Continuous gravitational waves I

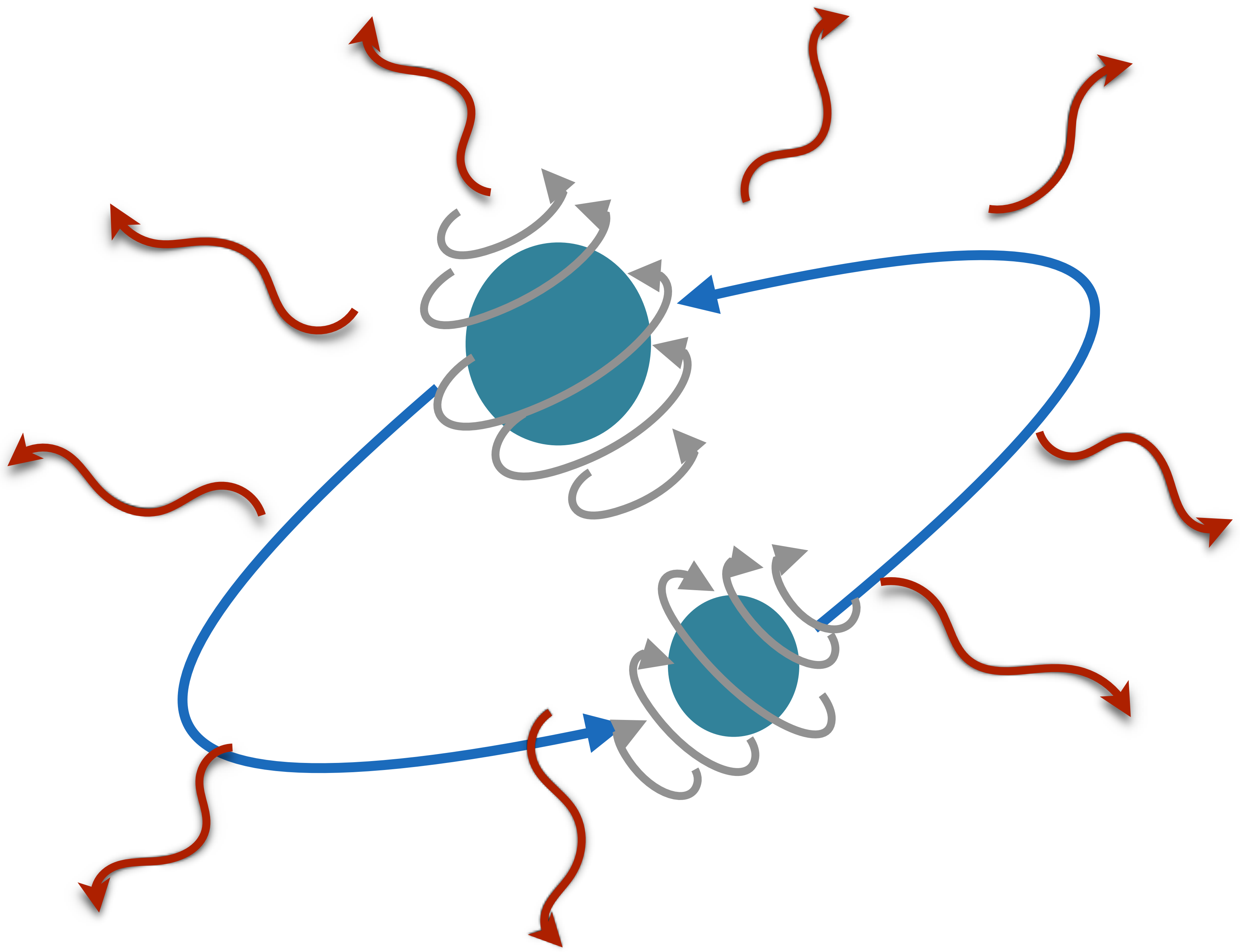
Detection methods

M. Alessandra Papa - New Horizons for Psi School - Lisbon 1-5 July 2024
Max Planck Institute for Gravitational Physics and Leibniz Univ. Hannover

Plan

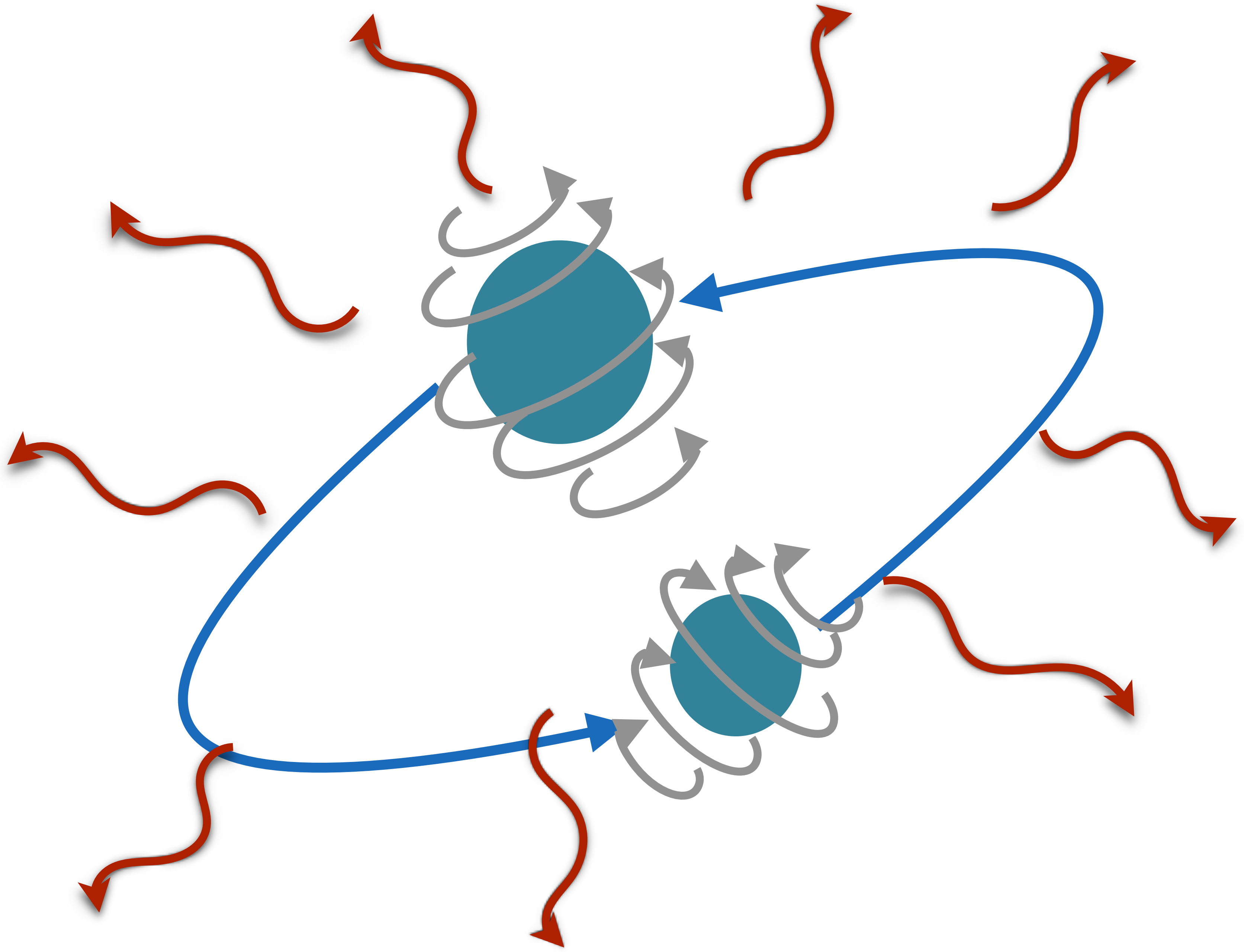
- Lecture I
  - Continuous Gravitational Waves: how are they emitted and why should we care
  - The signal
  - Detection methods
    - Coherent
    - Semi-coherent
- Lecture II
  - actual search results

Binary merger

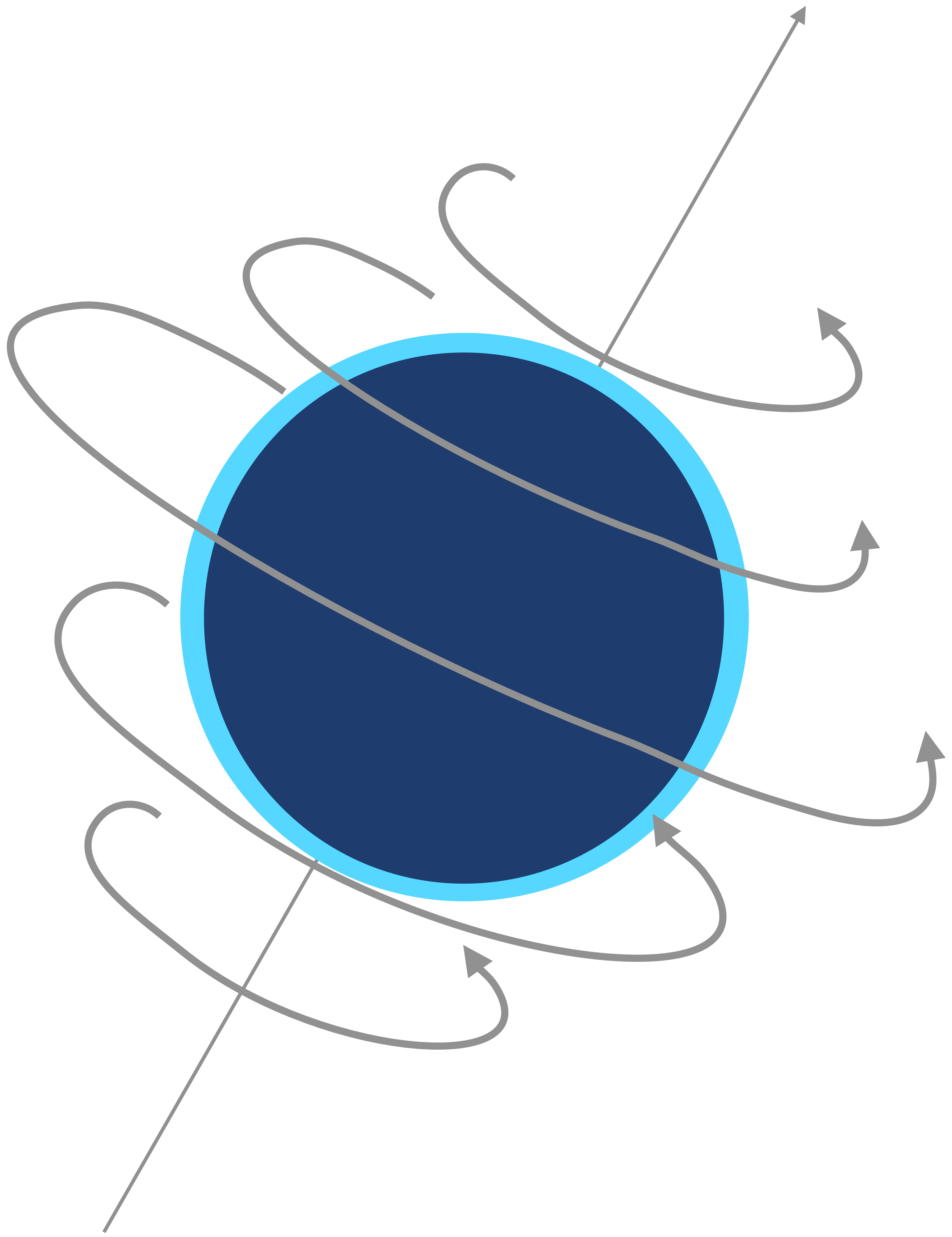


Gravitational waves
from the varying
mass quadrupole

Binary merger

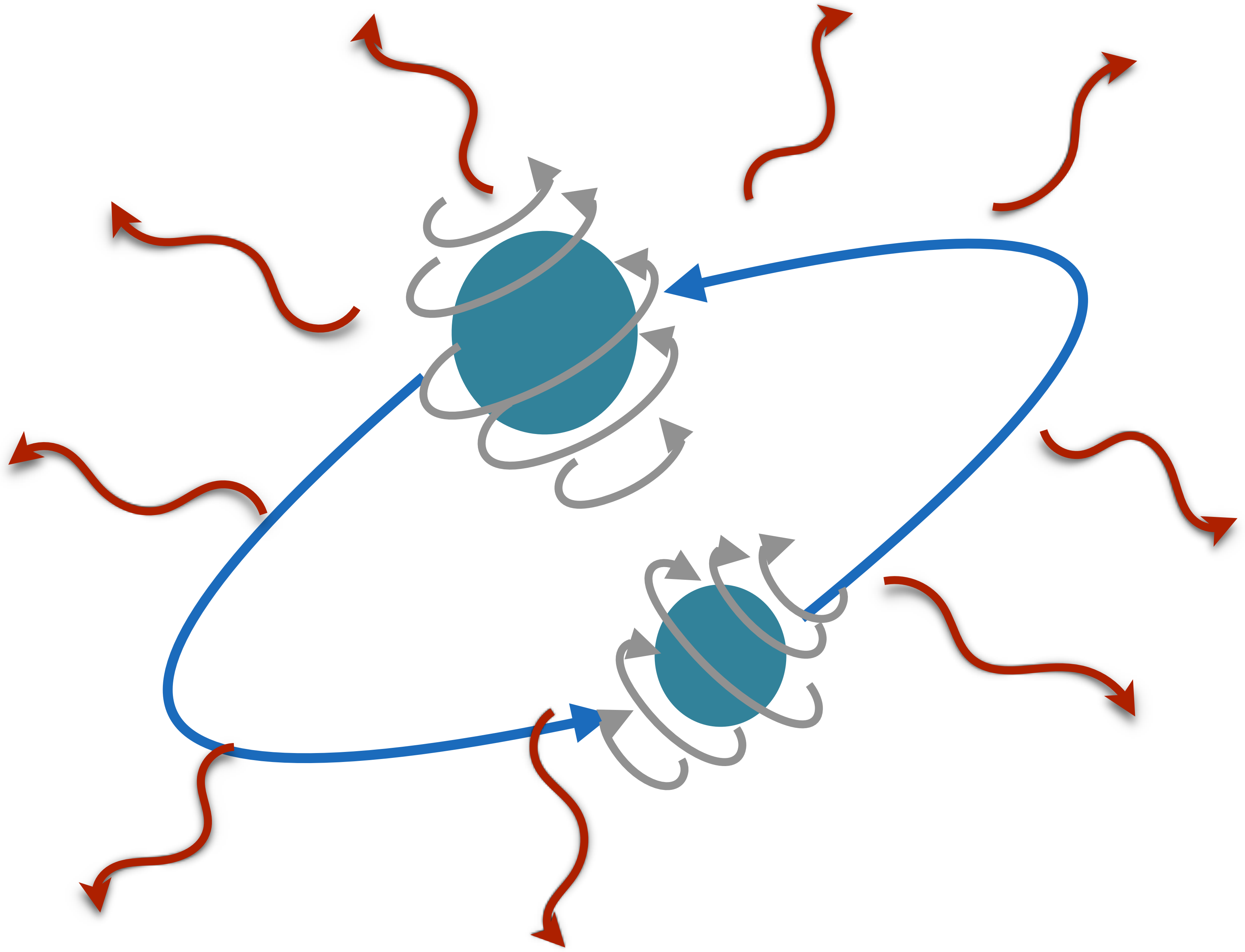


Spinning neutron star



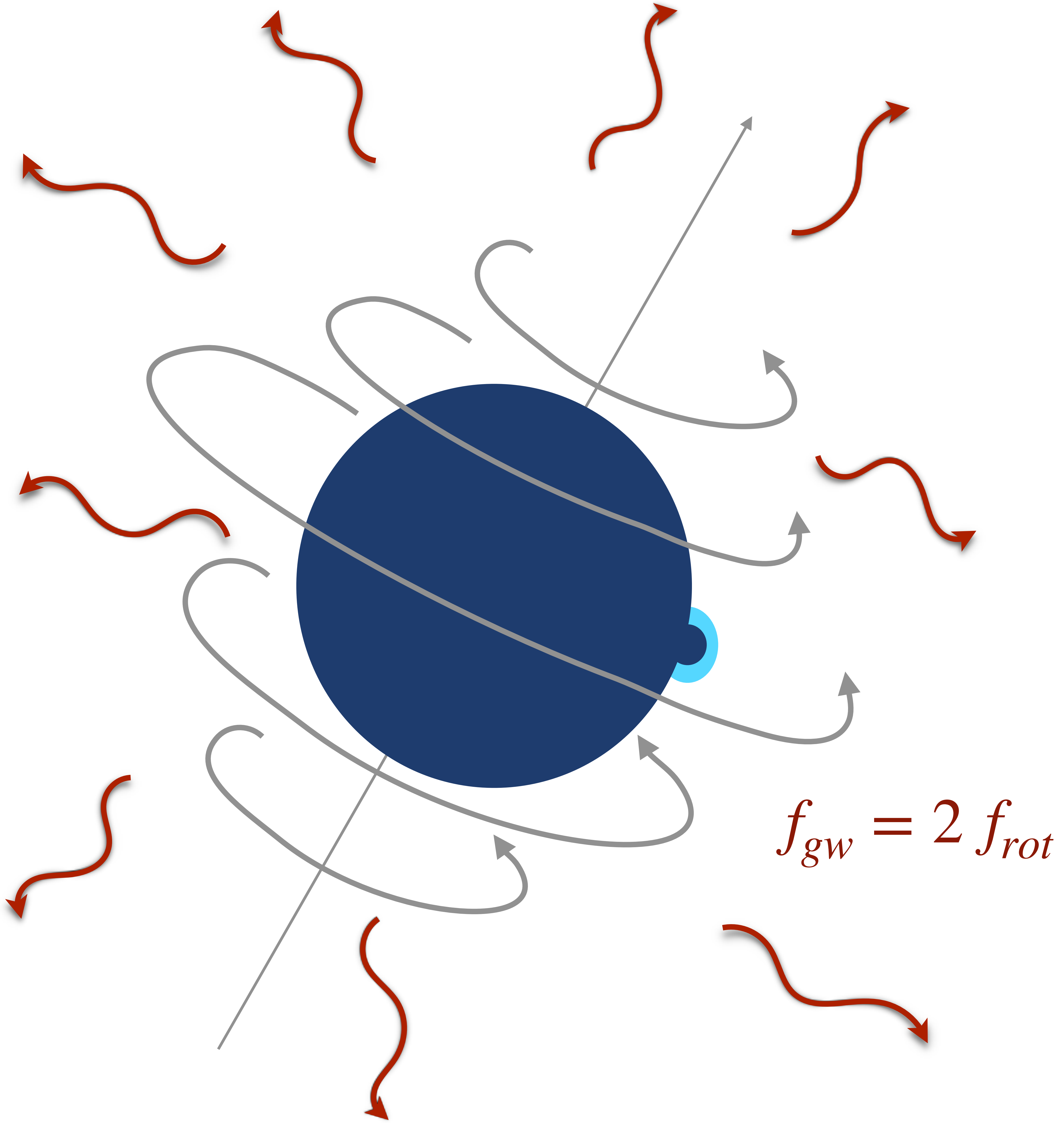
no gravitational waves

Binary merger



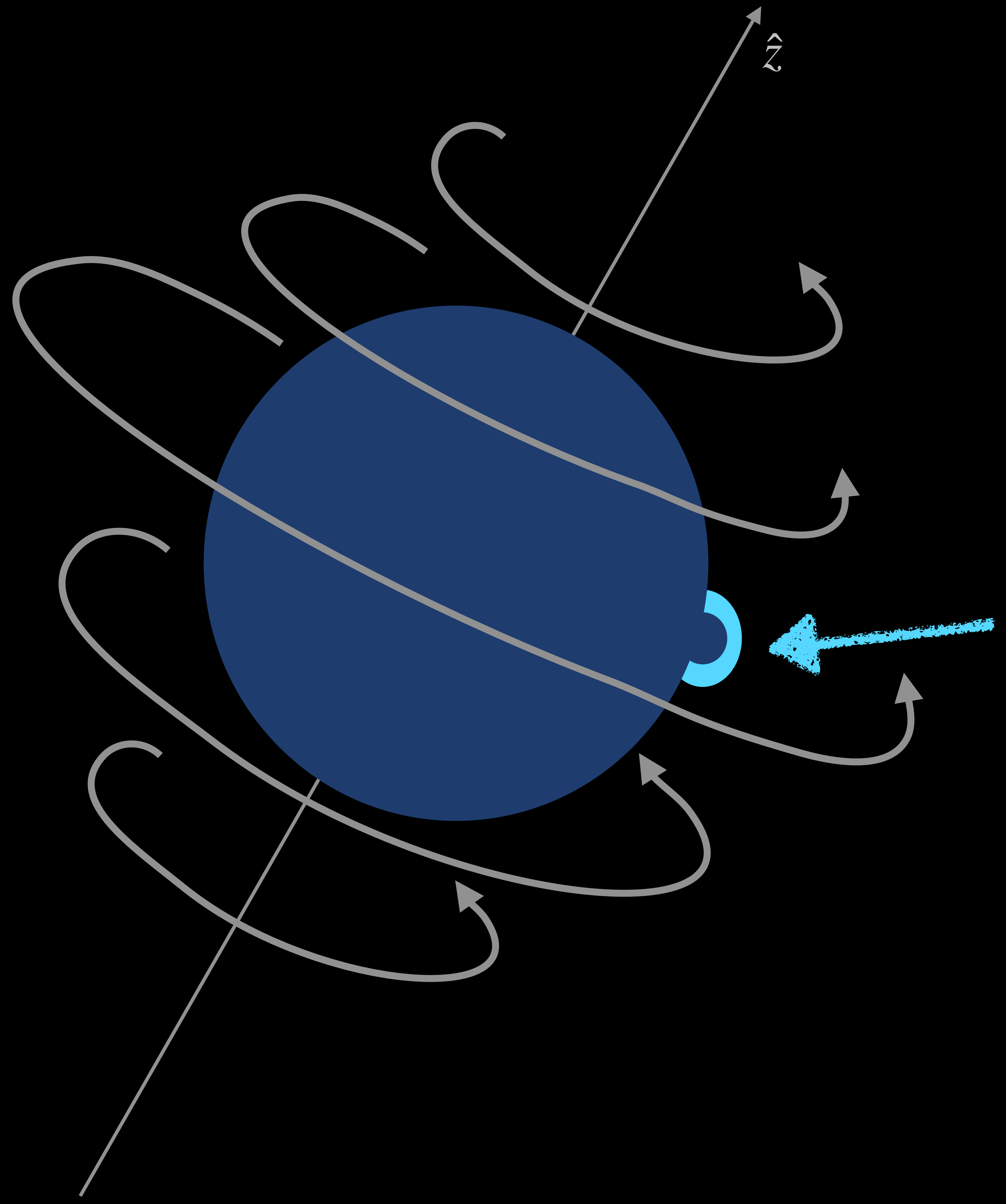
chirp signal lasting $\lesssim$ s

Spinning neutron star



continuous sinusoidal signal

GRAVITATIONAL WAVES FROM SPINNING TRI-AXIAL NEUTRON STARS

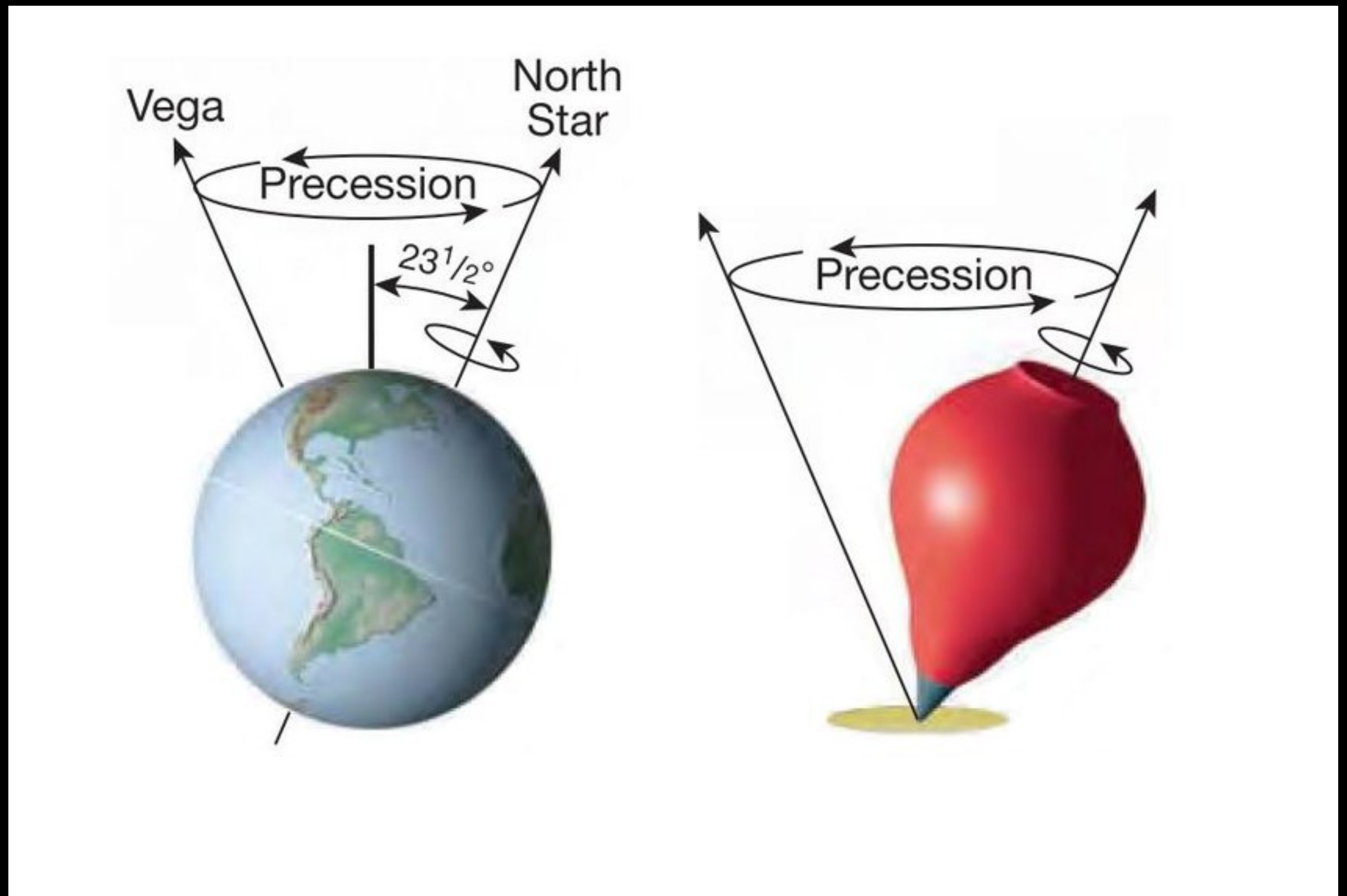


ellipticity $\varepsilon := \frac{I_{xx} - I_{yy}}{I_{zz}}$

NEUTRON STAR EMISSION MECHANISMS

image by freepik

"mountains"



Essentials of Geology (p. 283) by Lutgens et al (2012)

"wobble"

Bekki et al, Scince Advances (2024)

oscillations

EMISSION MECHANISMS

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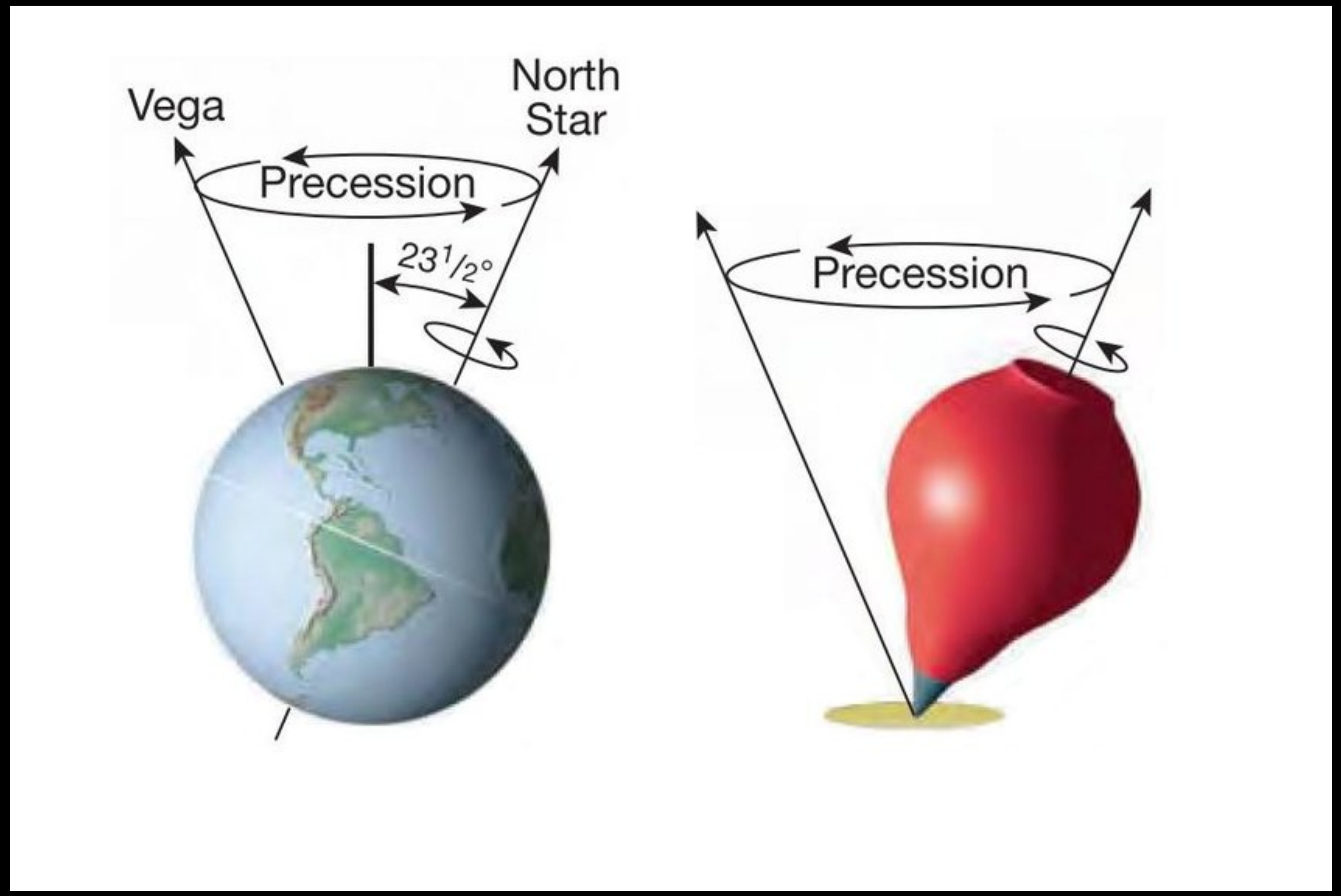

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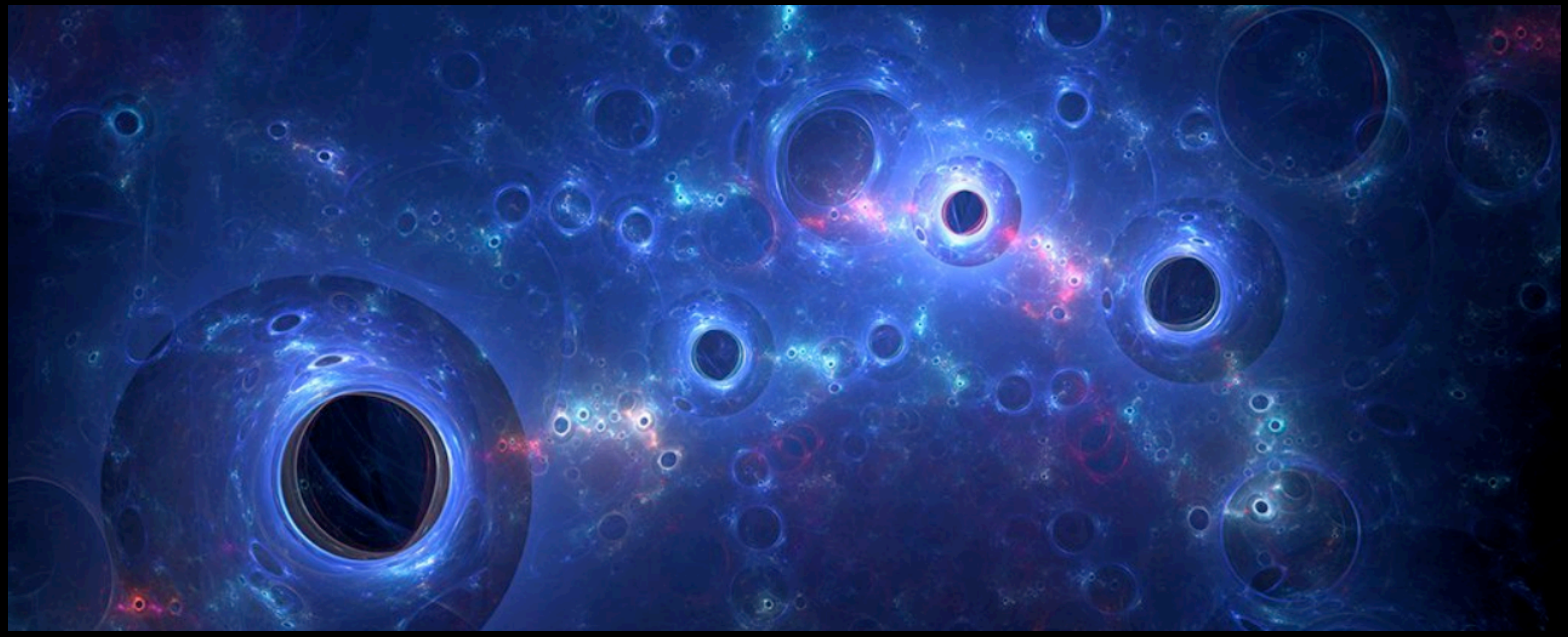


Sakkmesterke/iStock

exotic

WHAT COULD GENERATE SIGNAL ?

- what could source ellipticity ?
  - deformation frozen-in at birth
  - star-quakes
  - hot-spot (in accreting systems, very interesting)
  - *internal* magnetic fields*

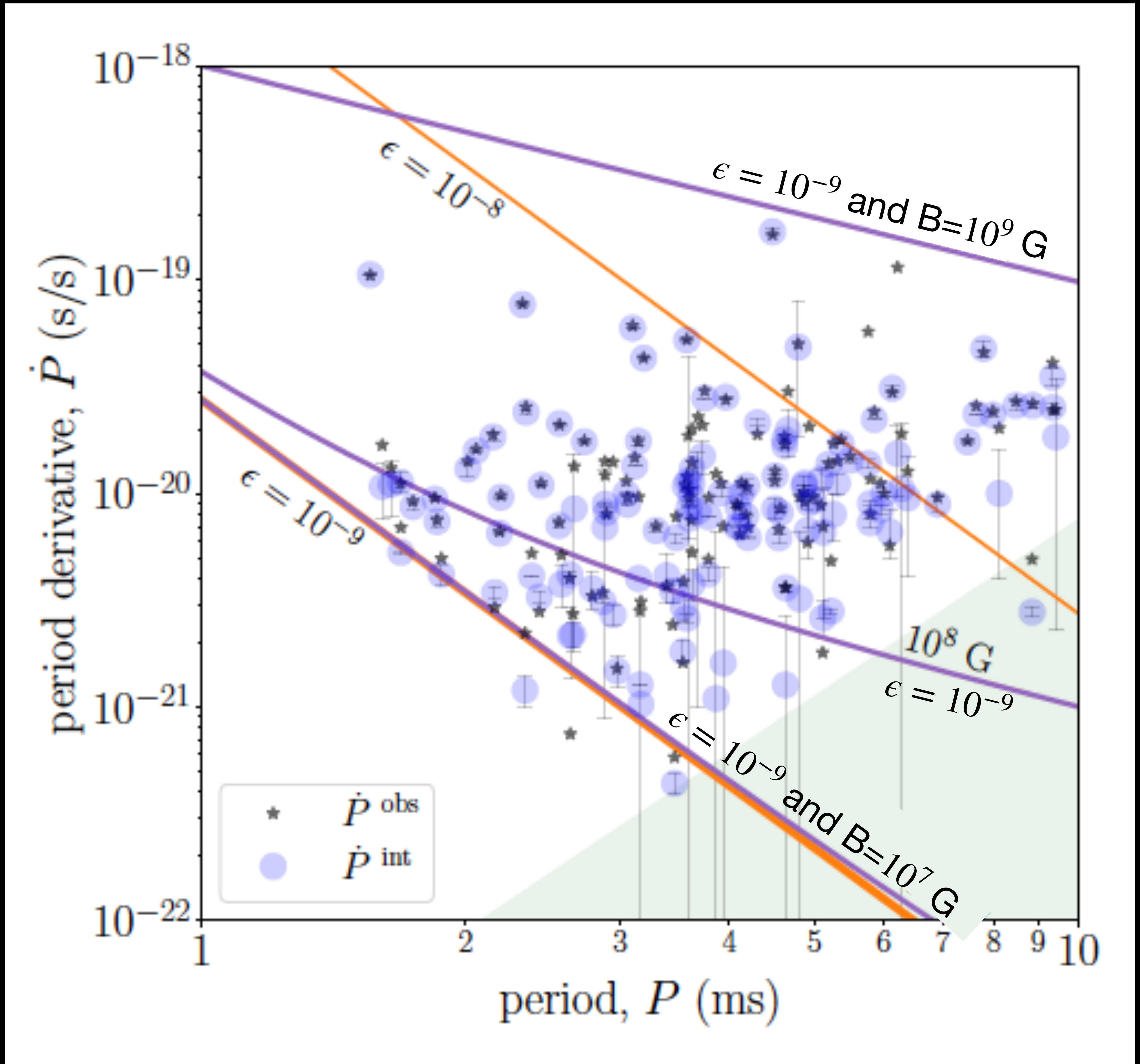
* Mastrano et al, MNRAS 417 (2011)

HOW LARGE COULD THE ELLIPTICITY BE ?

- difficult question
- maximum ellipticity**
  - i.e. before crust breaks, very uncertain $\approx (10^{-3})10^{-5} - 10^{-8}$
- smallest ellipticity
  - magnetic fields, very low $\approx 10^{-14}$

* **Johnson-McDaniel & Owen, PRD 88 (2013) - Gittins et al, PRD 101 (2020), Gittins & Andersson, MNRAS 500 (2020), MNRAS 507 (2021) - Morales & Horowitz, MNRAS 517 (2022)

EVIDENCE OF MINIMUM ELLIPTICITY $\varepsilon \sim 10^{-9}$?



Woan et al, ApJ 863 (2018)

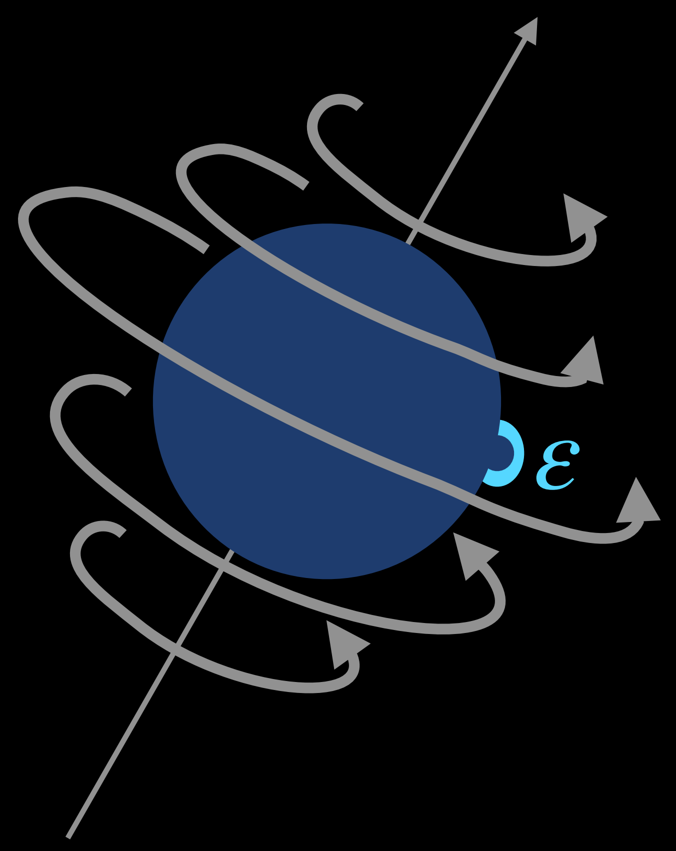
Spin evolution only due to GW emission

Spin evolution due to magnetic field and GW emission

WHAT COULD WE LEARN ?

- ellipticity of object, internal structure of NS
- access to invisible NS population
- tests of GR (non-GR polarisations)
- if in conjunction with EM timings
  - emission mechanism
  - differential rotation ?
- even more intriguing, if signal does not come from a neutron star

VERY WEAK SIGNALS



- signal always there

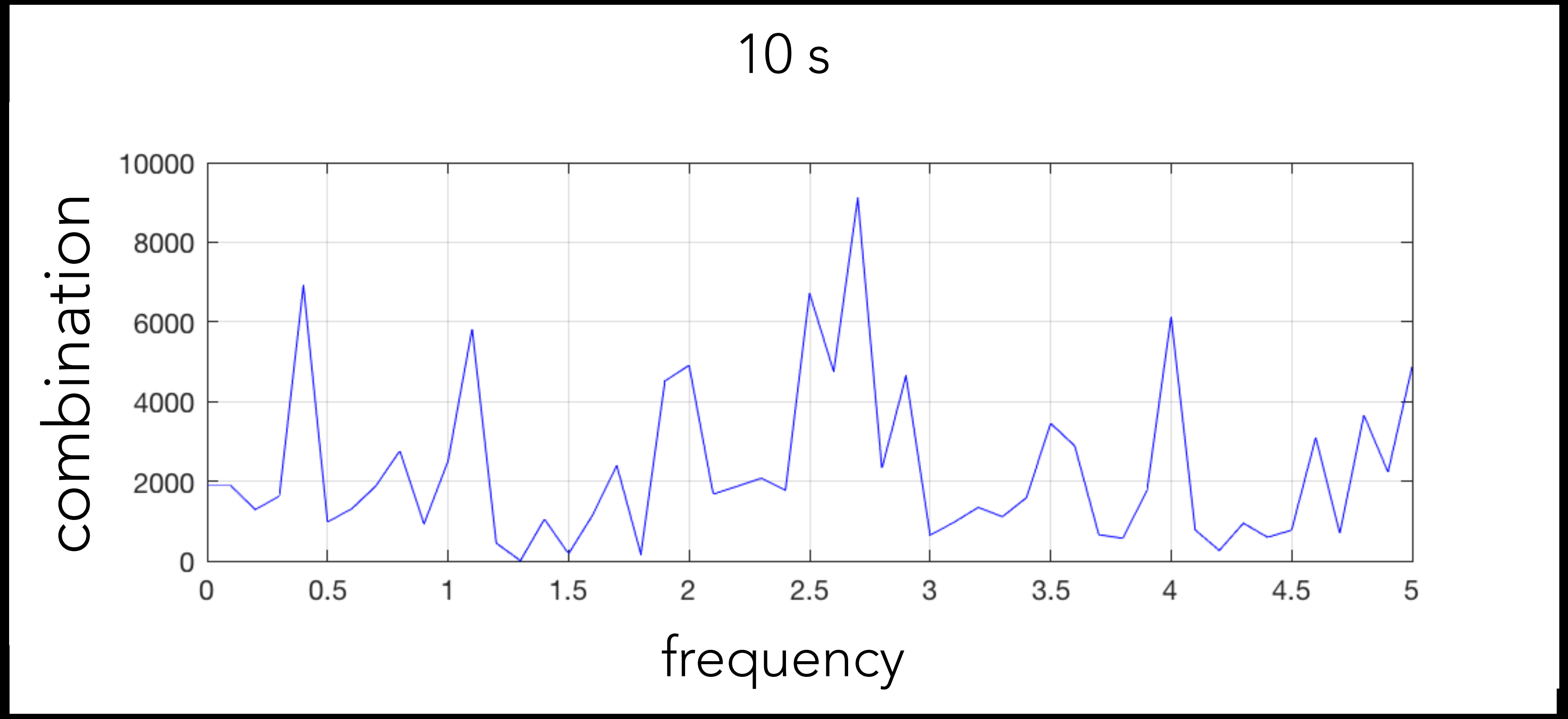
- very weak:

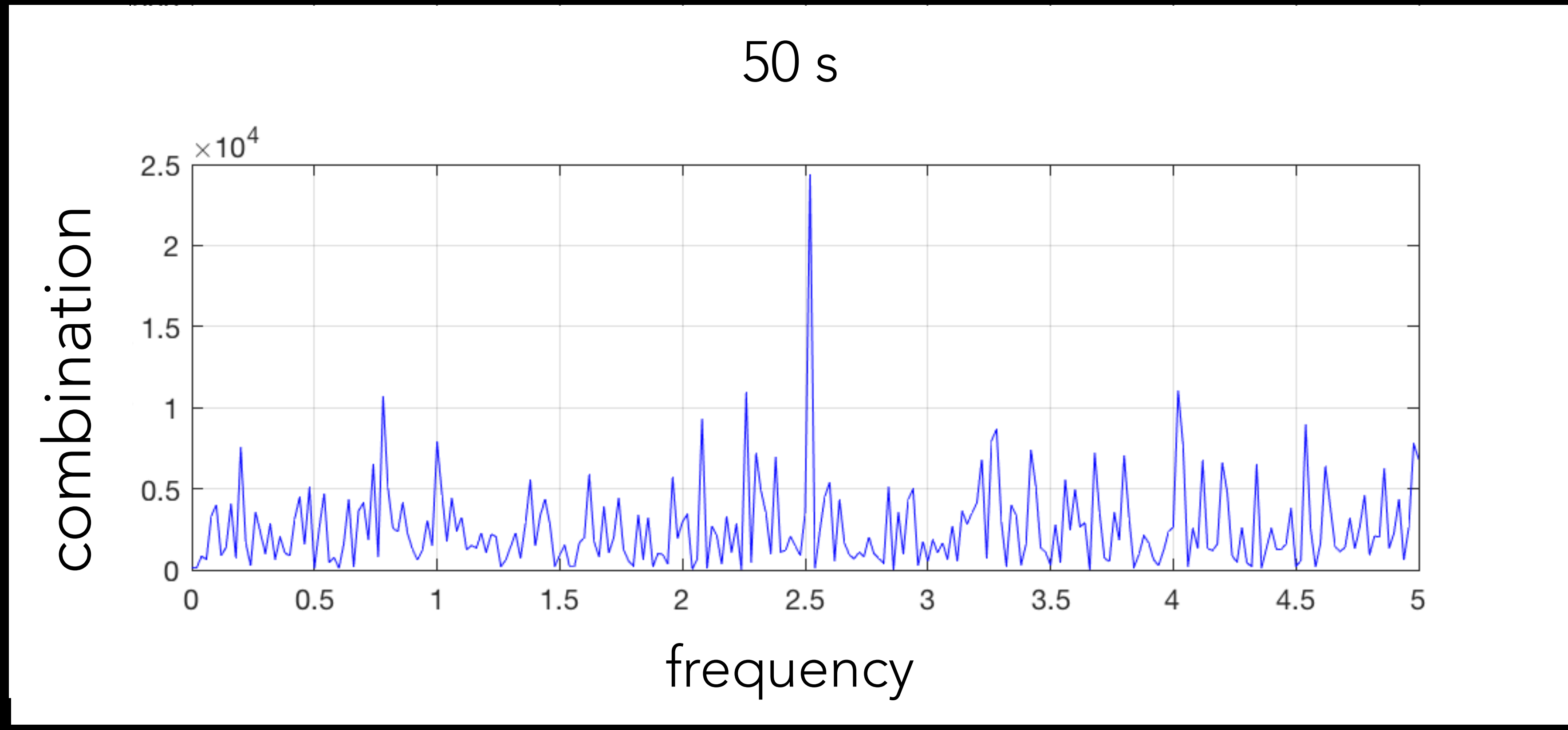
$$h_0 = \frac{2\pi^2 G}{c^4}\frac{I\varepsilon f_{gw}^2}{D} = 2\times 10^{-25}\left[\frac{I}{10^{38}\ \text{kg m}^2}\right]\left[\frac{\varepsilon}{10^{-6}}\right]\left[\frac{f_{gw}}{10^3\ \text{Hz}}\right]^2\left[\frac{1\ \text{kpc}}{D}\right]$$

compare: $h_0^{binaries} \approx 10^{-21}$

THE LONGER THE OBSERVATION IS, THE BETTER

basic idea: combine the data (think FFT power), the signals adds coherently, the noise does not (matched filtering)

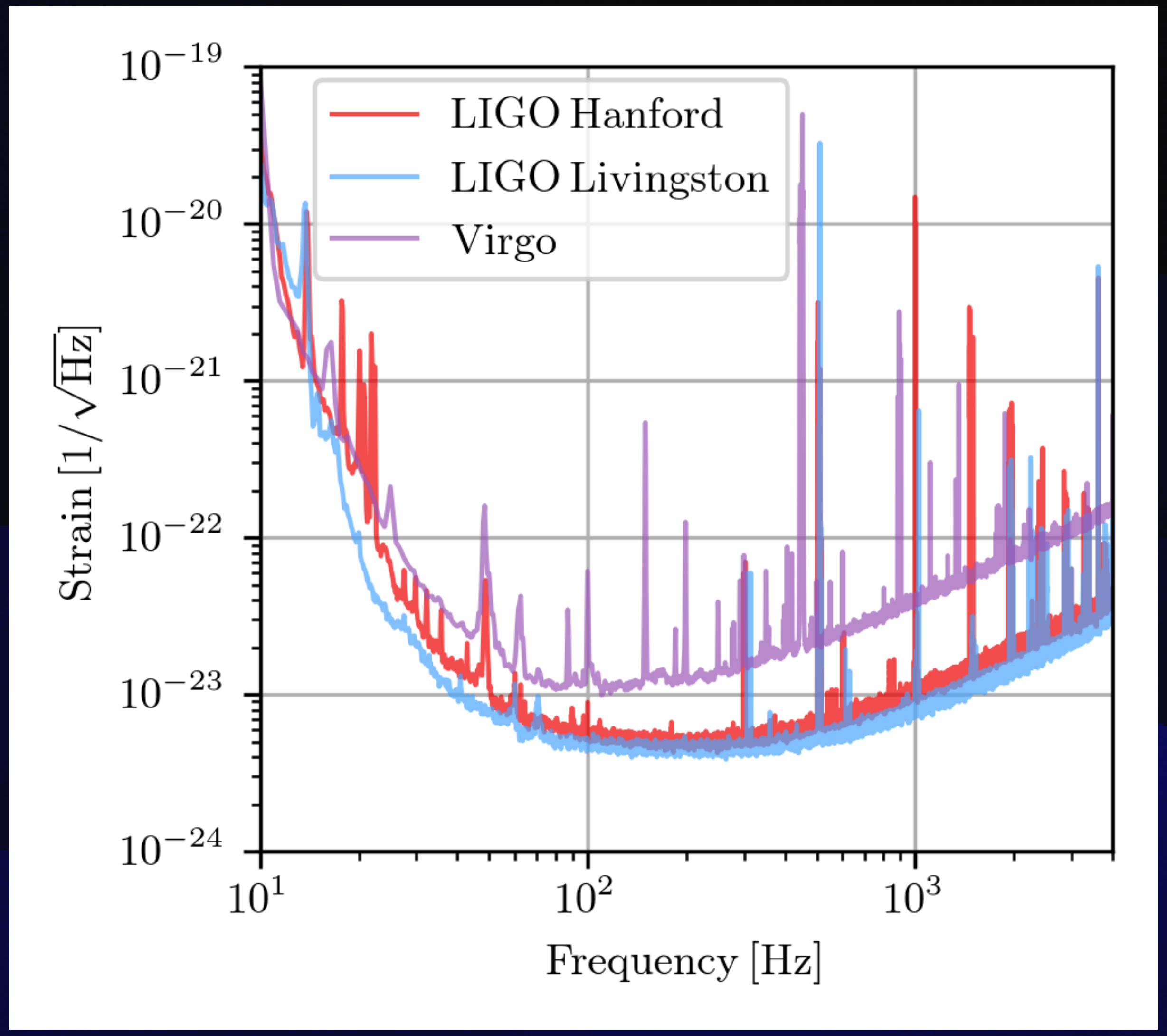




The longer is the time baseline, the higher is the SNR

HOW FAR WE HAVE TO DIG BELOW NOISE LEVEL ?

detector noise



+ target signal amplitude h_0

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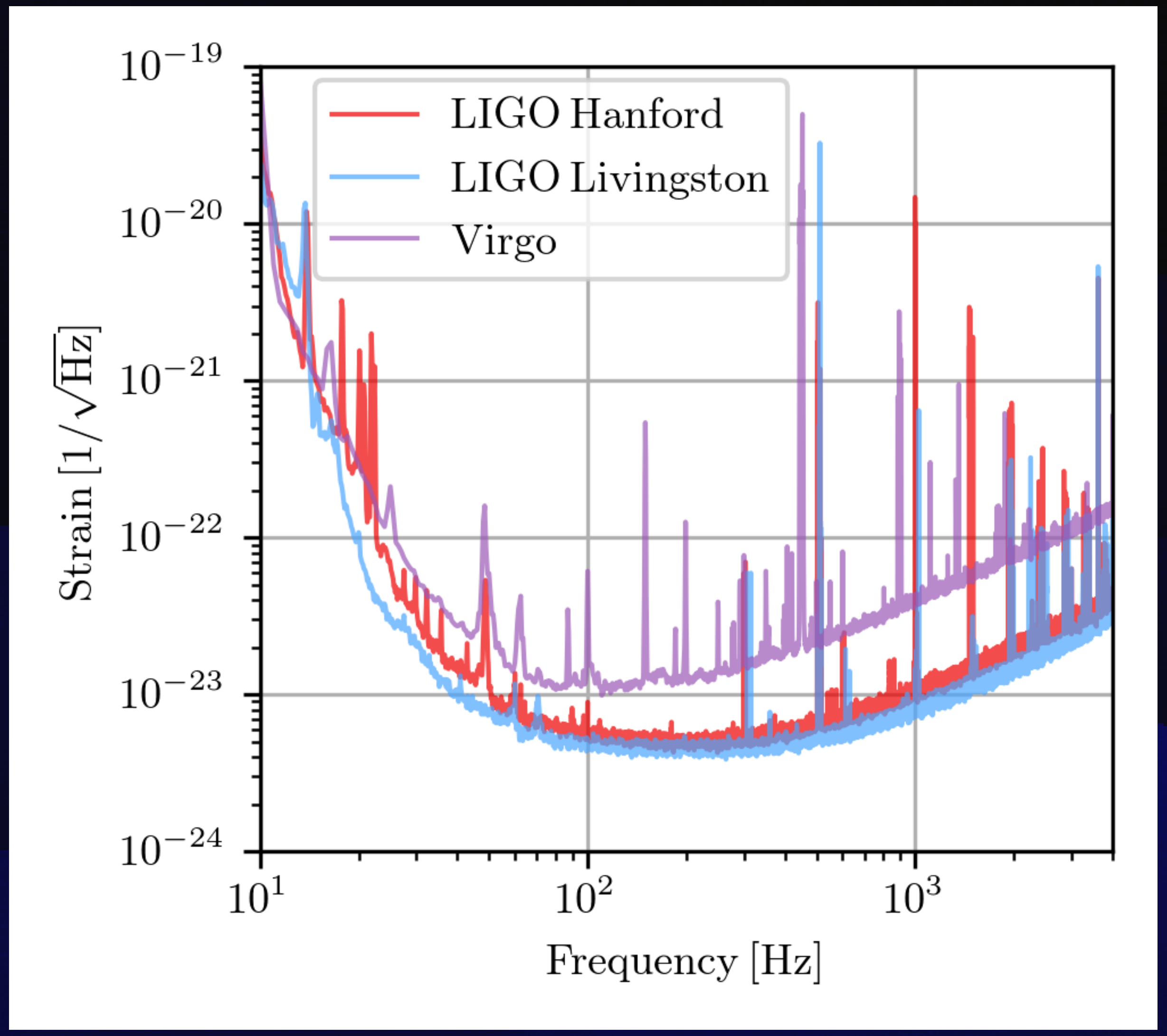
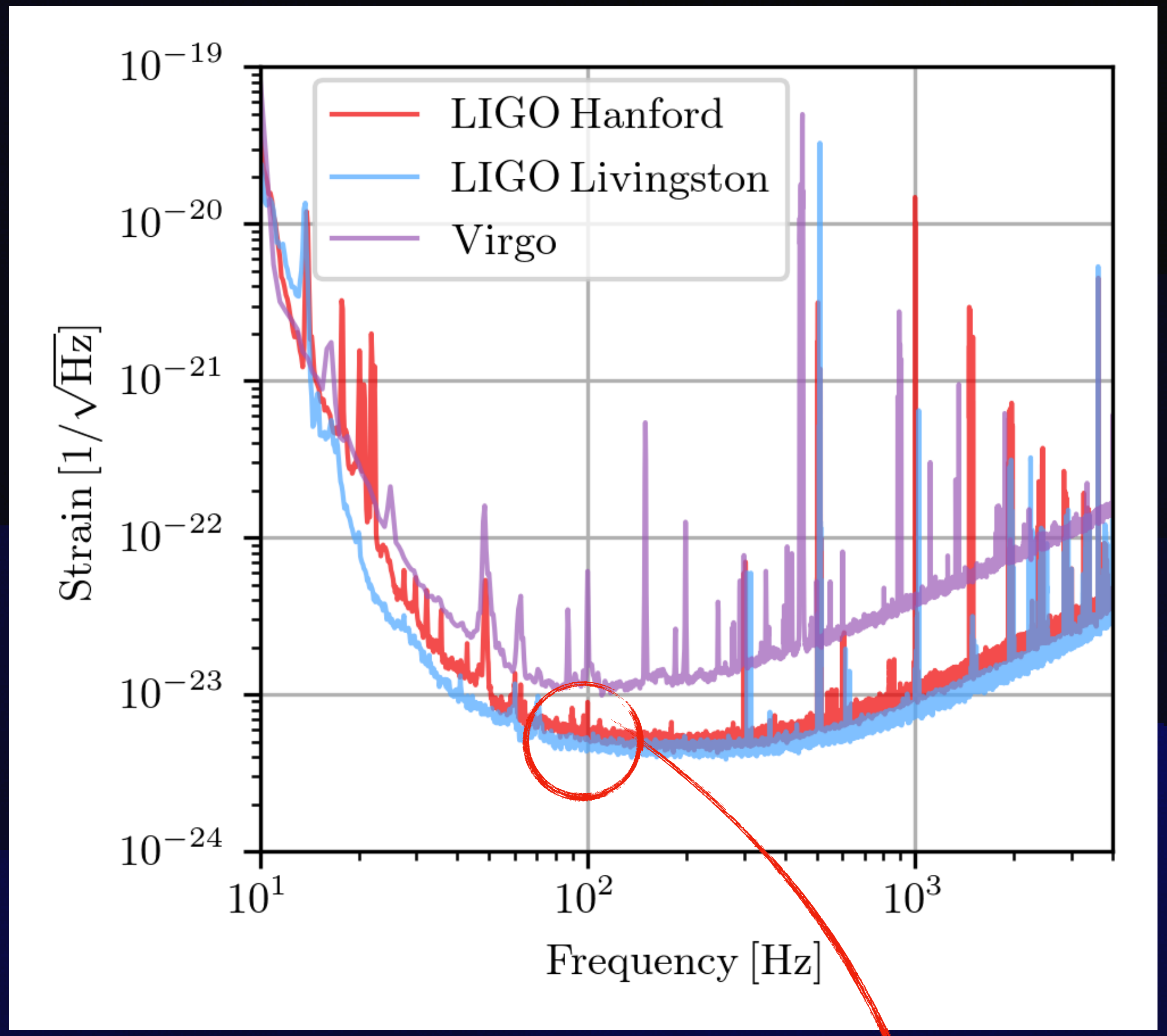


+ target signal amplitude h_0 ⇒ $\mathscr{D} := \frac{\sqrt{S_n}}{h_0} \quad \left[\sqrt{\frac{1}{\text{Hz}}}\right]$

sensitivity depth

HOW FAR WE HAVE TO DIG BELOW NOISE LEVEL ?

detector noise



$\approx 6 \times 10^{-24}$ $[1/\sqrt{\text{Hz}}]$

+ target signal amplitude h_0

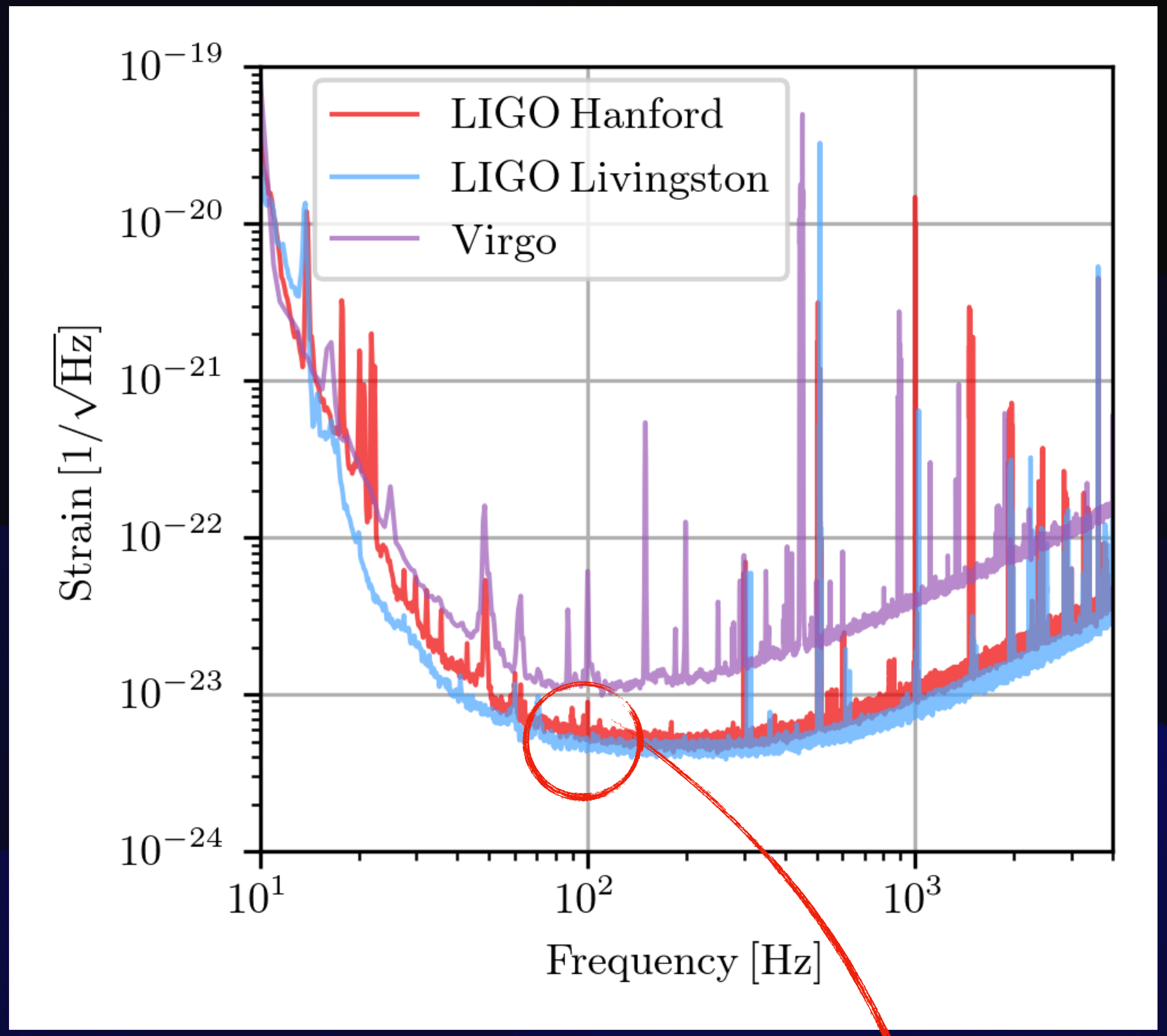
2×10^{-25}

$$\mathscr{D} := \frac{\sqrt{S_n}}{h_0} \quad \left[\sqrt{\frac{1}{\text{Hz}}}\right]$$

sensitivity depth

HOW FAR DO WE HAVE TO DIG BELOW NOISE LEVEL ?

detector noise



+ target signal amplitude h_0 ⇒ $\mathscr{D} := \frac{\sqrt{S_n}}{h_0} \left[\sqrt{\frac{1}{\text{Hz}}}\right]$

sensitivity depth

2×10^{-25}

$\approx 6 \times 10^{-24}\ [1/\sqrt{\text{Hz}}]$

$$\Rightarrow \mathscr{D} = 30 \left(\frac{\sqrt{S_n}}{6 \times 10^{-24}}\right)\left(\frac{2 \times 10^{-25}}{h_0}\right)\left[\frac{1}{\sqrt{\text{Hz}}}\right]$$

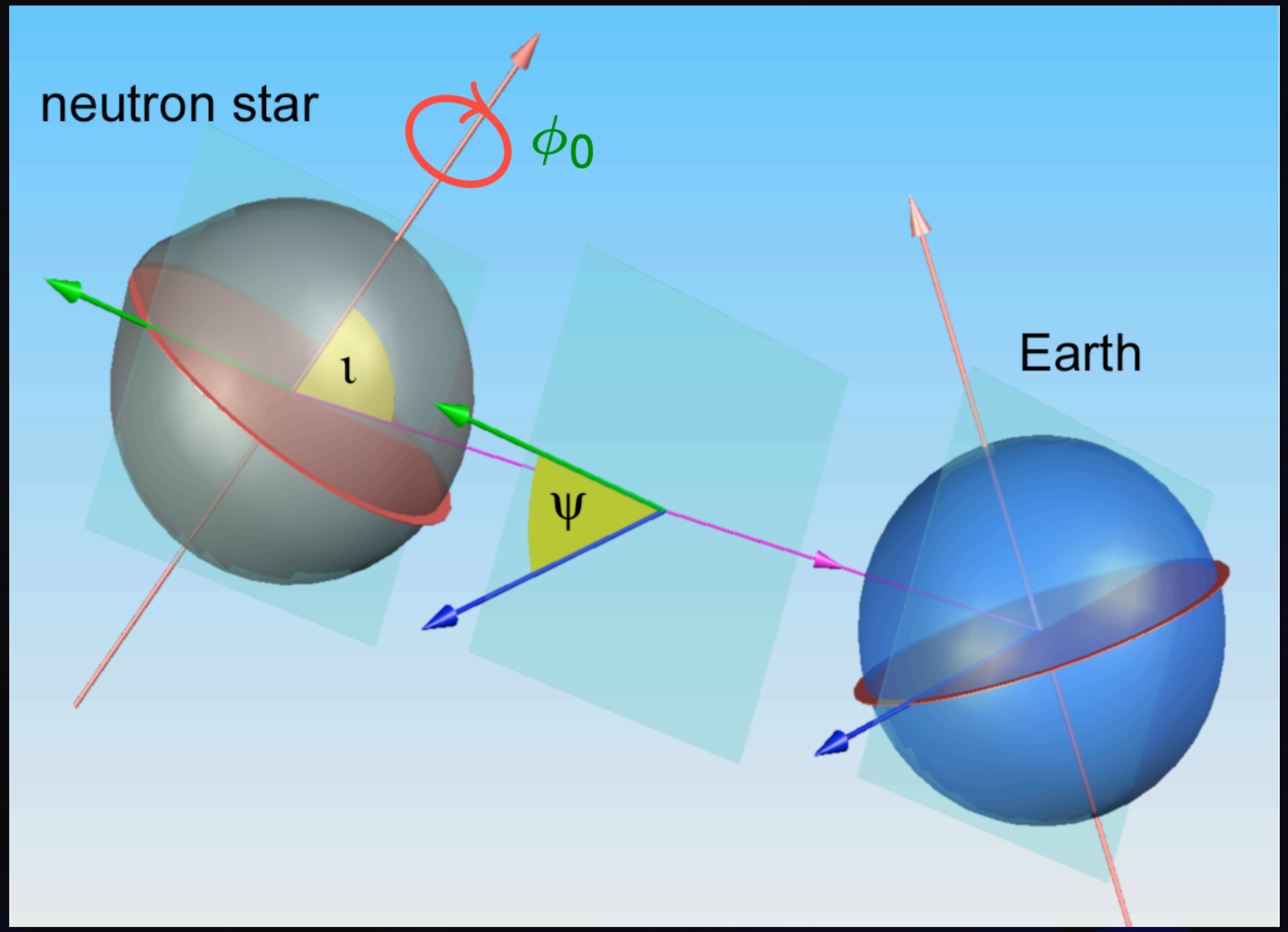
Detection methods

Detector response $h(t)$

for plane wave and detector-size << reduced wavelength

$$h(t) = F_+(t)\ h_+(t) + F_\times(t)\ h_\times(t)$$

Detector response $h(t)$



for plane wave and size detector << reduced wavelength

$$h(t) = F_+(t)\,h_+(t) + F_\times(t)\,h_\times(t)$$

the two polarisations

$$\begin{cases} h_+(t) = \frac{1}{2}h_0\ (1+\cos^2\iota)\ \ \cos 2\Phi(t) \\ h_\times(t) = h_0\ \ \cos\iota\ \ \sin 2\Phi(t) \end{cases}$$

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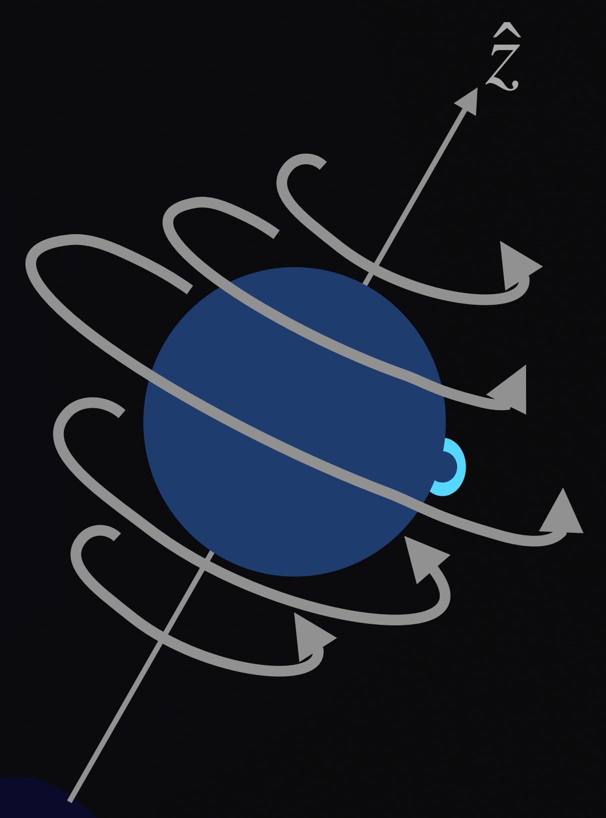
signal phase

Phase



$$\begin{cases} h_+(t) = \frac{1}{2} h_0 \, (1 + \cos^2 \iota) \, \cos 2\Phi(t) \\ h_\times(t) = h_0 \, \cos \iota \, \sin 2\Phi(t) \end{cases}$$

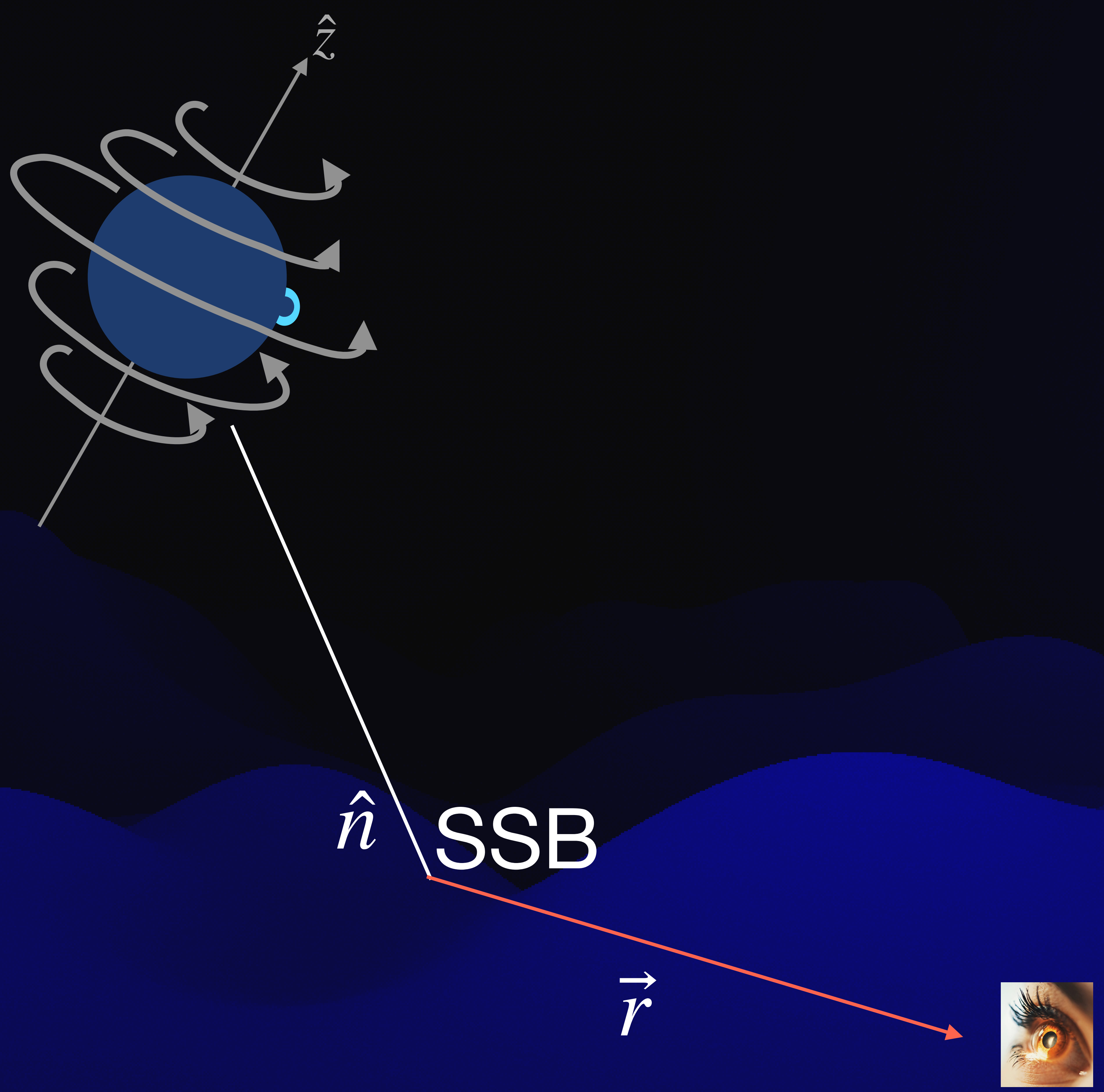
At rest with respect to the source, i.e. at SSB :

$f(\tau) = f_0 + f_1 \tau + \frac{1}{2} f_2 \tau^2$, reference time $\tau_0 = 0, f_0 = f(\tau_0), f_n = \frac{d^n f_0}{d\tau}$

$$\longrightarrow \Phi'(\tau) = \Phi_0 + 2\pi \left(f_0 \tau + \frac{1}{2} f_1 \tau^2 + \frac{1}{6} f_2 \tau^3 + \cdots \right)$$

Phase

$$\begin{cases} h_+(t) = \frac{1}{2}h_0\ (1+\cos^2\iota)\ \cos 2\Phi(t) \\ h_\times(t) = h_0\ \cos\iota\ \sin 2\Phi(t) \end{cases}$$



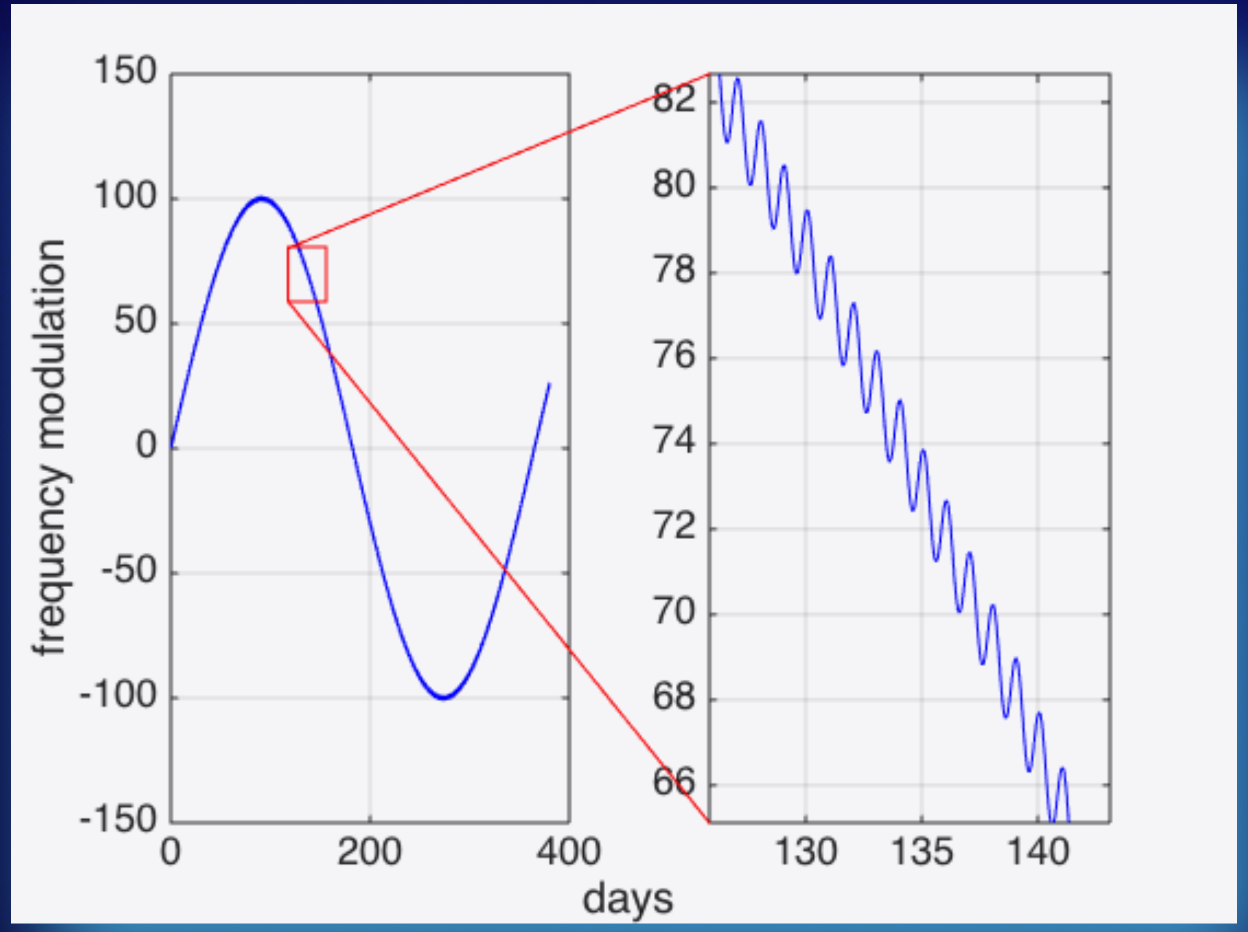
at the detector the observed frequency is not constant

τ at SSB, t at detector : $t(\tau)$

$$\Phi'(\tau(t)) = \Phi(t) \longleftrightarrow t(\tau) = \tau + \frac{\hat{n}\cdot\vec{r}}{c} + \cdots$$

Roemer, Shapiro Einstein delay

Ephemerides data



Detector response $h(t)$

for plane wave and size detector << reduced wavelength

$$h(t) = \boxed{F_+(t)}\, h_+(t) + \boxed{F_\times(t)}\, h_\times(t)$$

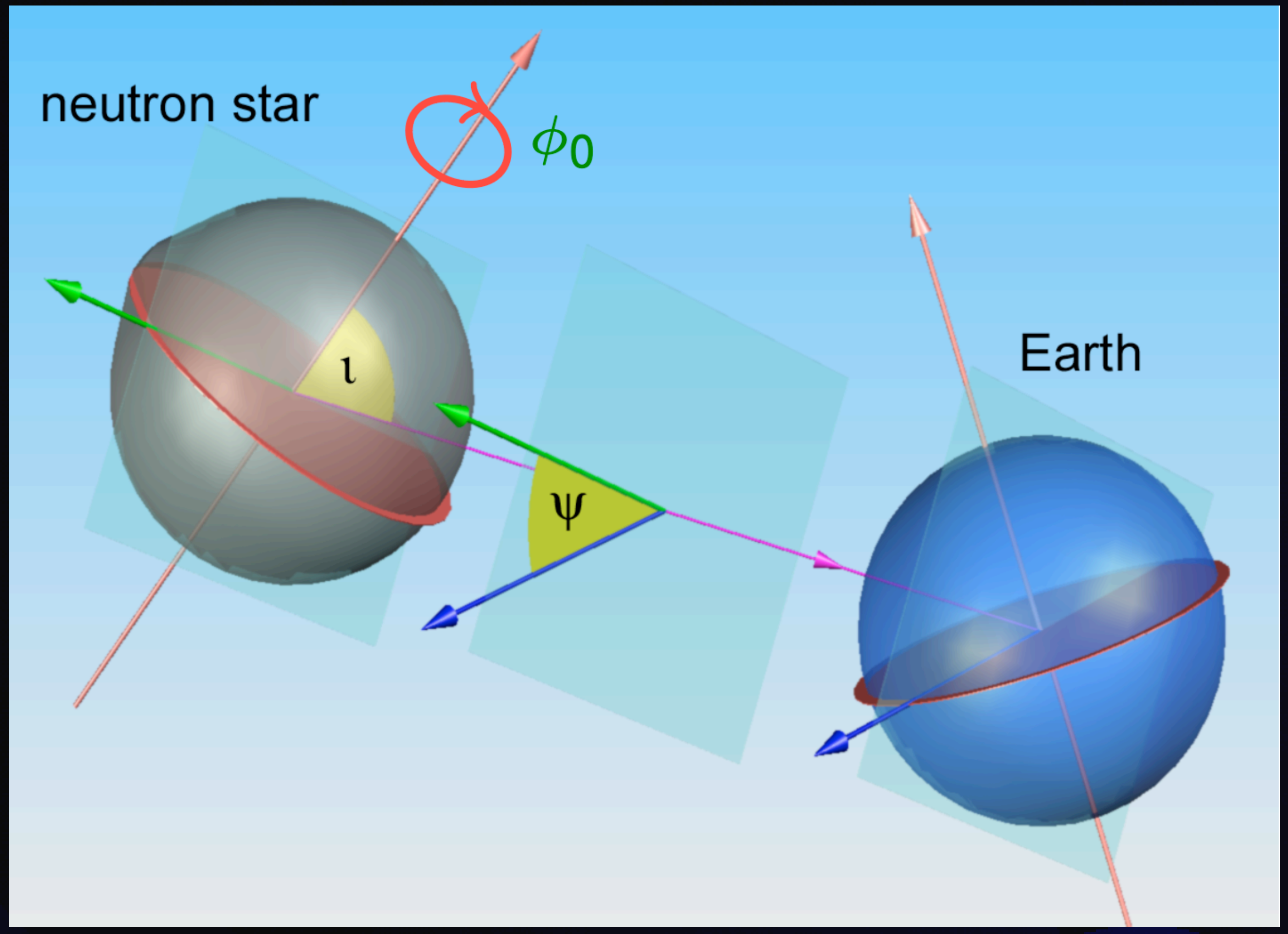
beam-pattern functions : the coupling of the wave with the detector

- Depends on mutual orientation of wave and detector (detector not equally sensitive to all directions and polarisations)
  - Depends on position of source and of detector
  - For a fixed sky position varies with time
  - Depends on the polarisation of wave

Beam pattern functions F_+ and $F_\times$

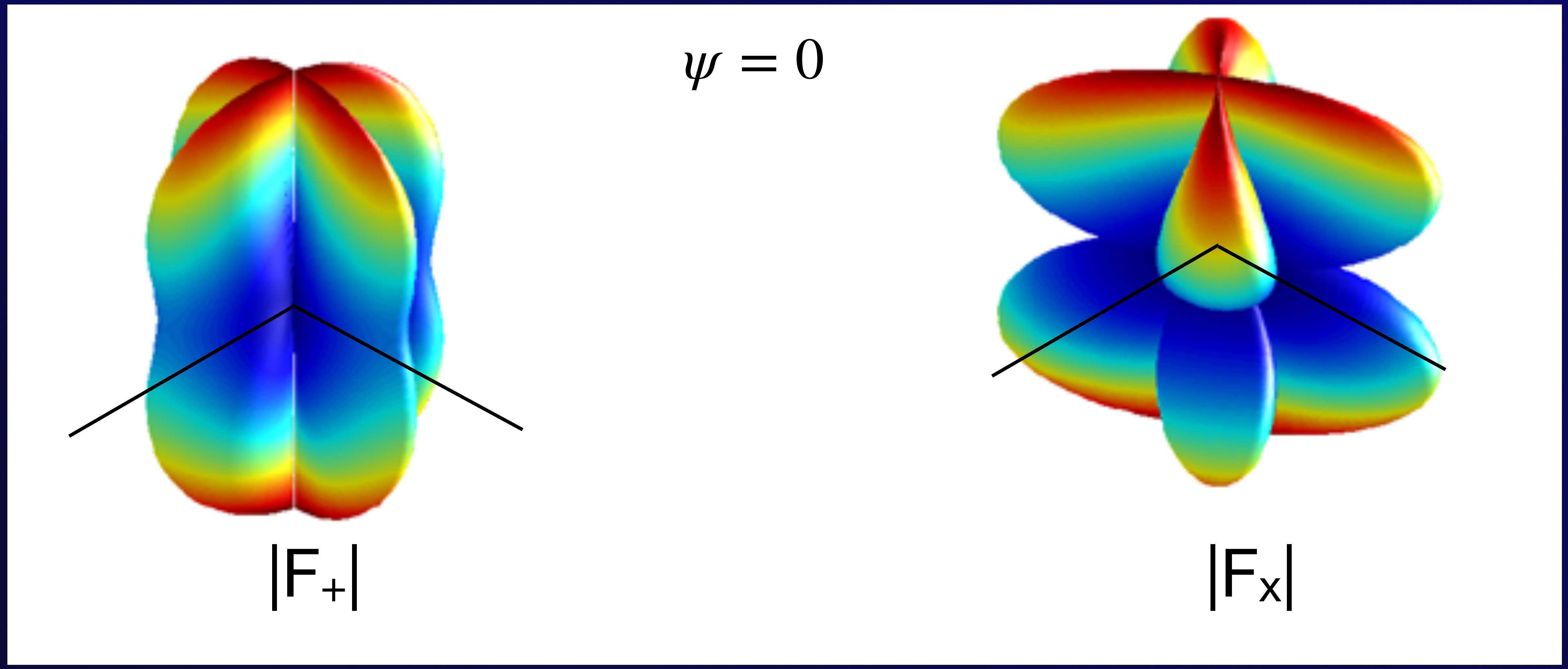
$$F_+(t) = \boxed{a(t)} \cos 2\psi + \boxed{b(t)} \sin 2\psi$$
$$F_\times(t) = \boxed{b(t)} \cos 2\psi - \boxed{a(t)} \sin 2\psi$$

(assuming perpendicular interferometer arms)



$$\begin{cases} a(t) = a(t; \text{det position, source position}) \\ b(t) = b(t; \text{det position, source position}) \end{cases}$$

have a periodicity of 1/2 sidereal day



Yunes and Siemens, Living Reviews in GR

so far:

$$h(t) = F_+(t)\ h_+(t) + F_\times(t)\ h_\times(t)$$

with

$$\begin{cases} F_+(t) = a(t)\ \cos 2\psi + b(t)\ \sin 2\psi \\ F_\times(t) = b(t)\ \cos 2\psi - a(t)\ \sin 2\psi \end{cases}$$

and

$$\begin{cases} h_+(t) = \frac{1}{2} h_0\ (1 + \cos^2 \iota)\ \cos 2\Phi'(t) \\ h_\times(t) = h_0\ \cos \iota\ \sin 2\Phi'(t) \end{cases}$$

so far:

$$h(t) = F_+(t)\ h_+(t) + F_\times(t)\ h_\times(t)$$

Re-parametrize →

with

$$\begin{cases} F_+(t) = a(t)\ \cos 2\psi + b(t)\ \sin 2\psi \\ F_\times(t) = b(t)\ \cos 2\psi - a(t)\ \sin 2\psi \end{cases}$$

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so far:

$$h(t) = F_+(t)\ h_+(t) + F_\times(t)\ h_\times(t)$$

Re-parametrize →

$$h(t) = \sum_{i=1\cdots4} A_i\ h_i(t)$$

with

$$\begin{cases} F_+(t) = a(t)\ \cos 2\psi + b(t)\ \sin 2\psi \\ F_\times(t) = b(t)\ \cos 2\psi - a(t)\ \sin 2\psi \end{cases}$$

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"amplitudes", as they only depend on h_0, ι, ψ, Φ_0

time-dependent part. Depends on $a(t), b(t), \Phi(t)$ so, fixed the detector, it depends on source's $\lambda = (\alpha, \delta, f_0, f_1, \ldots)$, the "phase-evolution" parameters.

JKS: Jaranowski, Krolak and Schutz, PRD 58 (1998)

Detection problem
frequentist approach

data $x(t) = h(t) + n(t)$

optimal (Neymann Pearson) detection statistic is any monotonic function of the likelihood Λ :

$$\Lambda(x; signal) = \frac{prob(x \,|\, signal)}{prob(x \,|\, noise)}$$

so also optimal $\log \Lambda$ is an optimal statistic.

A little reminder : the likelihood Λ is a random variable, with PDF $f_{\Lambda}(\lambda)$

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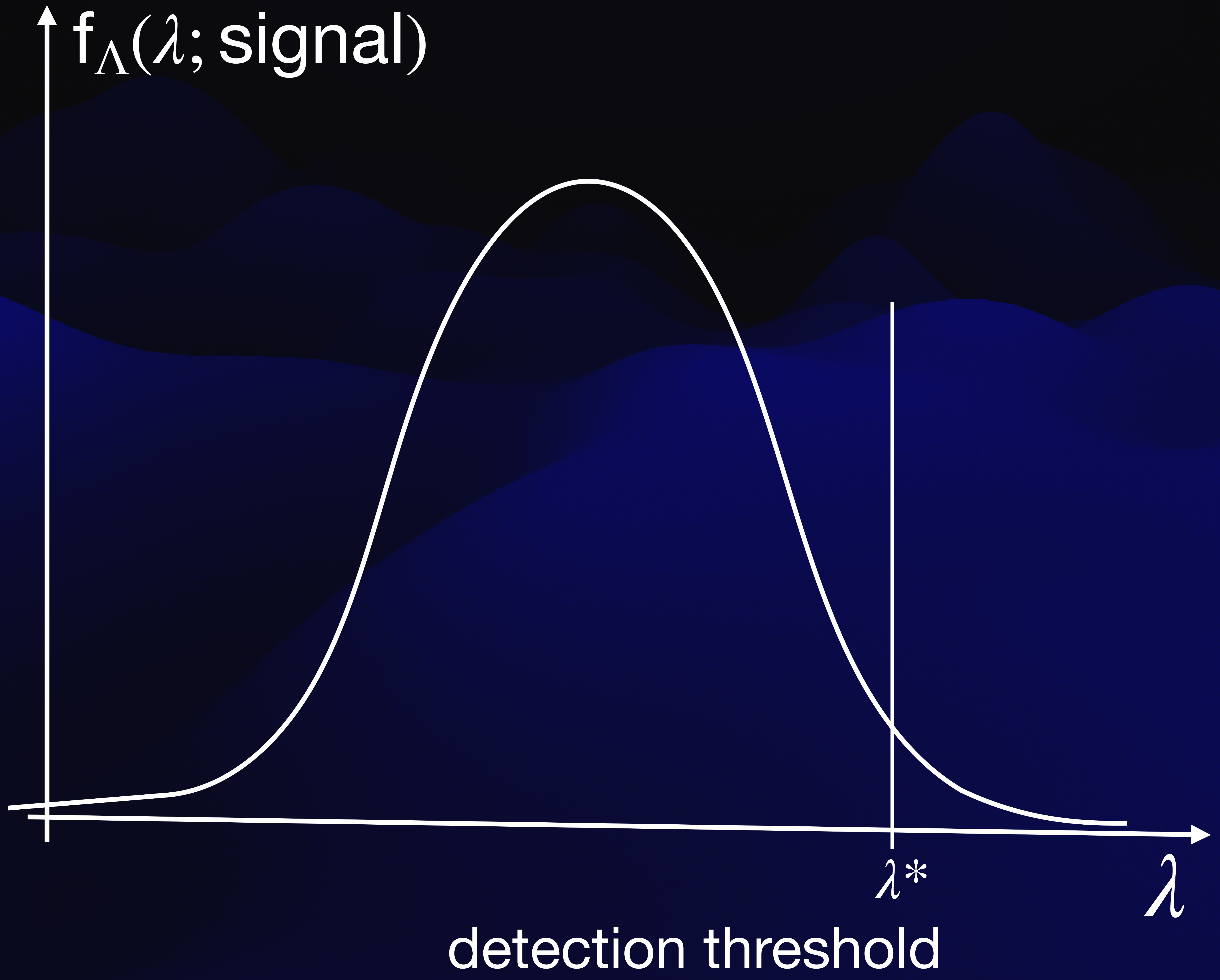
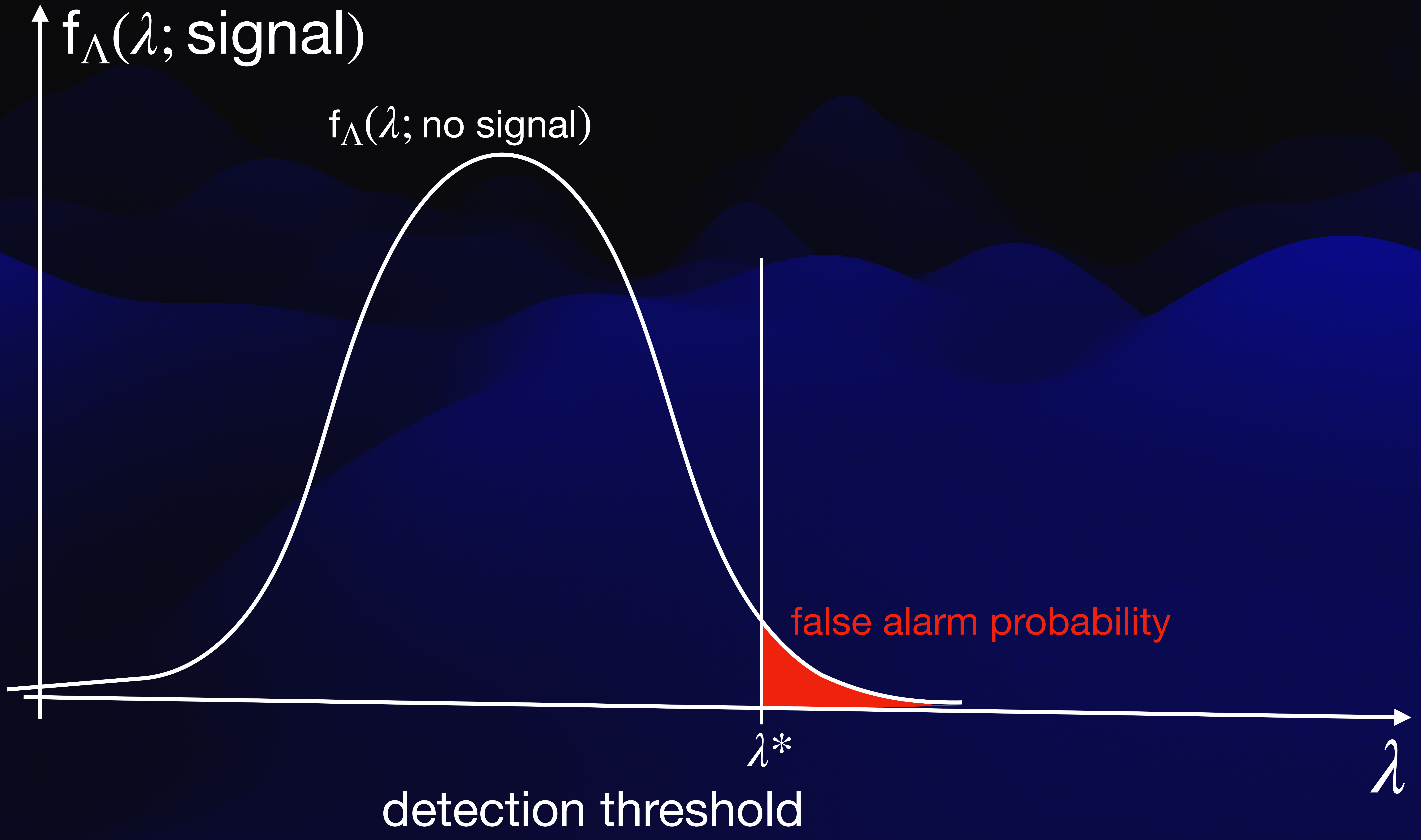


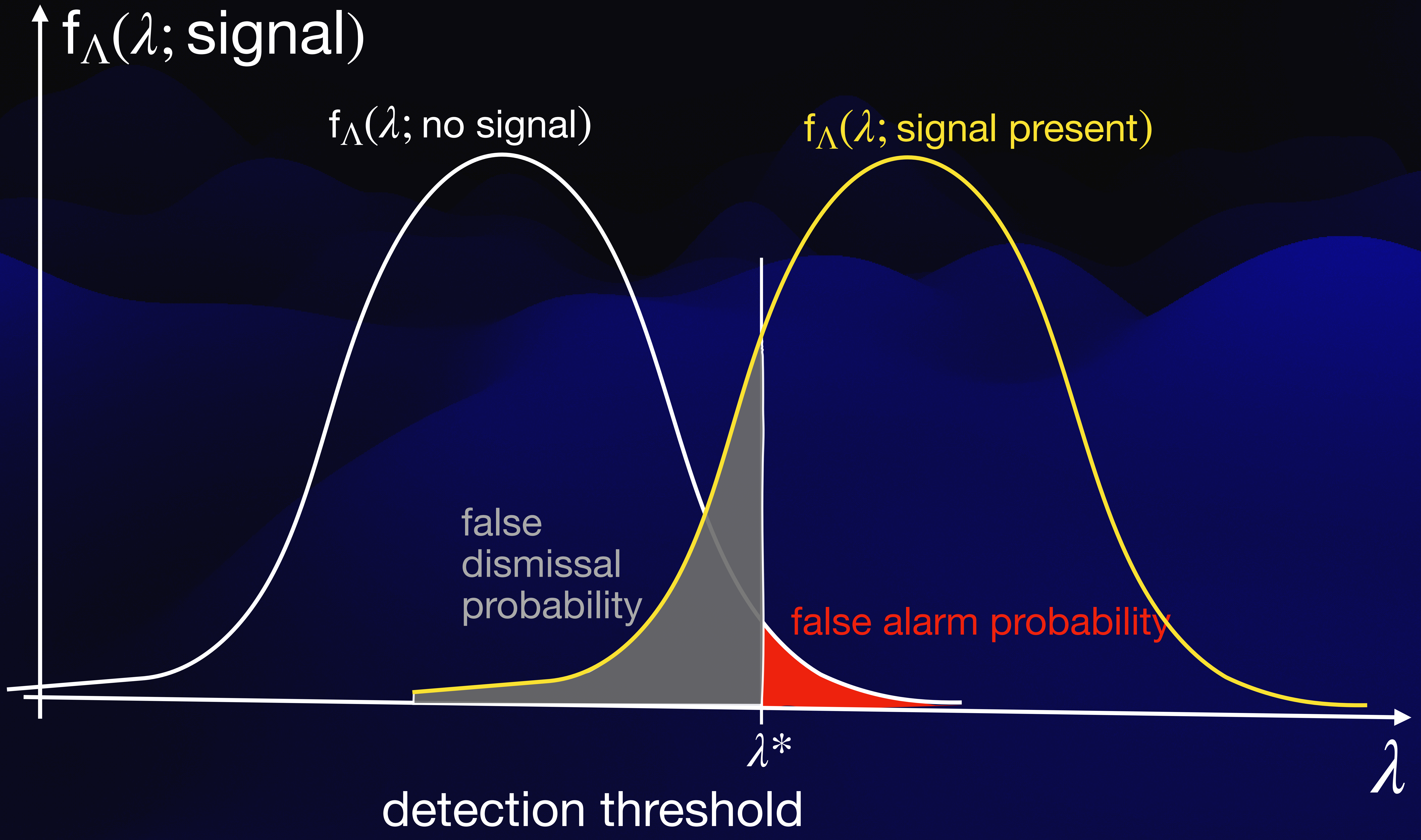
Neymann-Pearson detection

A little reminder



Neymann-Pearson detection

Optimal: smallest false dismissal at fixed false alarm



The log-likelihood

For stationary, zero-mean Gaussian noise

$$\log \Lambda(x;h) = (x\,|\,h) - (h\,|\,h)$$

For observation T, data containing noise with one-sided noise spectral density $S_h(f)$ and a narrow-band signal with frequency $\approx f_0$

$$(x\,|\,h) \simeq \frac{2}{S_n(f_0)} \int_T dt\ x(t)h(t)$$

The log-likelihood

Remember that

$$h(t) = \sum_{i=1\cdots4} A_i h_i(t)$$

"amplitude parameters" as they only depend on $A_i = A_i(h_0, \iota, \psi, \Phi_0)$

time-dependent "phase-evolution" parameters $\lambda = (\vec{n}_{sky}, f_0, f_1, \ldots)$

$$\Rightarrow \log \Lambda = A^i x_i - \frac{1}{2} A^i M_{ij} A^j$$

in Gaussian stationary noise

matched filters output $x_i(\lambda) := (x \,|\, h_i)$

antenna-pattern matrix $M_{ij}(\vec{n}_{sky}) := (h_i \,|\, h_j) = \gamma \begin{pmatrix} A & C & 0 & 0 \\ C & B & 0 & 0 \\ 0 & 0 & A & C \\ 0 & 0 & C & B \end{pmatrix}$

$\gamma = \frac{\mathrm{T}}{S_n}, A = < a^2 > , B = < b^2 > , C = < ab >$

The maximum log-likelihood
With respect to the amplitude parameters

$$\log \Lambda(x; A_i, \lambda) = A^i x_i(\lambda) - \frac{1}{2} A^i M_{ij}(\vec{n}_{sky}) A^j$$

maximising wrt A_i

$$2\mathcal{F}(x;\lambda) := \max_{\{A_i\}}[\log \Lambda(x; A_i, \lambda)]$$

re-plugging in the maximum likelihood estimators $\hat{A}^i = M^{ij} x_j$:

$$= x_i M^{ij} x_j =$$

$$= 2D^{-1}[B\mathcal{F}_A + A\mathcal{F}_B - 2C^2\mathcal{F}_C] \quad \text{with} \quad \begin{cases} \mathcal{F}_A = |F_a|^2 = |(x_1 - ix_3)/\gamma|^2 \\ \mathcal{F}_B = |F_b|^2 = |(x_2 - ix_4)/\gamma|^2 \\ \mathcal{F}_C = \mathcal{R}(F_a^* F_b)| \\ D = AB - C^2 \end{cases}$$

The max likelihood ratio

$$2\mathscr{F} = 2D^{-1}[B\mathscr{F}_A + A\mathscr{F}_B - 2C^2\mathscr{F}_C]$$

The max likelihood ratio

$$2\mathscr{F} = 2D^{-1} \ldots A\mathscr{F}_B - 2C^2\mathscr{F}_C]$$

The max likelihood ratio

$$2\mathcal{F} = 2D^{-} \quad \mathcal{F}_A \quad A\mathcal{F}_B - 2C^2\mathcal{F}_C]$$

$$\mathcal{F}_a = |F_A|^2 \sim |x_1 - ix_3|^2$$

$$\begin{cases} x1 = (x|h_1) & h_1(t) = a(t)\cos 2\Phi(t) \\ x3 = (x|h_3) & h_3(t) = a(t)\sin 2\Phi(t) \end{cases} \quad \longrightarrow \quad F_A = \sim \left| \int_T x(t)\ a(t)\ e^{-i2\Phi(t)}\ dt \right|^2$$

Distribution of $2\mathscr{F}$

- Chi-square distribution with 4 degrees of freedom, χ^2_4

sin, cos, + 2 polarisations

- When a signal at the searched parameters is present, a non-centrality parameter arises: $\rho^2_{opt} = (h^{sig} \,|\, h^{sig})$, that JKS call the optimal SNR²

depends on amplitude parameters,
on relative source-detector position,
on detector noise, on amount of data

- If the search template does not exactly match signal parameters the resulting non-centrality parameter $\rho^2 = (h^{sig} \,|\, h^{templ}) \leq \rho^2_{opt} = (1-\mu)\ \rho^2_{opt}$ where μ is the mismatch

The optimal ρ^2

relation with h_0

$$\rho^2_{opt} = \frac{h_0^2}{S_n}\left[\frac{1}{4}(1+\cos^2\iota)^2\int_{\mathrm{T}} F_+^2\, dt + \cos^2\iota \int_{\mathrm{T}} F_\times^2\, dt\right]$$

$$\simeq \frac{h_0^2}{S_n}\,[G_1(\delta,\psi,\iota)\mathrm{T} + G_2(\alpha,\delta,\psi,\iota;\mathrm{T})]$$

$$< \rho^2_{opt} >_{\alpha,\delta,\psi,\iota} \simeq \frac{4}{25}\frac{h_0^2\,\mathrm{T}}{S_n}$$

With 1% false alarm and 10% false dismissal $h_0^{detectable} \approx 11.4\sqrt{\frac{S_n}{\mathrm{T}}}$ in a 1-template search.

In practice

- Data (Short-Fourier-Trasforms, SFTs)
- Signal model (template):
  $\alpha, \delta, f_0, f_1, \ldots,$ binary parameters

lalapps_ComputeFstat

→ $2\mathscr{F}(data; signal)$

In practice

- Data (Short-Fourier-Trasforms, SFTs)
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lalapps_ComputeFstat → $2\mathscr{F}(data; signal)$

You get “for free” the search and maximisation over the amplitude parameters. You just have to search explicitly over phase-evolution parameters.

Short digression

on other methods and detection confidence

COHERENT SEARCHES — METHODOLOGIES

14 Aug 2003

Setting upper limits on the strength of periodic gravitational waves using the first science data from the GEO 600 and LIGO detectors

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First LSC paper put on the arXiv was this CW paper !

COHERENT SEARCHES — METHODOLOGIES

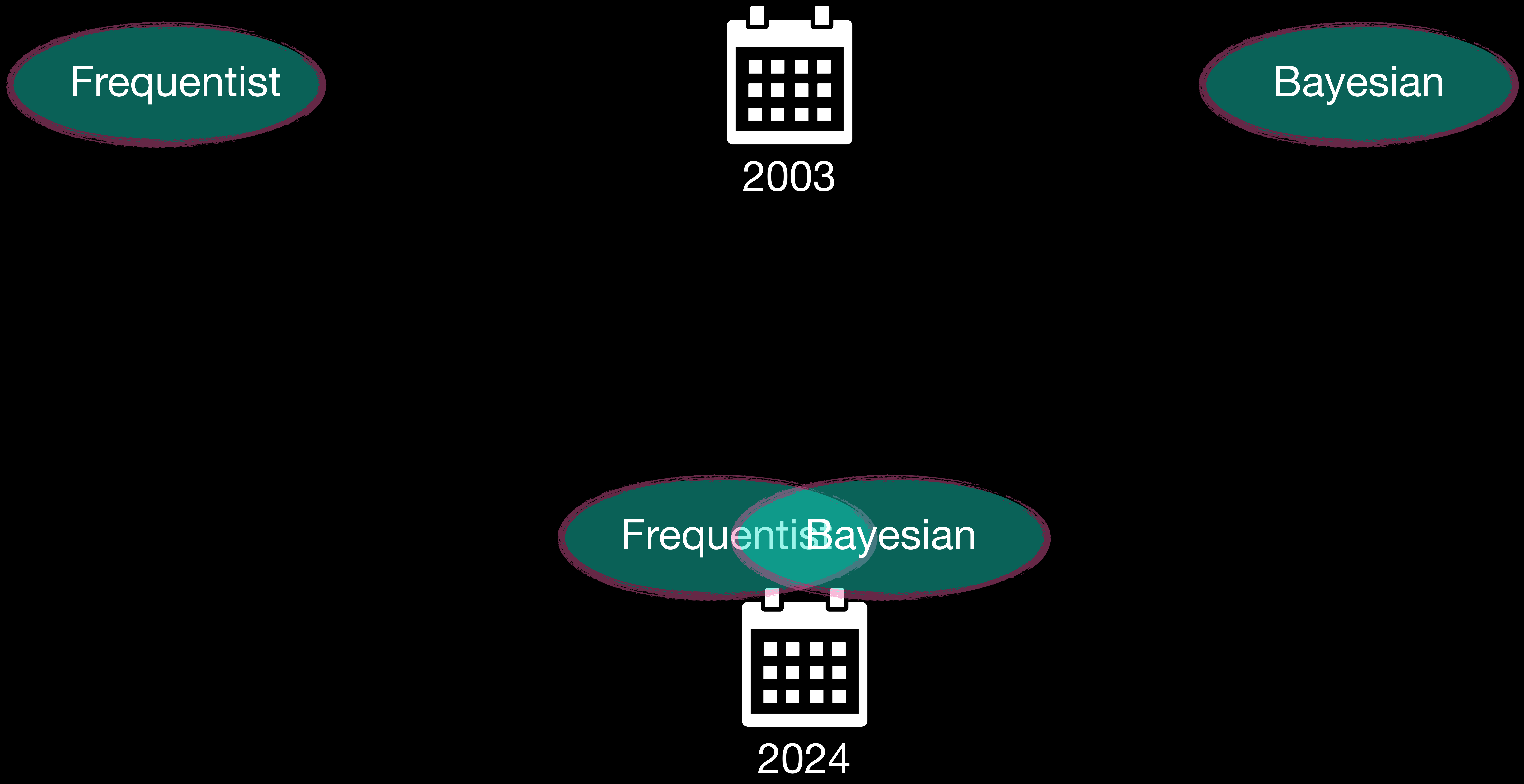
Setting upper limits on the strength of periodic gravitational waves using the first science data from the GEO 600 and LIGO detectors

Data collected by the GEO 600 and LIGO interferometric gravitational wave detectors during their first observational science run were searched for continuous gravitational waves from the pulsar J1939+2134 at twice its rotation frequency. Two independent analysis methods were used and are demonstrated in this paper: a frequency domain method and a time domain method. Both achieve consistent null results, placing new upper limits on the strength of the pulsar's gravitational wave emission. A model emission mechanism is used to interpret the limits as a constraint on the pulsar's equatorial ellipticity.

PACS numbers: 04.80.Nn, 95.55.Ym, 97.60.Gb, 07.05.Kf

First LSC paper put on the arXiv was this CW paper !

COHERENT SEARCHES — METHODOLOGIES



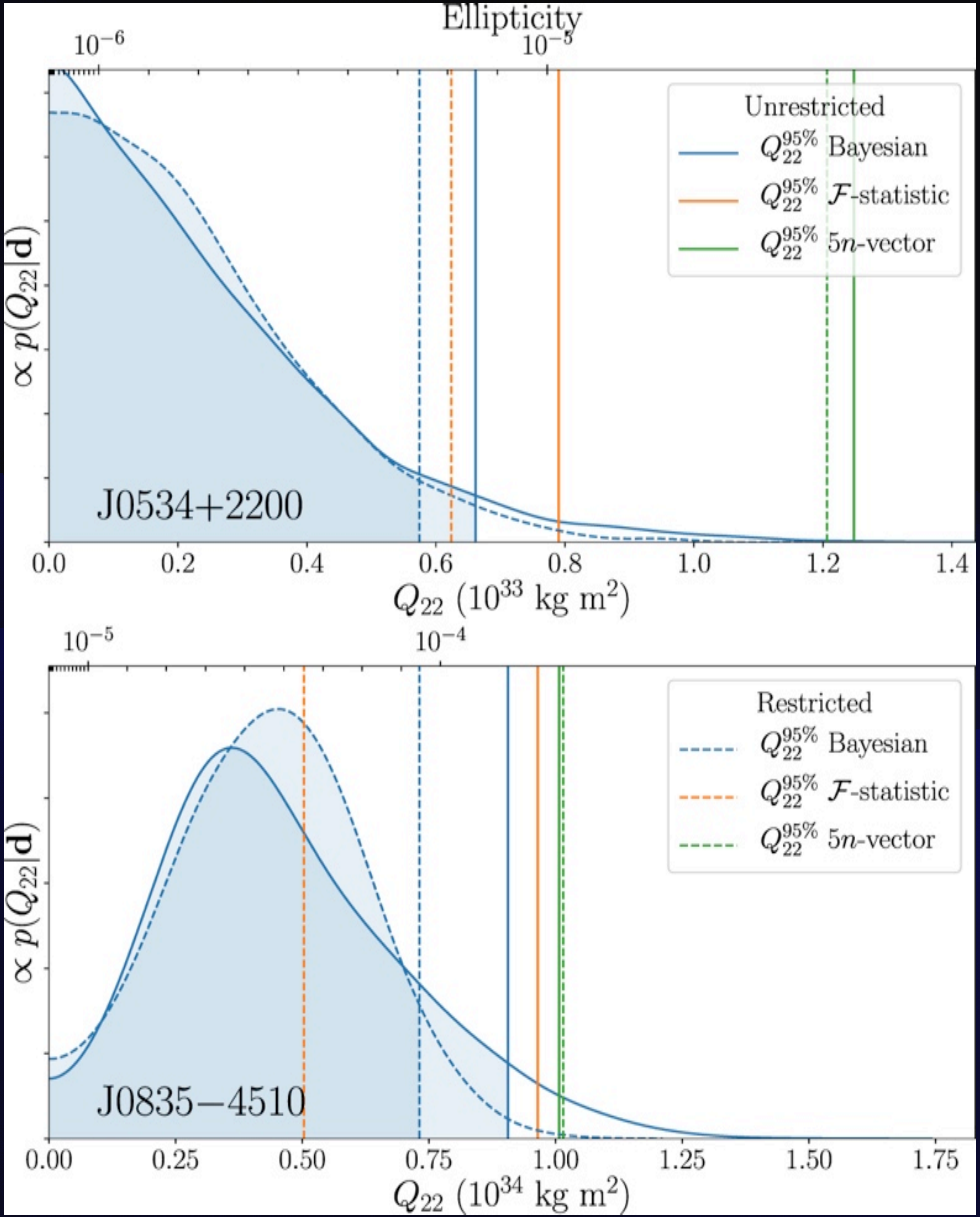
Bayesian

Posterior probability of a given signal h, given the data x

prob$(h; x) \propto$ prob$(s) \cdot$ prob$(x; s)$

- Heterodyne the data (according to phase parameters) and remove the frequency modulation, band-pass: $x \rightarrow x'$
- $p(h_0; x') \propto \iiint p(x'; h_0, \Phi_0, \psi, \cos\iota) \times p(\Phi_0)\ d\Phi_0\ p(\psi)\ d\psi\ p(\cos\iota)\ d\cos\iota$
- Explicitly have to evaluate likelihood for all values of the amplitude parameters, less efficient than $\mathscr{F}$-stat, so used for known-pulsar searches only.

Dupuis&Woan, PRD 72 (2005), Pitkin et al, arXiv:1603.00412 (2012), Pitkin et al arXiv:1705.08978 (2017), Pitkin et al, PRD 98 (2018)



Example posteriors

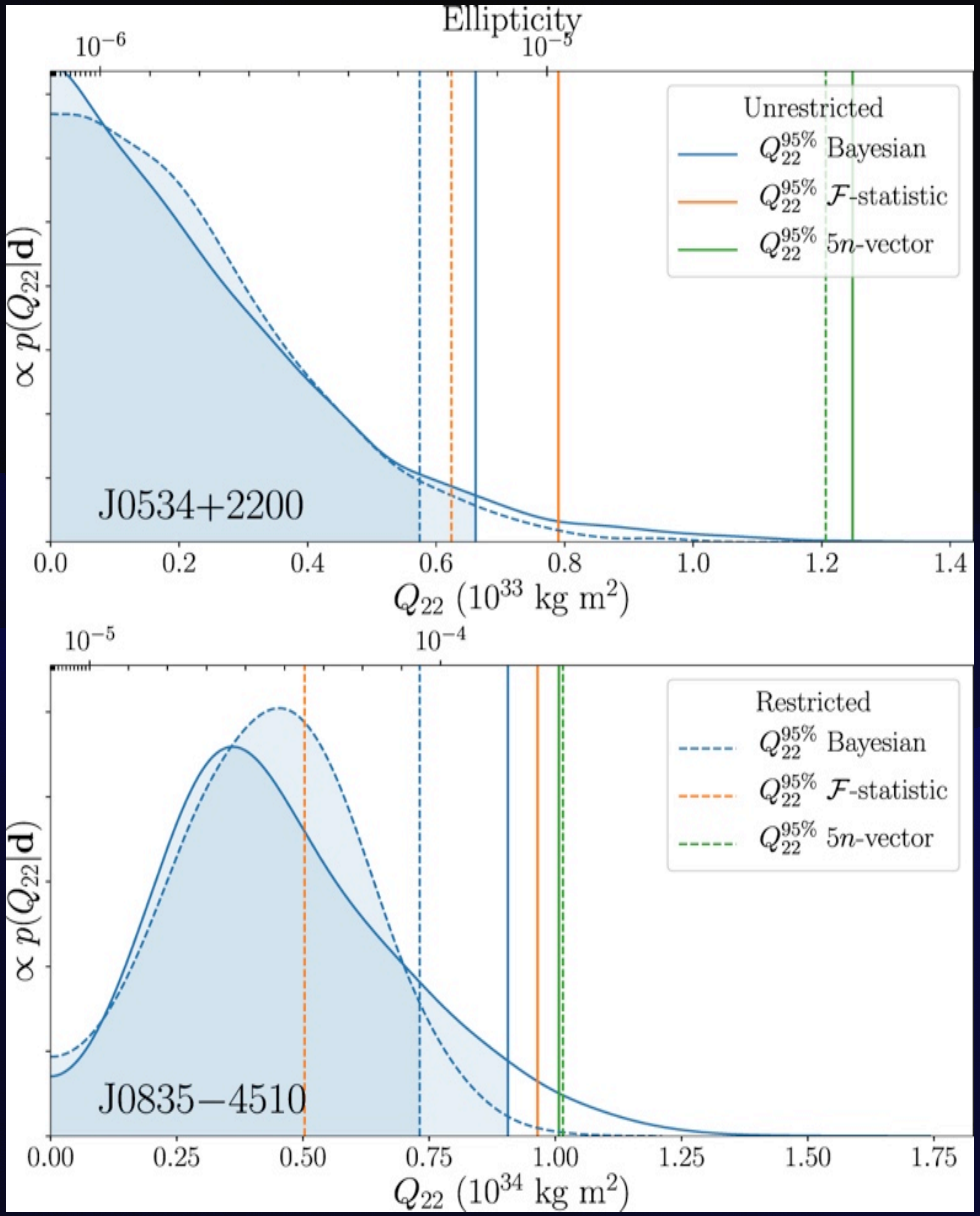
for the Crab and Vela pulsars

and actually $h_0 \longrightarrow Q_{22}$

LVC, ApJ Lett 902, L21 (2020)

Example posteriors

for the Crab and Vela pulsars

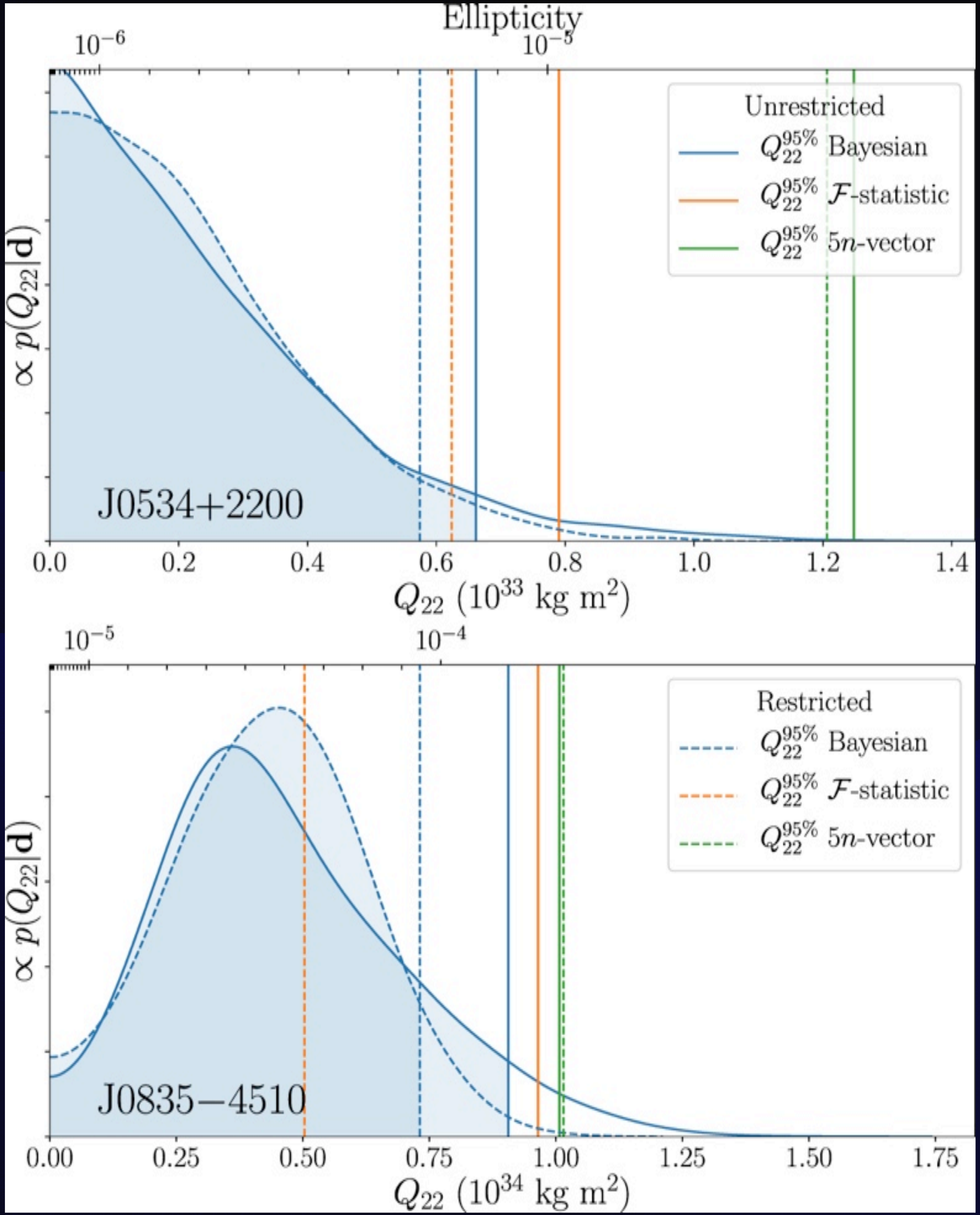


does this look like a signal ?

LVC, ApJ Lett 902, L21 (2020)

Establishing detection confidence

- Would it be significant in Gaussian noise ?
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
  - Estimating the background



"not disjoint from zero"
"not uncommon for pure Gaussian noise"

LVC, ApJ Lett 902, L21 (2020)

Establishing detection confidence

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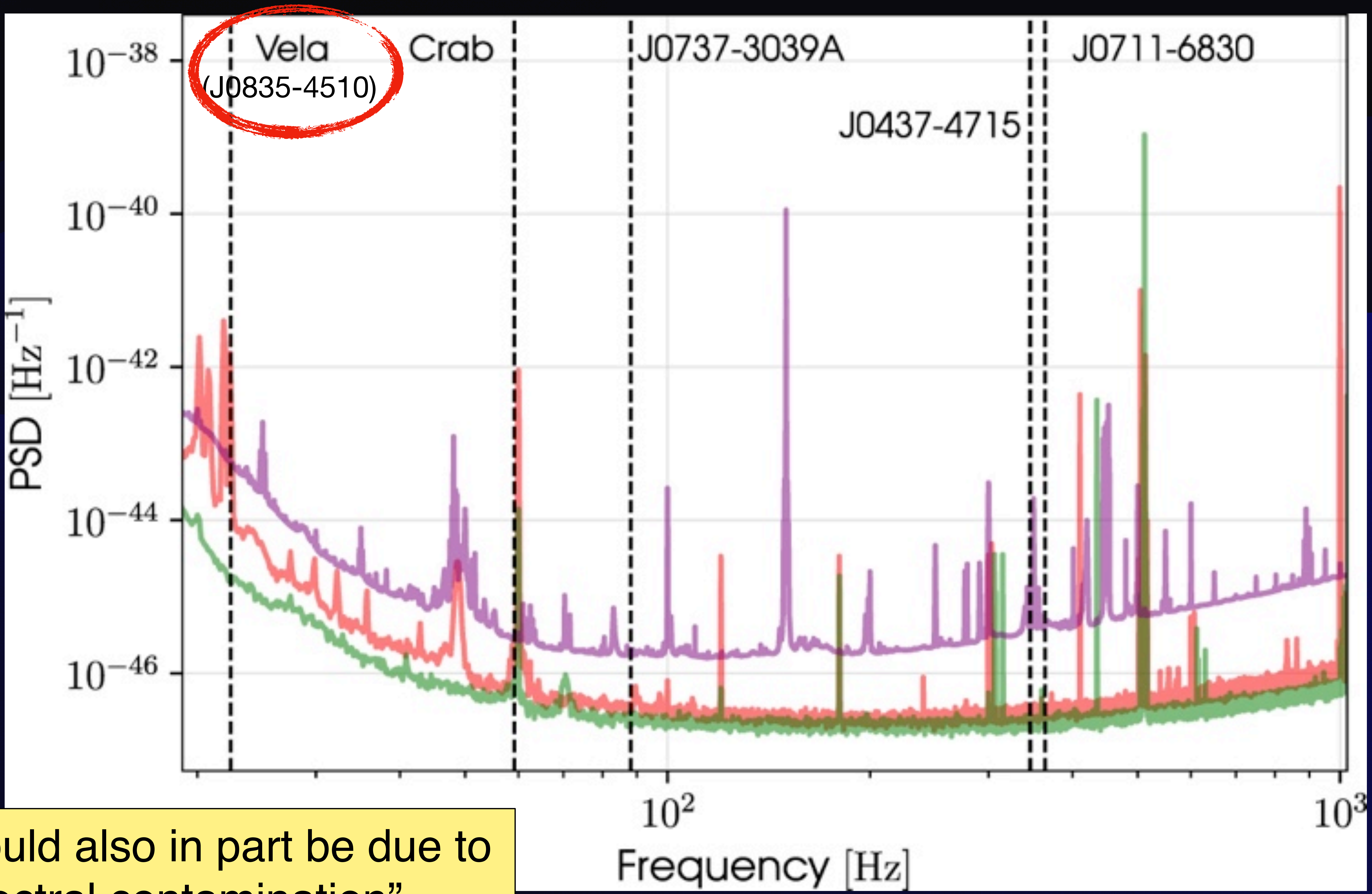
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
    - Estimating the background

Establishing detection confidence



"could also in part be due to spectral contamination"

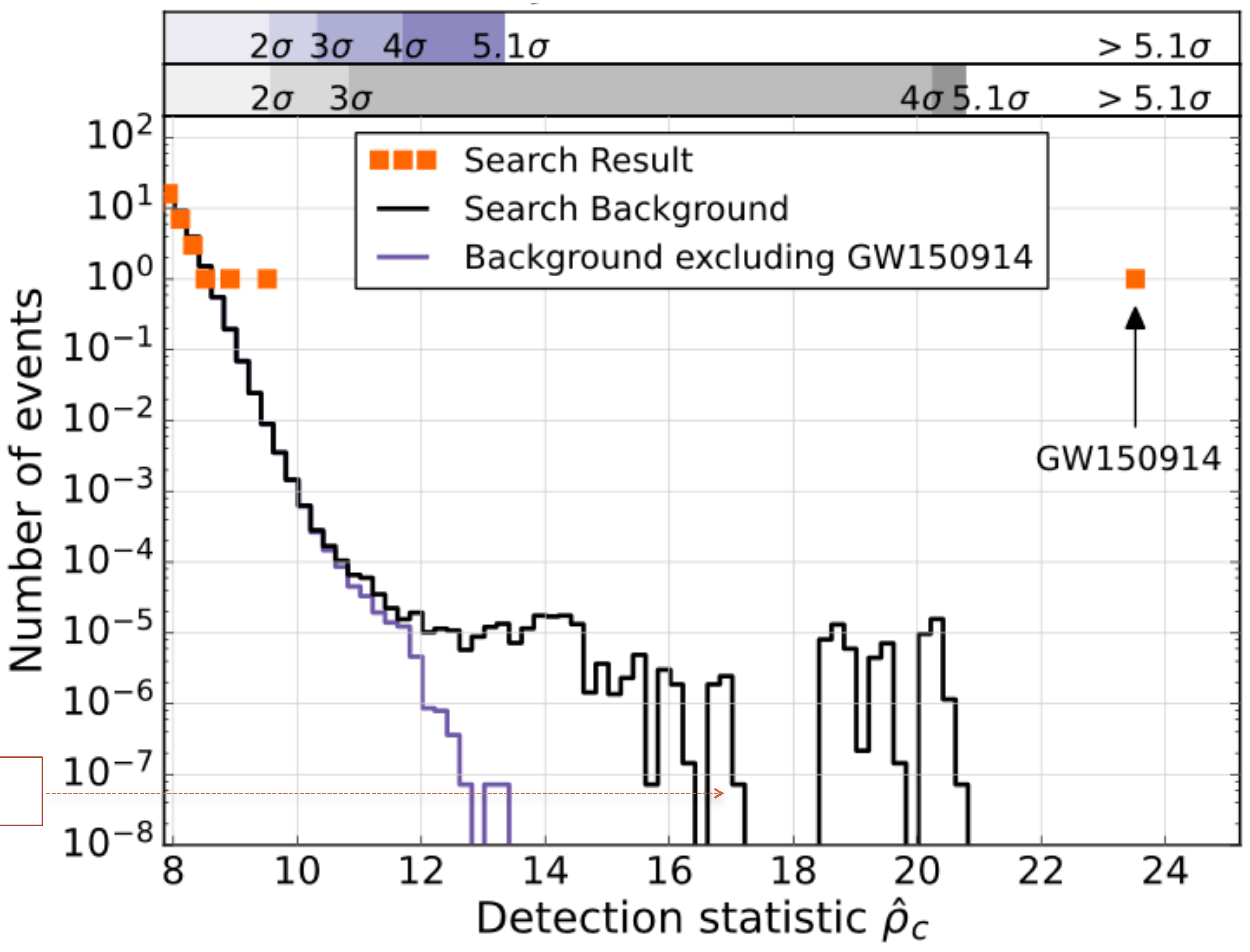
Establishing detection confidence

- Would it be significant in Gaussian noise ?
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
  - Estimating the background

The first GW detection

[Observation of Gravitational Waves from a Binary Black Hole Merger]
Phys.Rev.Lett. 116 (2016)

1.4 x 10^7 time slides corresponding to 608 000 yrs of simulated background.



$7 \times 10^{-8} \approx (1.4 \times 10^7)^{-1}$

LVC, GW discovery paper

Establishing detection confidence

Background estimate

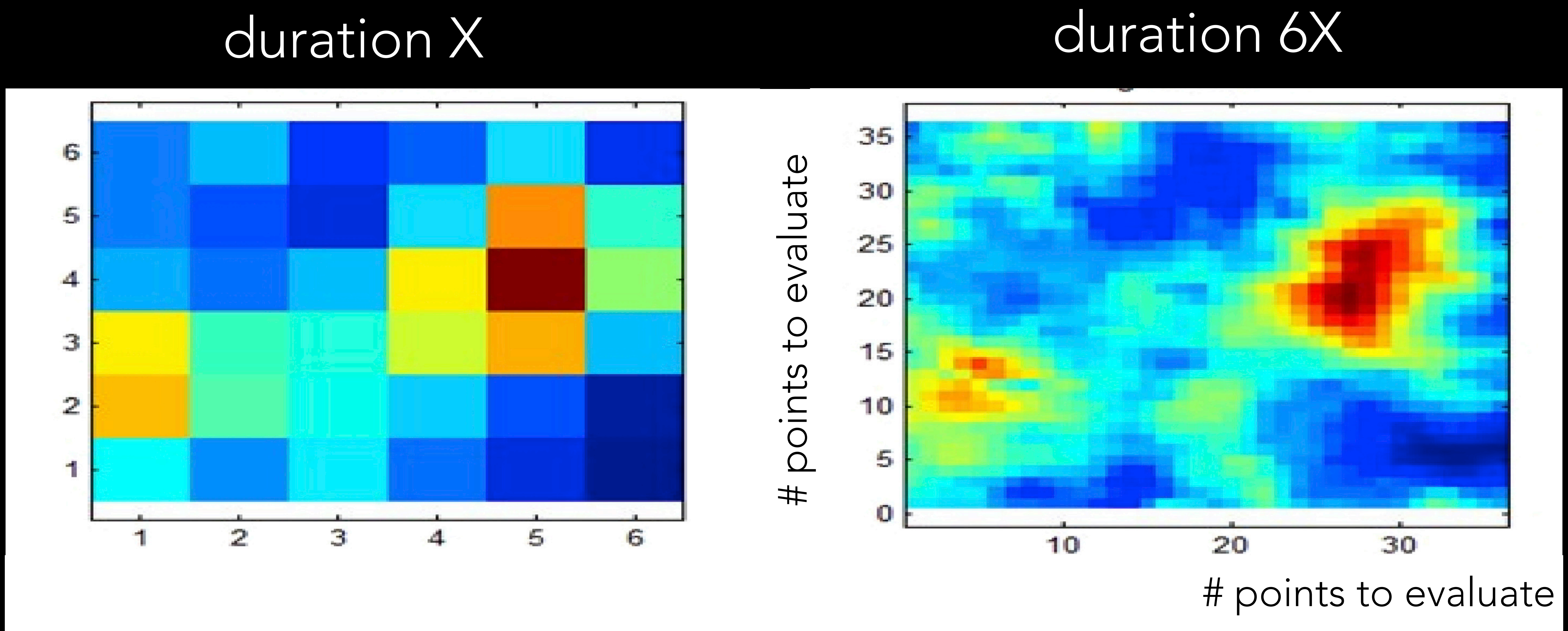
- For a search for emission from a known pulsar it should be possible to estimate the background:
  - Repeating the same search many times "off-source"
    - near-by frequencies (extensive literature)
    - different sky positions, Isi et al, arXiv:2010.12612 (2020)
- Not so simple for other types of continuous wave searches

IF I DO NOT KNOW SOURCE

- I assume a source position, spin, spin derivative
- Compute $\mathscr{F}$-stat
- Next set of possible source parameters, repeat

BUT....

- number of potential sources to evaluate quickly grows with observation time ($\propto T^5$ for a typical all-sky search)

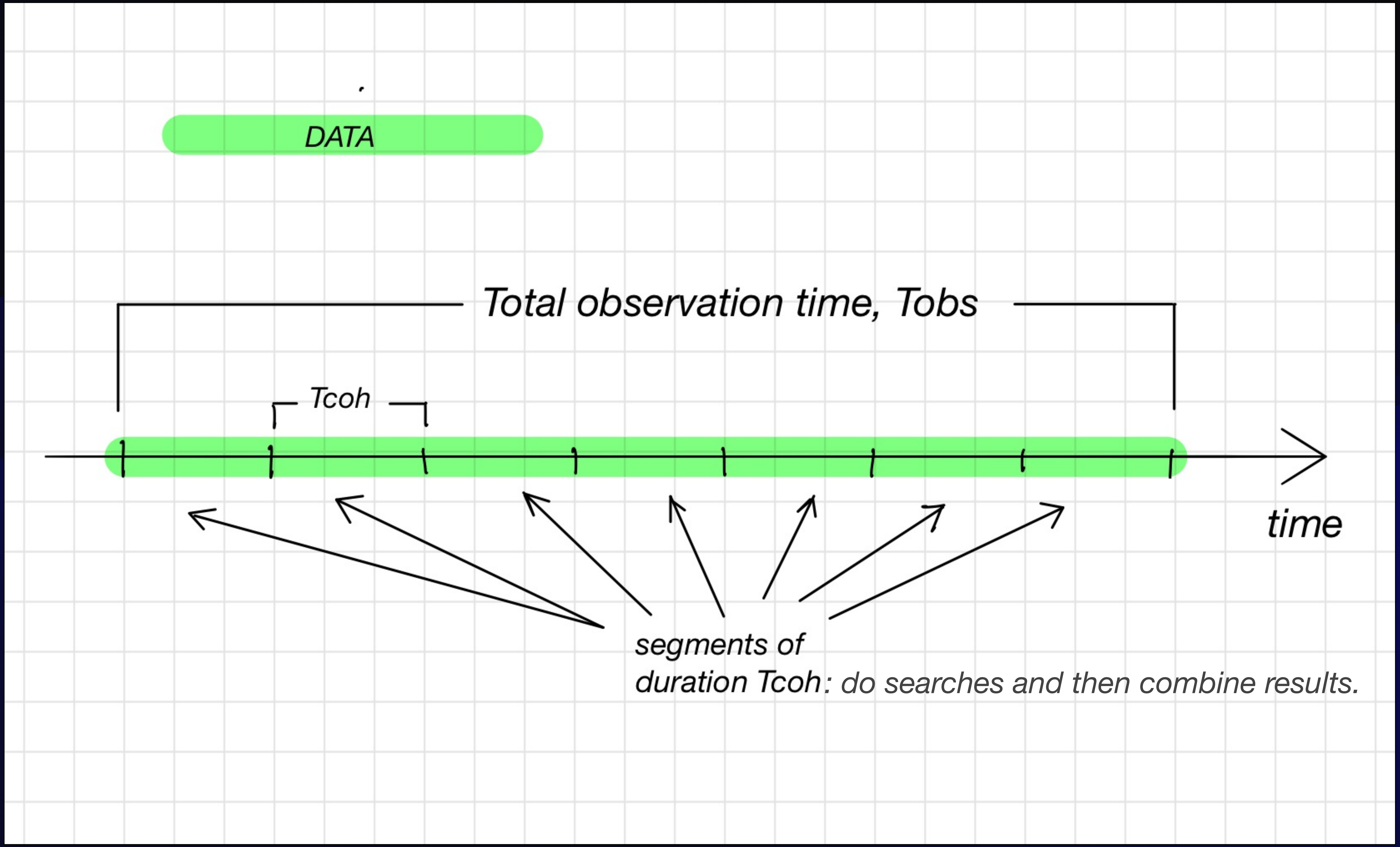


same patch of sky

$\approx 10^{28}$ WAVEFORMS RESOLVABLE WITH 6 MONTHS OF DATA

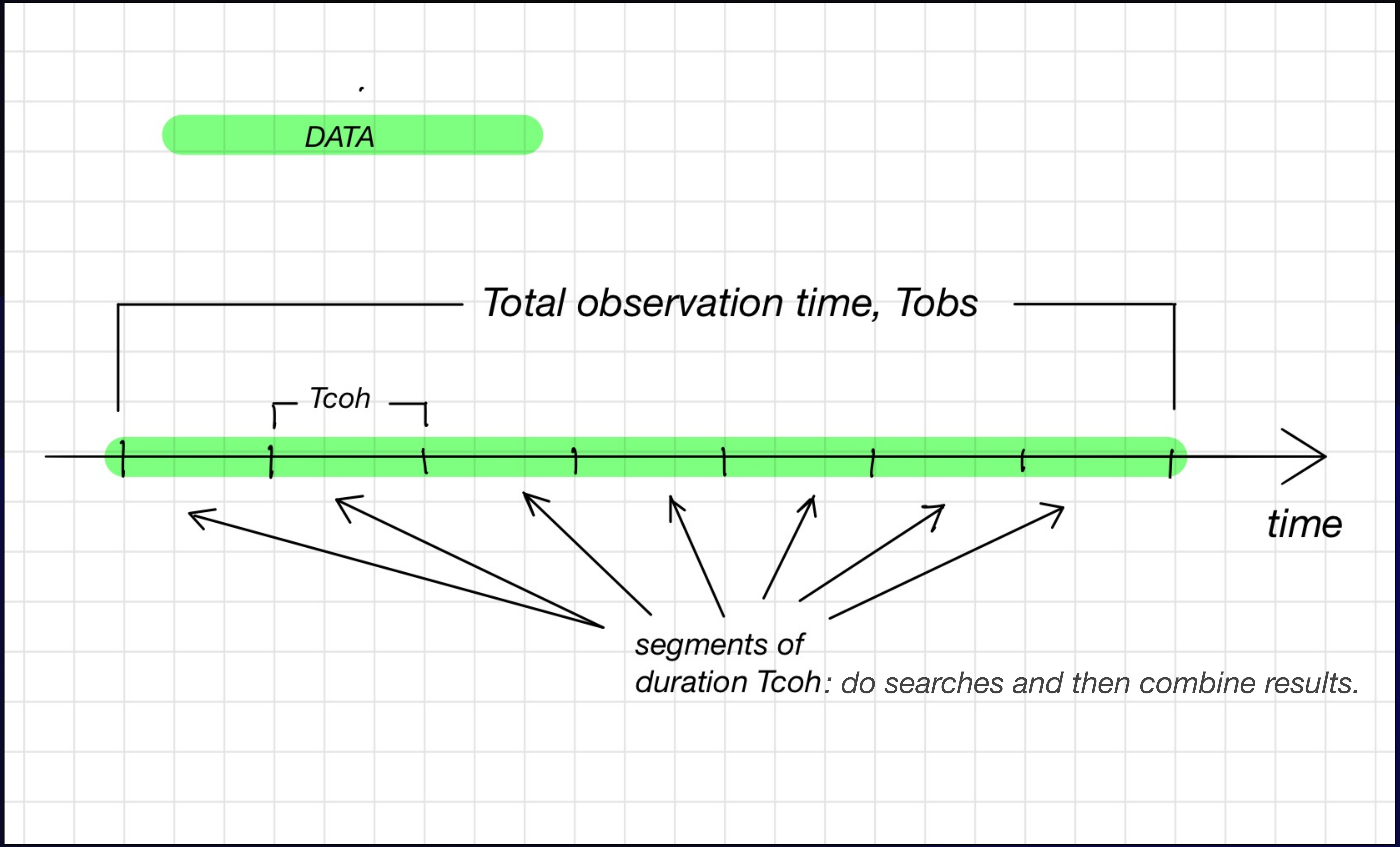
OPTIMAL (FULLY COHERENT) SEARCH METHODS CANNOT BE USED

Semi-coherent searches



Brady et al, PRD 57 (1998), Brady&Creighton, PRD 61 (2000), Krishnan et al, PRD70 (2004), Dhurandhar et al, PRD 77 (2008), Astone et al, PRD90 (2014), Walsh et al, PRD 94 (2016), O. Piccinni et al, CQG 36 (2019), Dergachev&Papa, PRL 123 (2019)

Semi-coherent searches



ROUGH ESTIMATES :

$$\mathrm{SNR}^{coh} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{\mathrm{T}_{coh}} \sqrt{\mathrm{N}_{seg}}$$

$$\mathrm{SNR}^{semi-coh} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{\mathrm{T}_{coh}} \sqrt[4]{\mathrm{N}_{seg}}$$

$$\text{Template-count}^{coh} \propto \mathrm{T}^5_{coh} \mathrm{N}^5_{seg}$$

$$\text{Template-count}^{semi\text{-}coh} \propto \mathrm{T}^5_{coh} \mathrm{N}_{seg}$$

180 1-day segments: 3.7 less sensitive and a billion times fewer templates.

SEARCH MORE WAVEFORMS

SENSITIVITY OF SEARCH DECREASES

POSSIBLE WAVE SHAPES ↑, SENSITIVITY ↓

Data + Signal model (template):

$\alpha, \delta, f_0, f_1, \ldots,$ binary parameters

lalapps_ComputeFstat

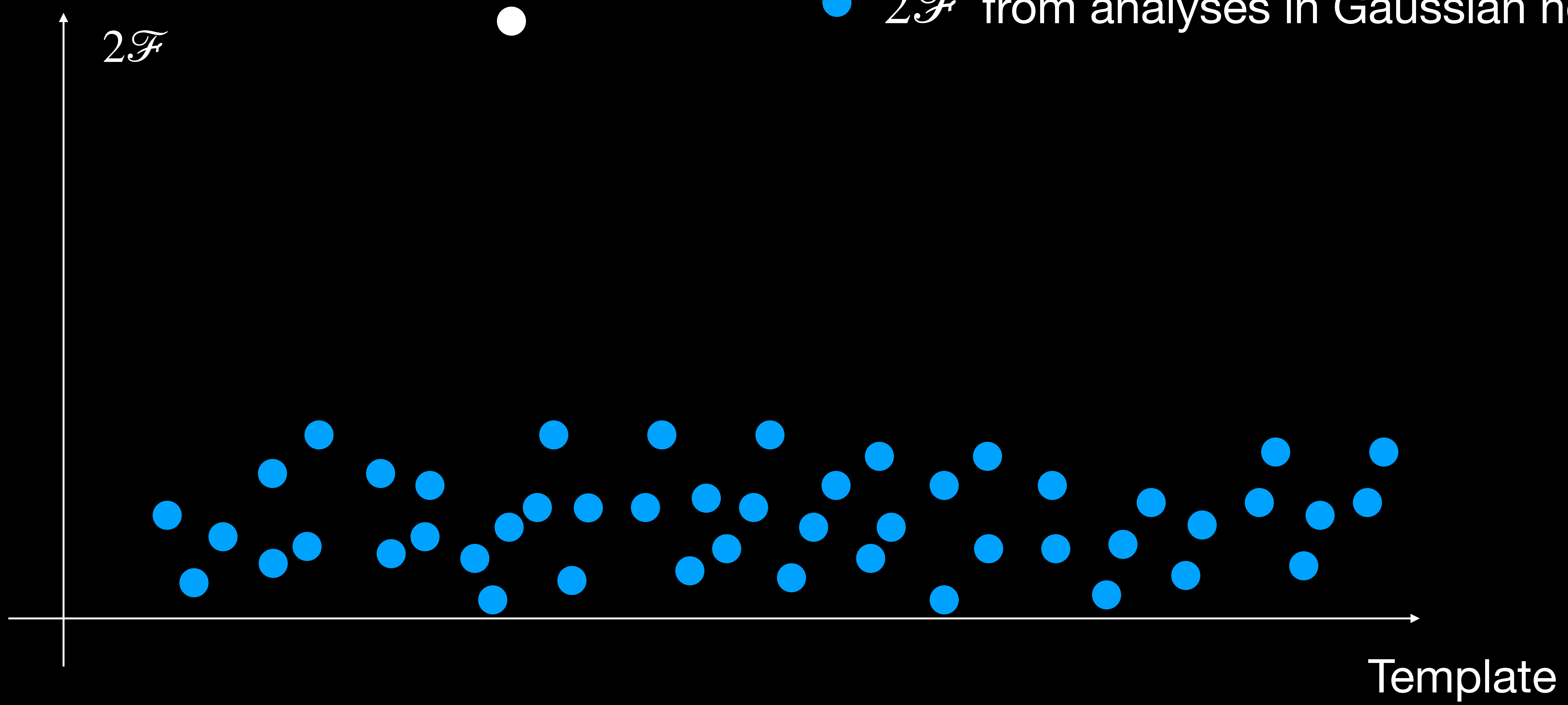
→ $2\mathscr{F}(data; signal)$

it's a score that tells you how likely is it that the data contains that signal. The higher it is, the more likely it is.

1-template search

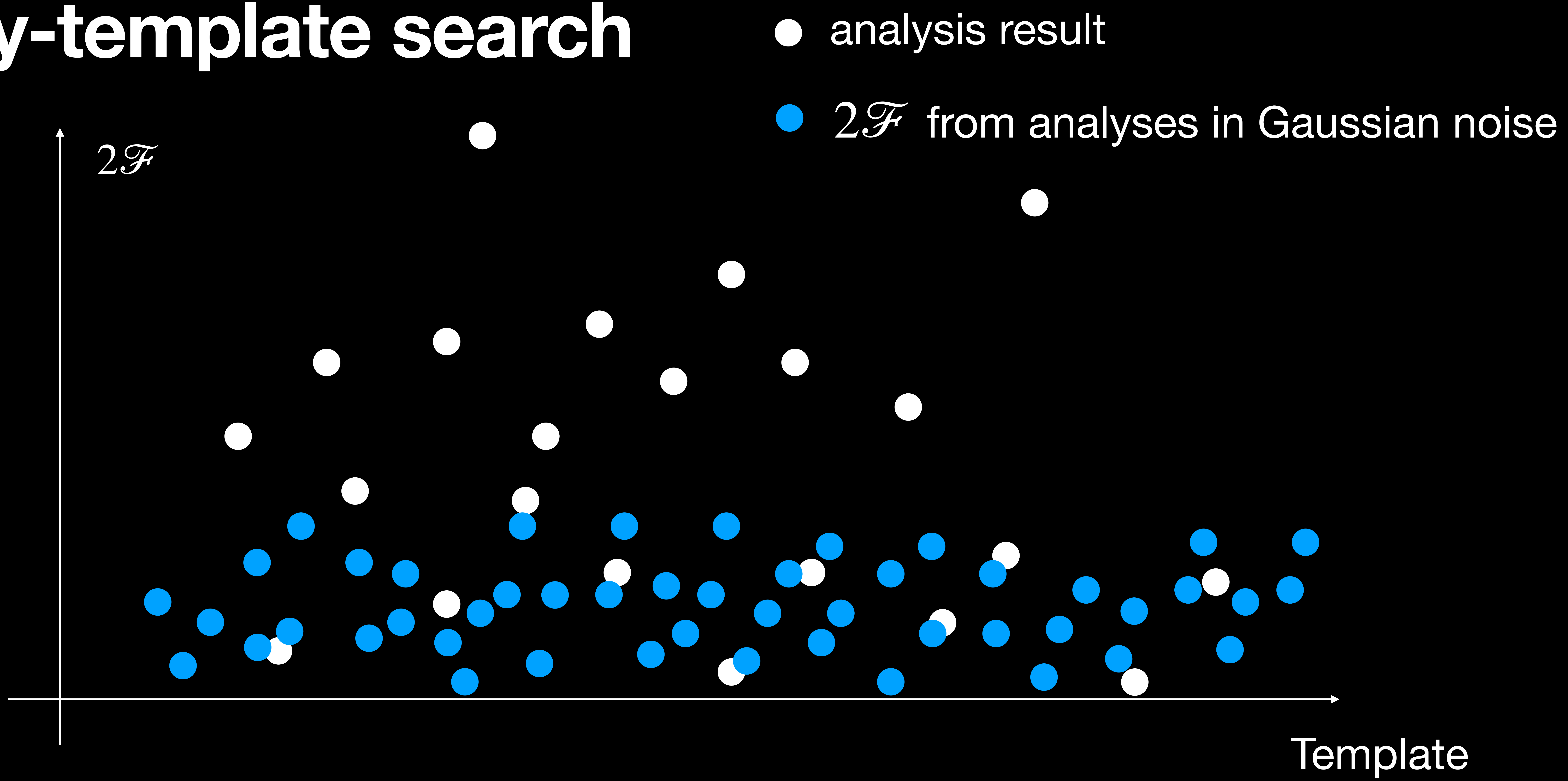
● analysis result

● $2\mathscr{F}$ from analyses in Gaussian noise



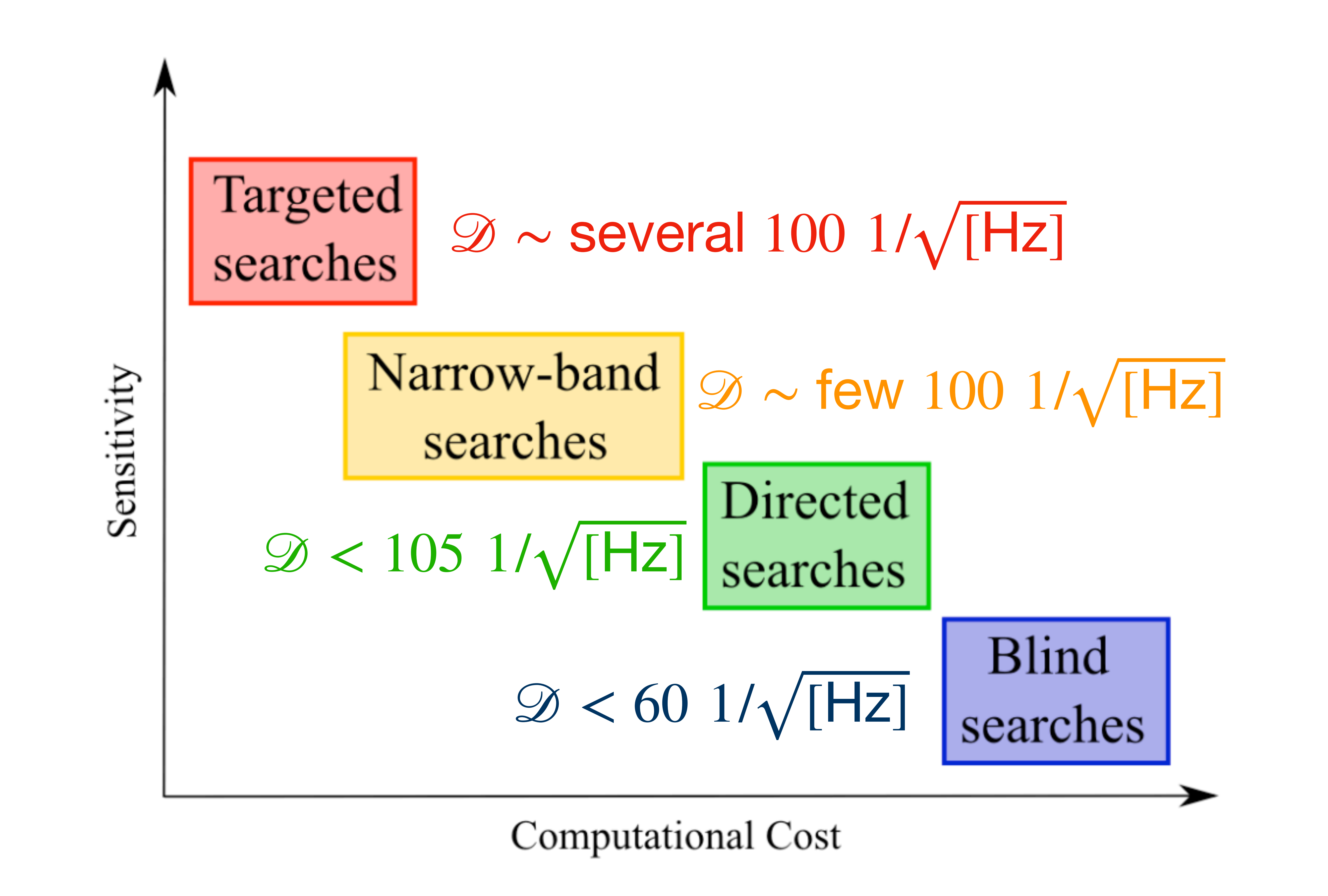
Search for weak/rare signal

many-template search



Search for weak/rare signal

POSSIBLE WAVE SHAPES ↑, SENSITIVITY ↓



End of Lecture I