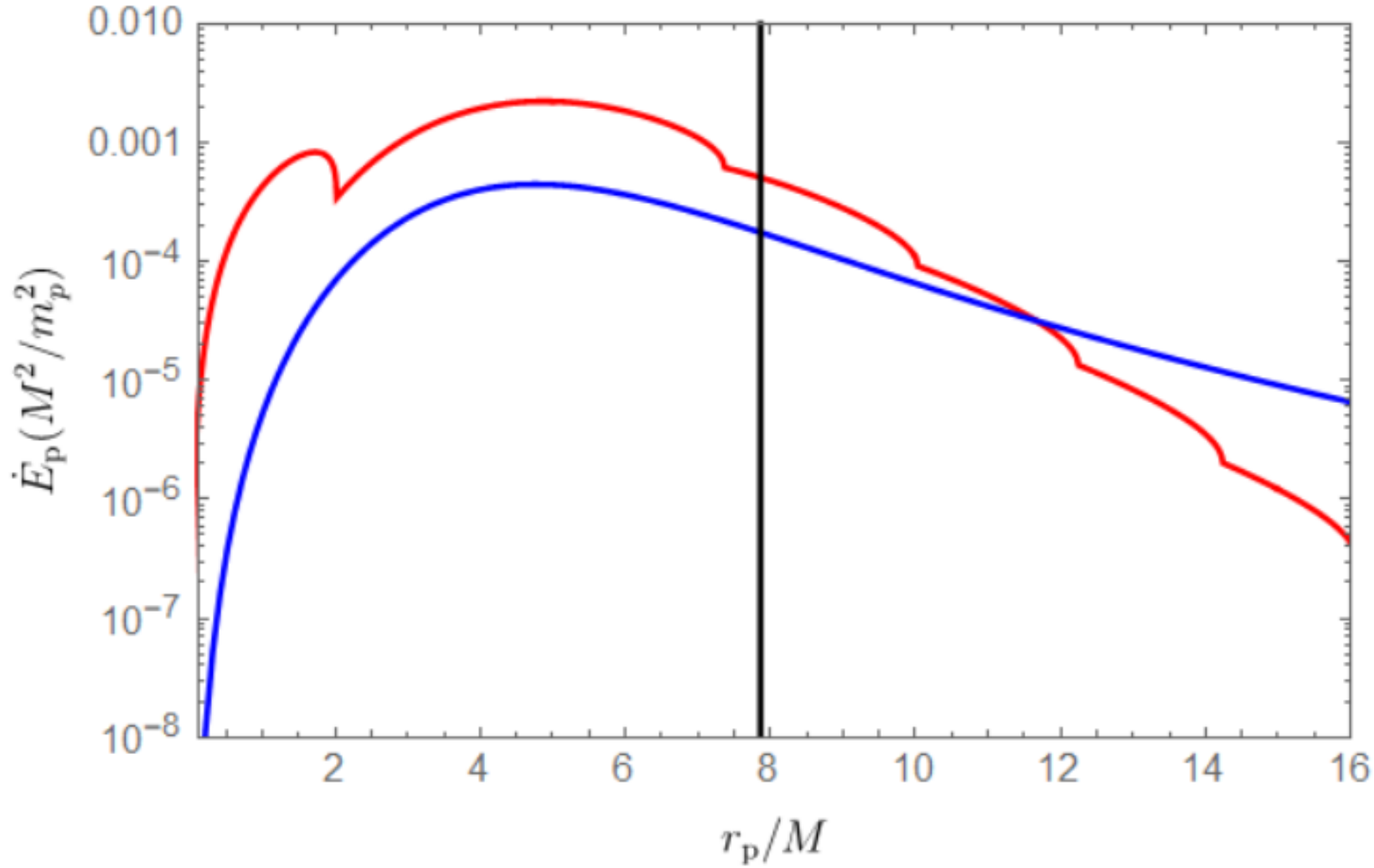
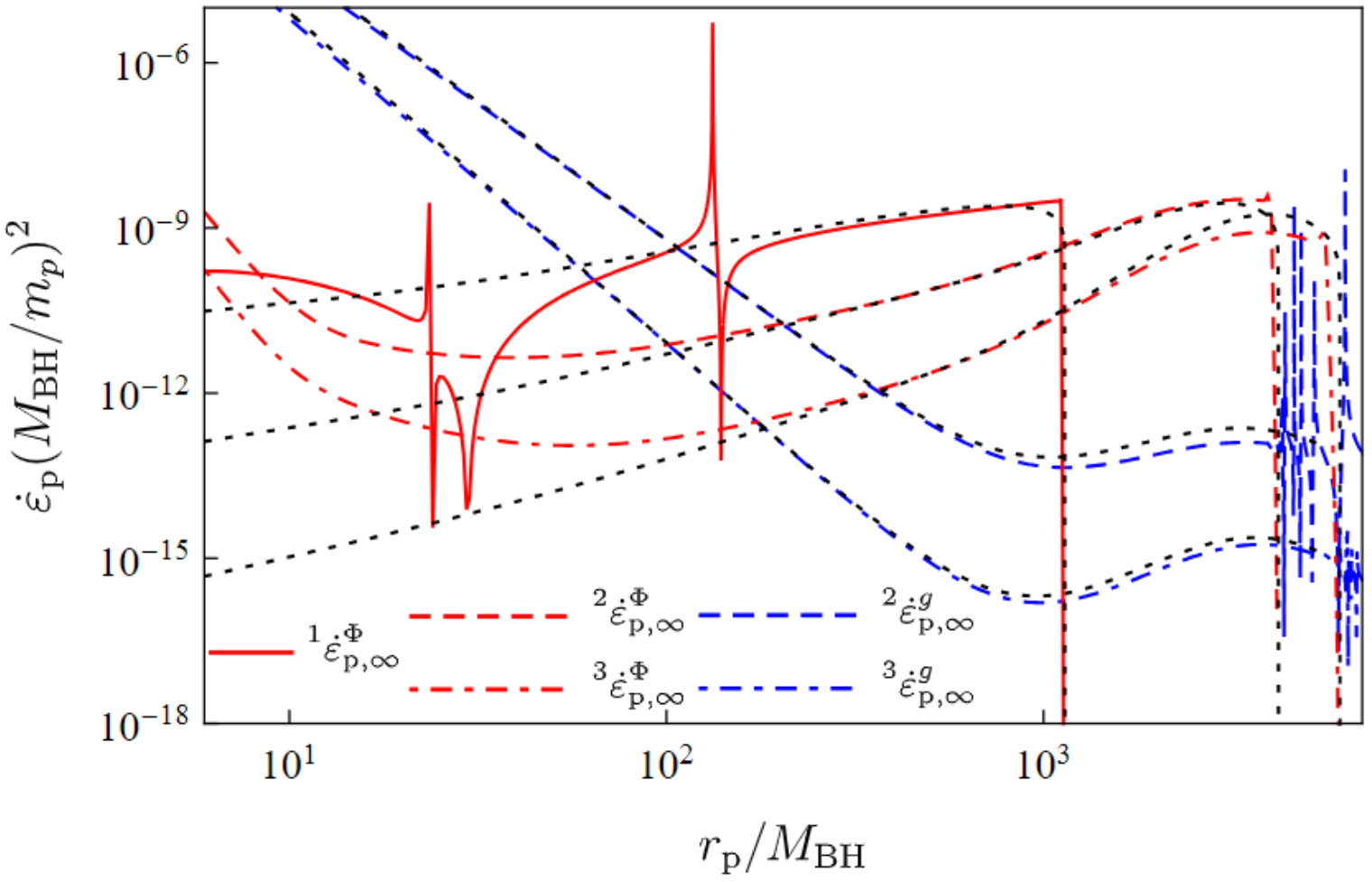


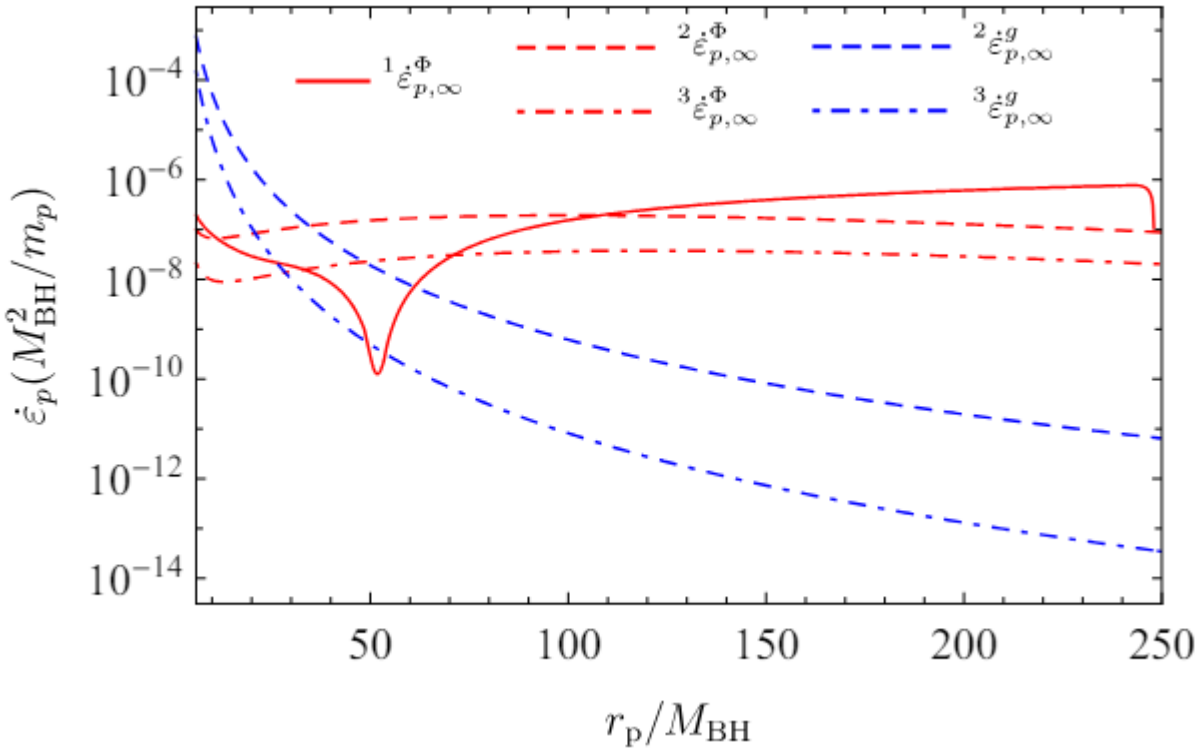
EMRI: BH surrounded by (scalar) dark matter

Duque et al. arXiv:2312.06767 (2023)



EMRI: BH surrounded by a cloud

Duque et al. arXiv:2312.06767 (2023). Brito, Shah Phys.Rev.D 108 8, 084019 (2023)



$$\delta_{\rm cyc} \sim 20\left(\frac{q}{10^{-5}}\right)\left(\frac{t_{\rm obs}}{4\,{\rm yr}}\right)\left(\frac{\Omega_{\rm p}/\pi}{0.3\,{\rm mHz}}\right)^{1/3}$$

Final remarks

- Binaries in environments can be remarkably different.
- Dynamical friction is important in eccentricity and CM boosts.
- Period shifts can be a discriminator.
- Axisymetric time-dependent environment introduces resonances.

Thank you!

I GRAVITAS SCHOOL

ON GRAVITY AND FIELDS

CASA DE CULTURA -FONTE DO CARANÃ SALINÓPOLIS/PA - BRAZIL, 02-06 DECEMBER 2024