Gravitational drag forces — the numerical way

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Based on works with Zipeng Wang, Katy Clough, Rodrigo Vicente, Emanuele Berti, Pedro Ferreira

Thomas Helfer and Lam Hui

Джа-джа

New horizons for Psi workshop,

Instituto Superior Técnico Lisbon,

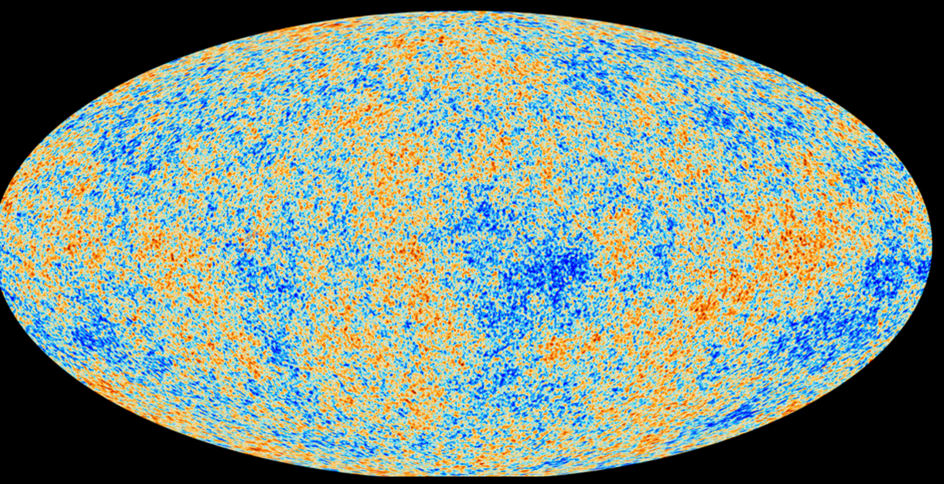
5th July 2024

Max-Planck-Institut für
Gravitationsphysik
Albert-Einstein-Institut

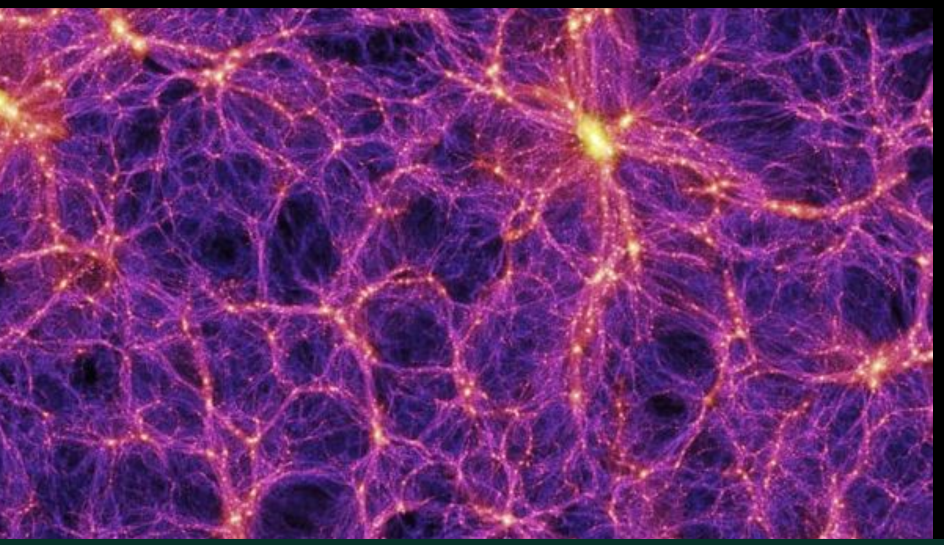
ΛCDM — the standard cosmological model

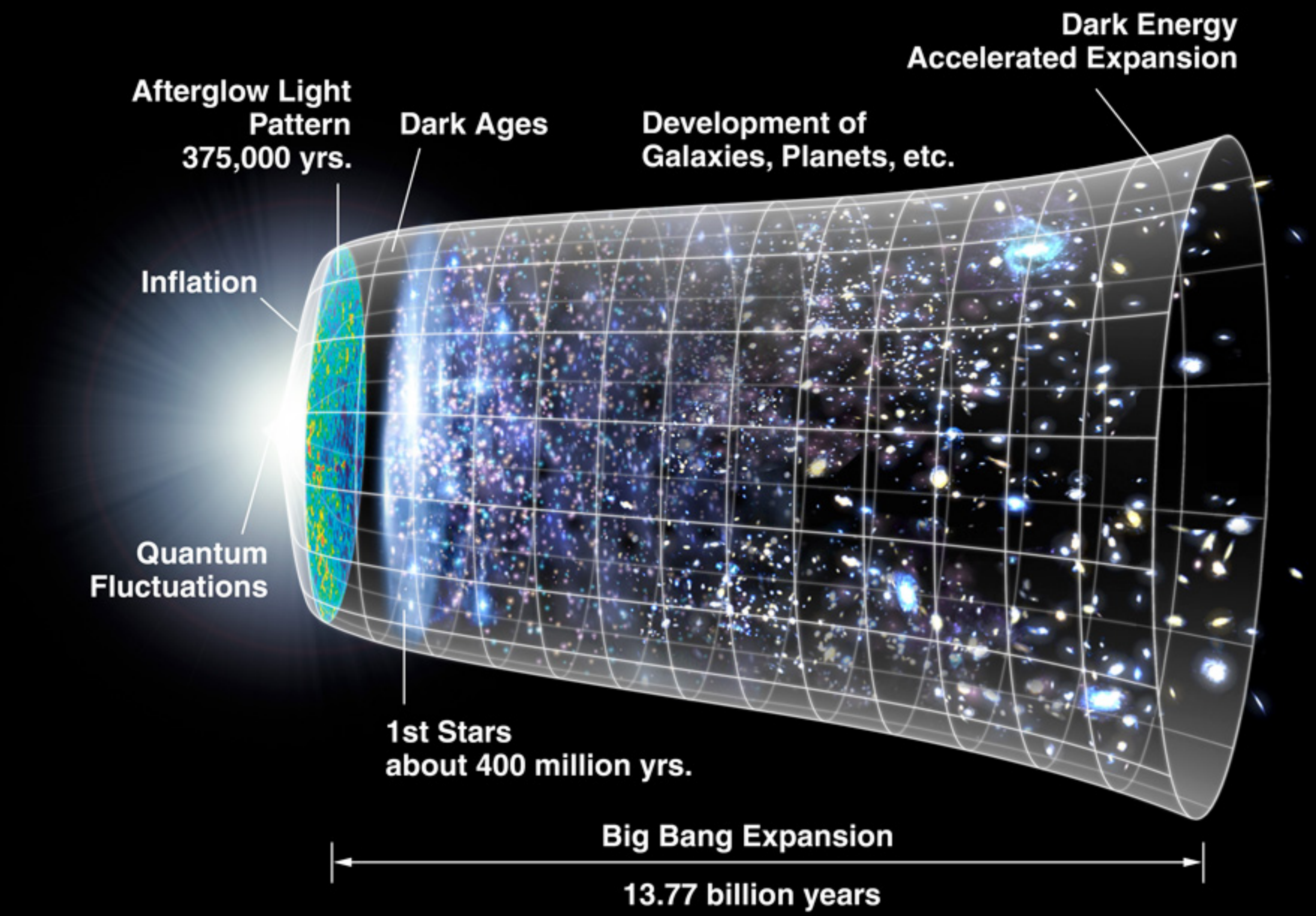
NASA/ESA

NASA/ESA/JPL

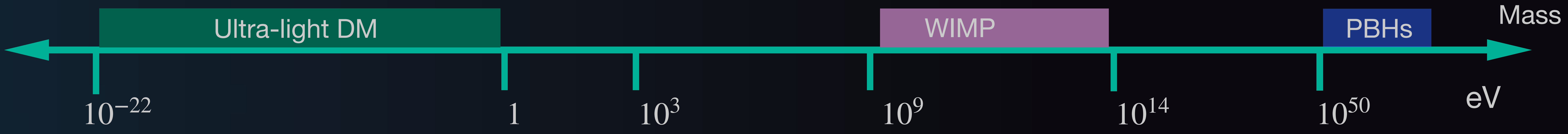


ESA/Planck Collaboration





Ultra-light scalar alternative?

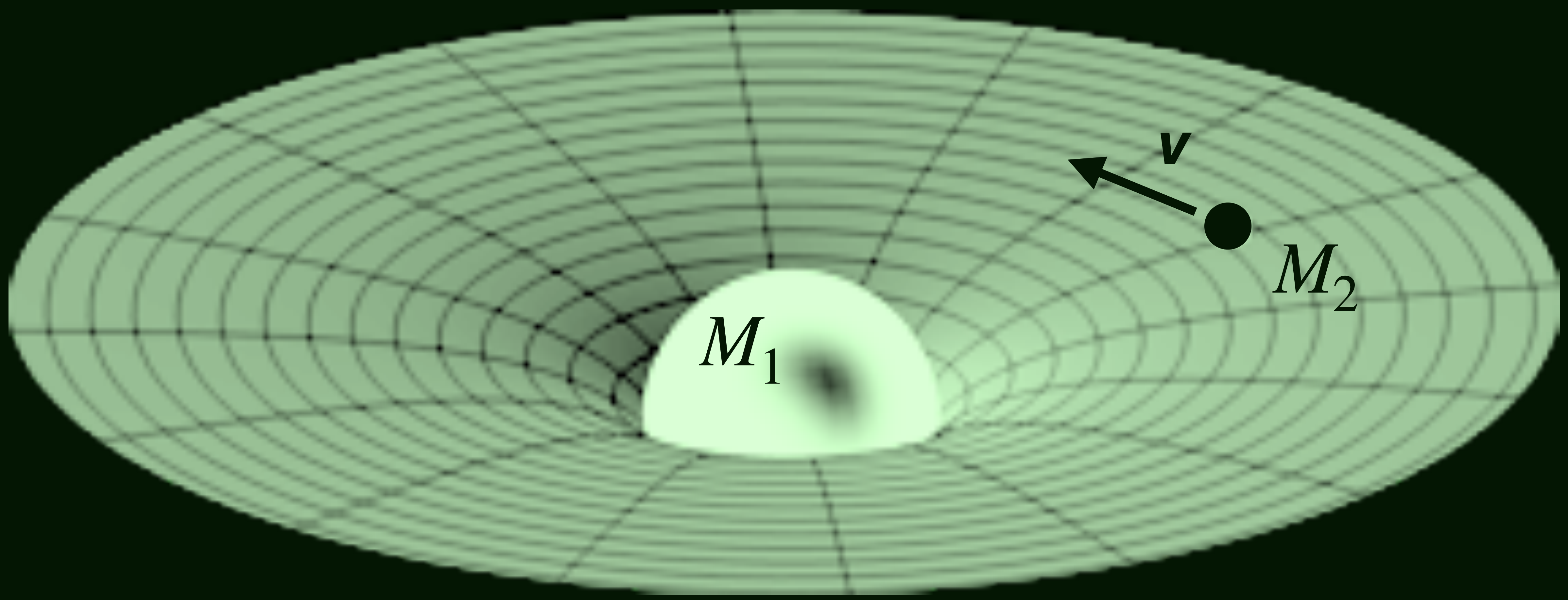


$\lambda_{\rm dB} \sim 1/mv$ —> $m \sim 10^{-22}{\rm eV} \to \lambda_{\rm dB} \sim 1\,{\rm kpc}$

- $d \gg \lambda_{\rm dB}$ Large scales — standard particle DM (CDM)
- $d \ll \lambda_{\rm dB}$ Small scales — ULDM behaves like a wave

—> Can explain challenges on small scales, but still fits cosmological data

Extreme mass ratio inspirals



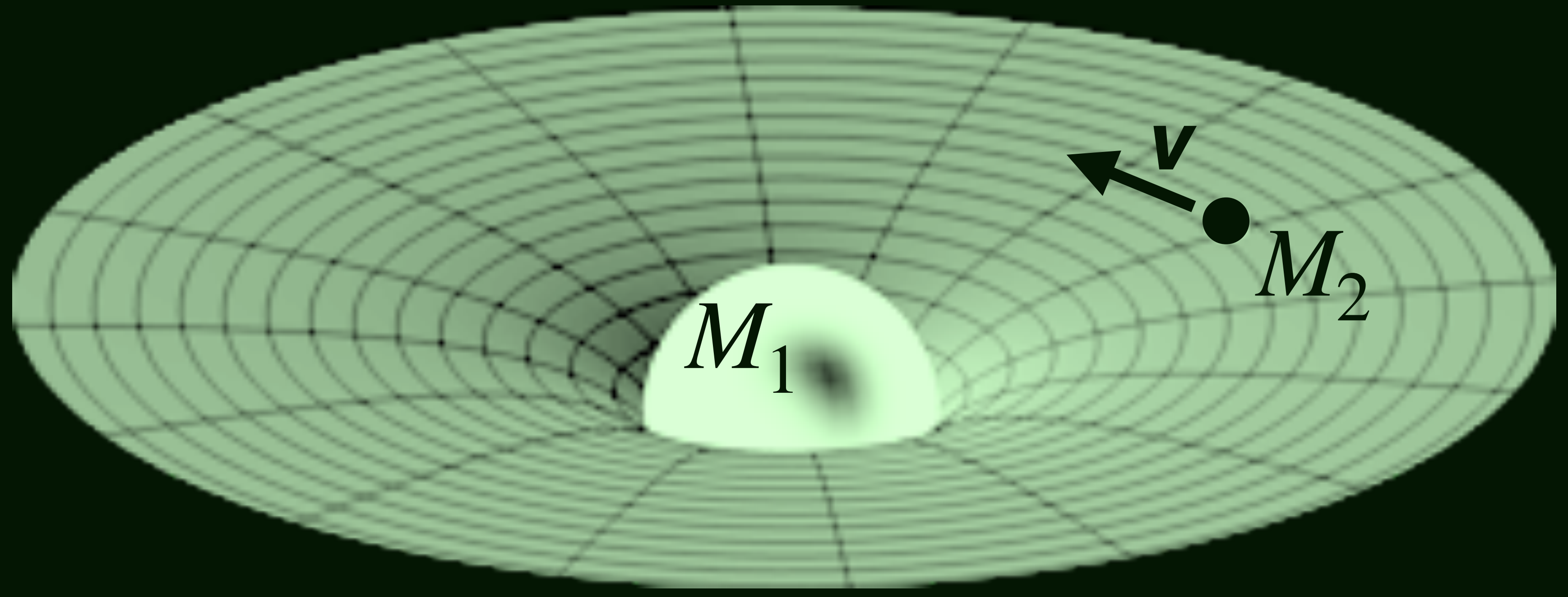
- $M_1 \sim 10^4 - 10^6 M_\odot, \; M_2 \sim 1 - 10^2 M_\odot$
- Frequency range $\sim 10^{-4} - 1$ Hz —> LISA
- Complete $\sim 10^4 - 10^5$ orbits

- Advantages of EMRI observations:

  Measure source masses and spins to great precision

  Longer timescales for dephasing to accumulate

  Environmental effects — stronger in more massive BHs



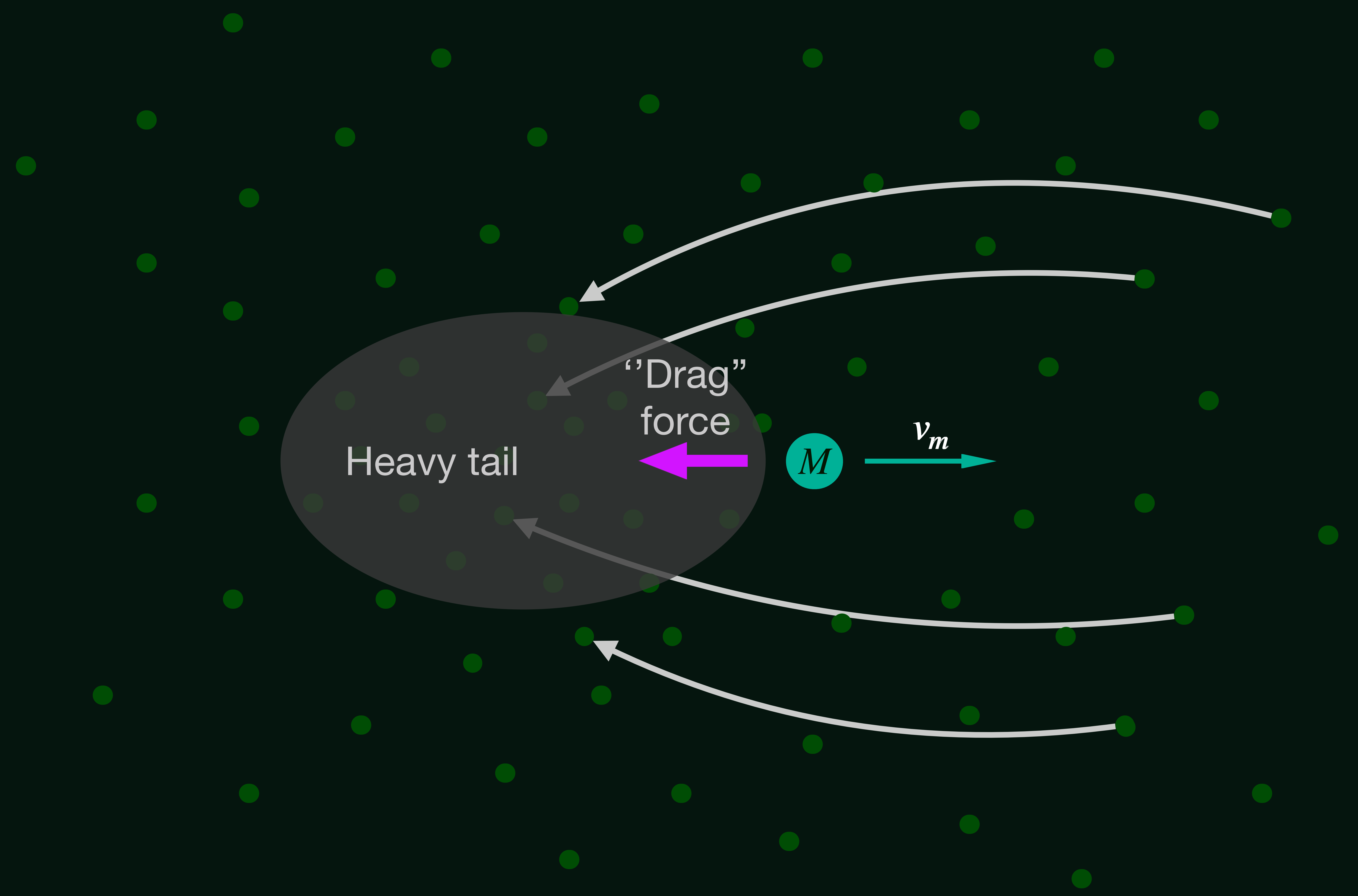
Can treat system as a single BH

- EMRIs difficult for NR
- $M_1 \gg M_2$:

  - simulate M_1 + a test particle

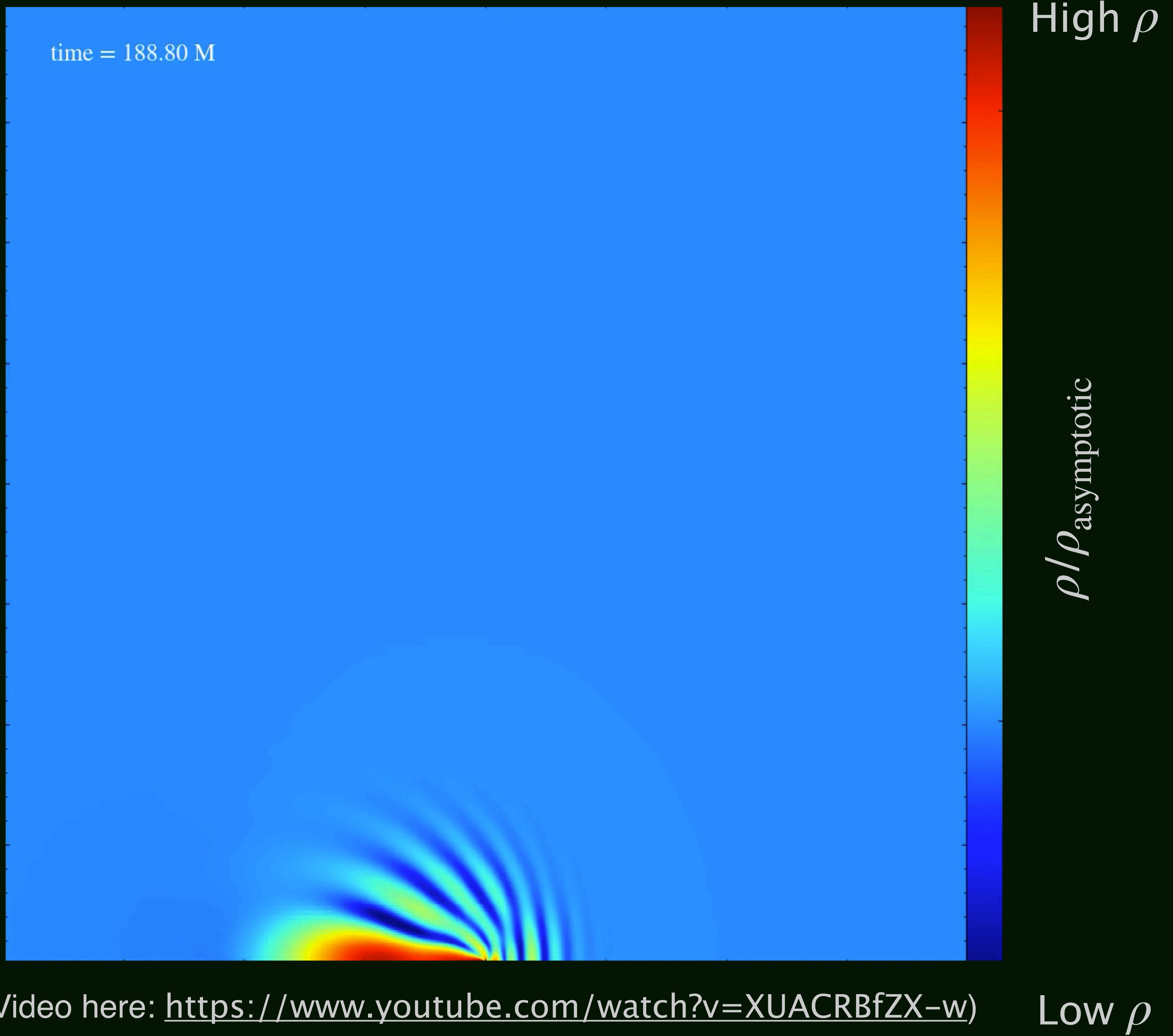
  - approximate satellite motion (M_2) as straight line
- Calculate accretion rate/drag force on a single object
- Implement expressions as a correction to the flux in a waveform generating code
- Integrate as an ODE

Dynamical friction



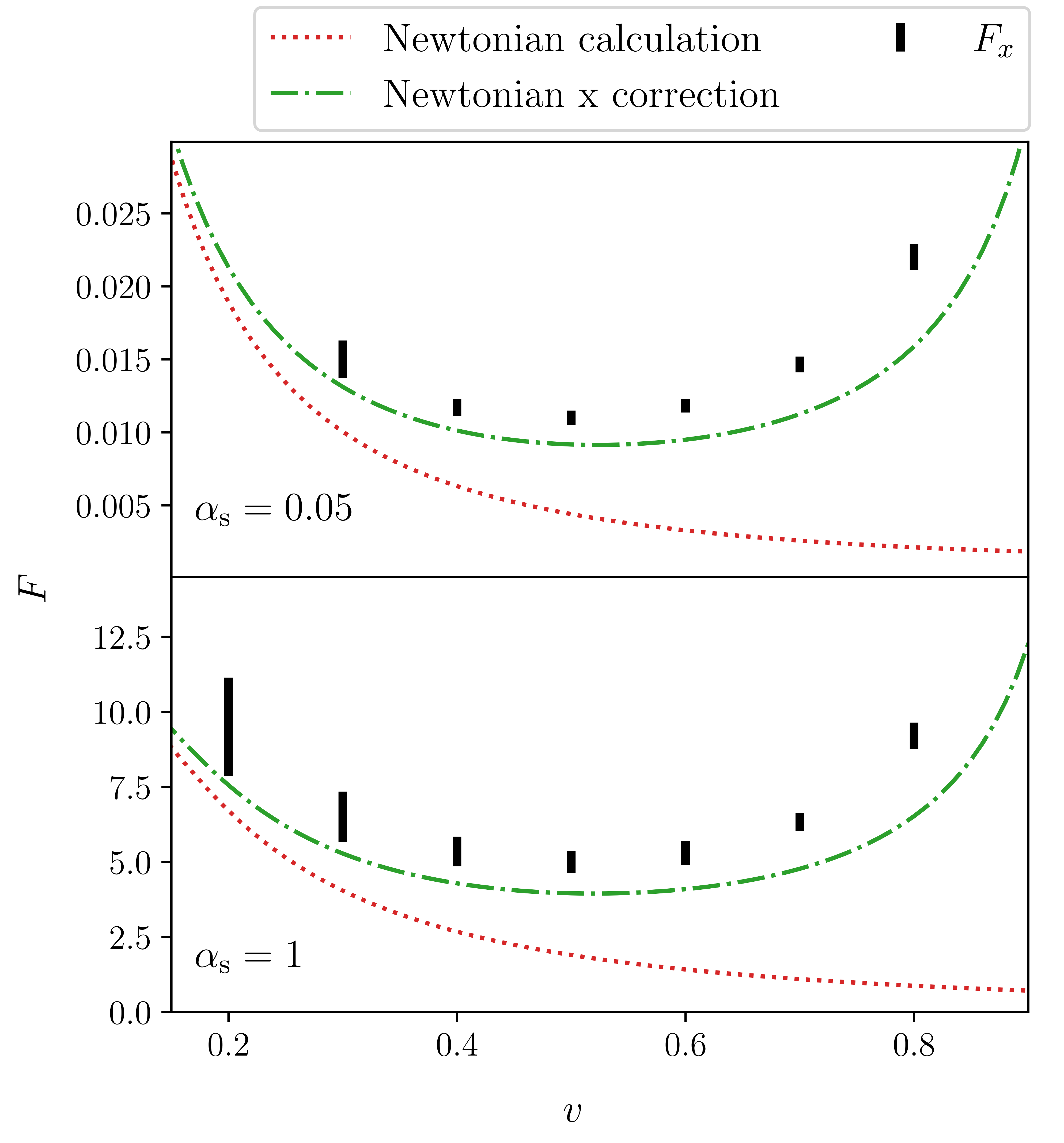
Density profile evolution

- Motion of satellite BH $\sim$ straight line
- In the rest frame of the field
- Single BH boosted $x-$dir.
- No backreaction
- Coordinates on BH



(Video here: https://www.youtube.com/watch?v=XUACRBfZX-w)

Numerical results



DT, K.Clough, T.Helfer, E.Berti, P.Ferreira, L.Hui
PRD 104, 103014 (2021)

- Classical DF:

$$F_{\text{Chandra}} \approx 4\pi\rho \left(\frac{M}{v}\right)^2 \ln\left(\frac{b_{\text{max}}}{b_{\text{min}}}\right)$$

[S.Chandrasekhar, Astrophys. J. 97, 255 (1943)]

- Newtonian light SF:

$$F_x = 4\pi\rho \left(\frac{M}{v}\right)^2 \left(\ln(2\mu v r) - 1 - \text{Re}[\Psi(1 + i\beta)]\right)$$

[L.Hui, J.Ostriker, S.Tremaine, E.Witten,
PRD 95, 043541 (2017)]

- Relativistic correction for fluids:

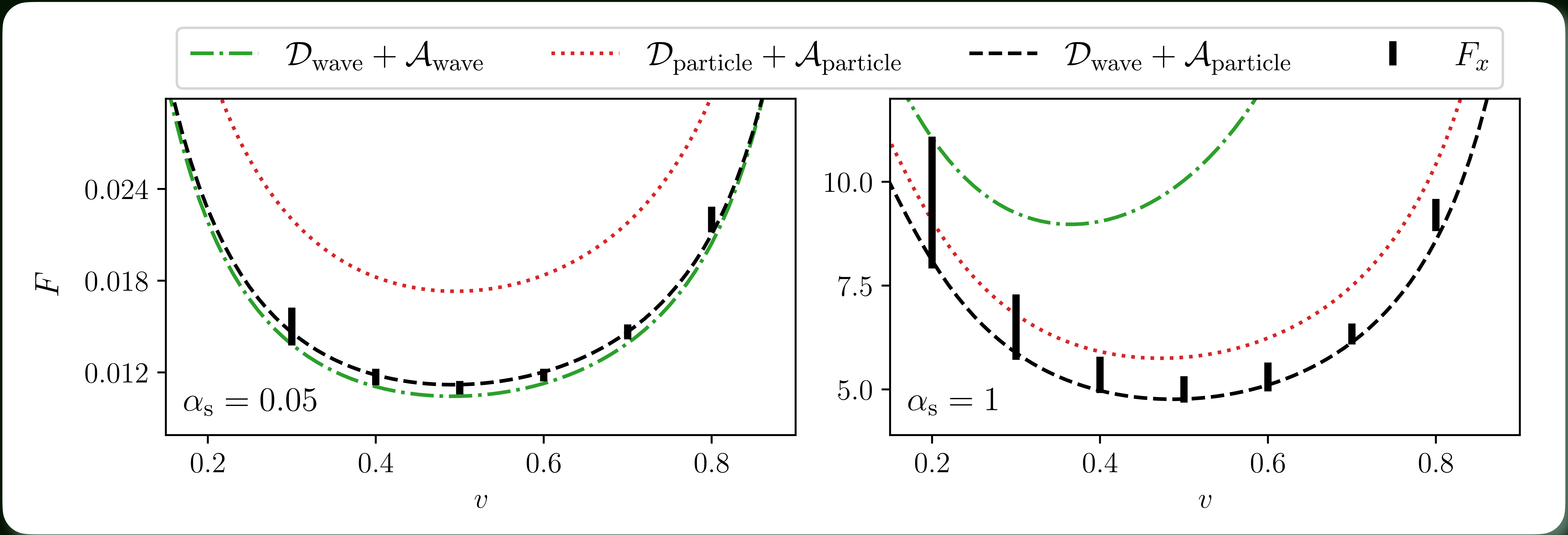
$$\gamma^2(1 + v^2)^2 \qquad , \ \gamma = 1/\sqrt{1 - v^2}$$

[L.Petrich, S.Shapiro, R.Stark, S.Teukolsky,
Astrophys. J. 336, 313 (1989)]

[E.Barausse, MNRAS 382, 826 (2007)]

Total drag force

Contributions from dynamical friction, $\mathcal{D}$ + momentum accretion, $\mathcal{A}$



Low mass → High mass

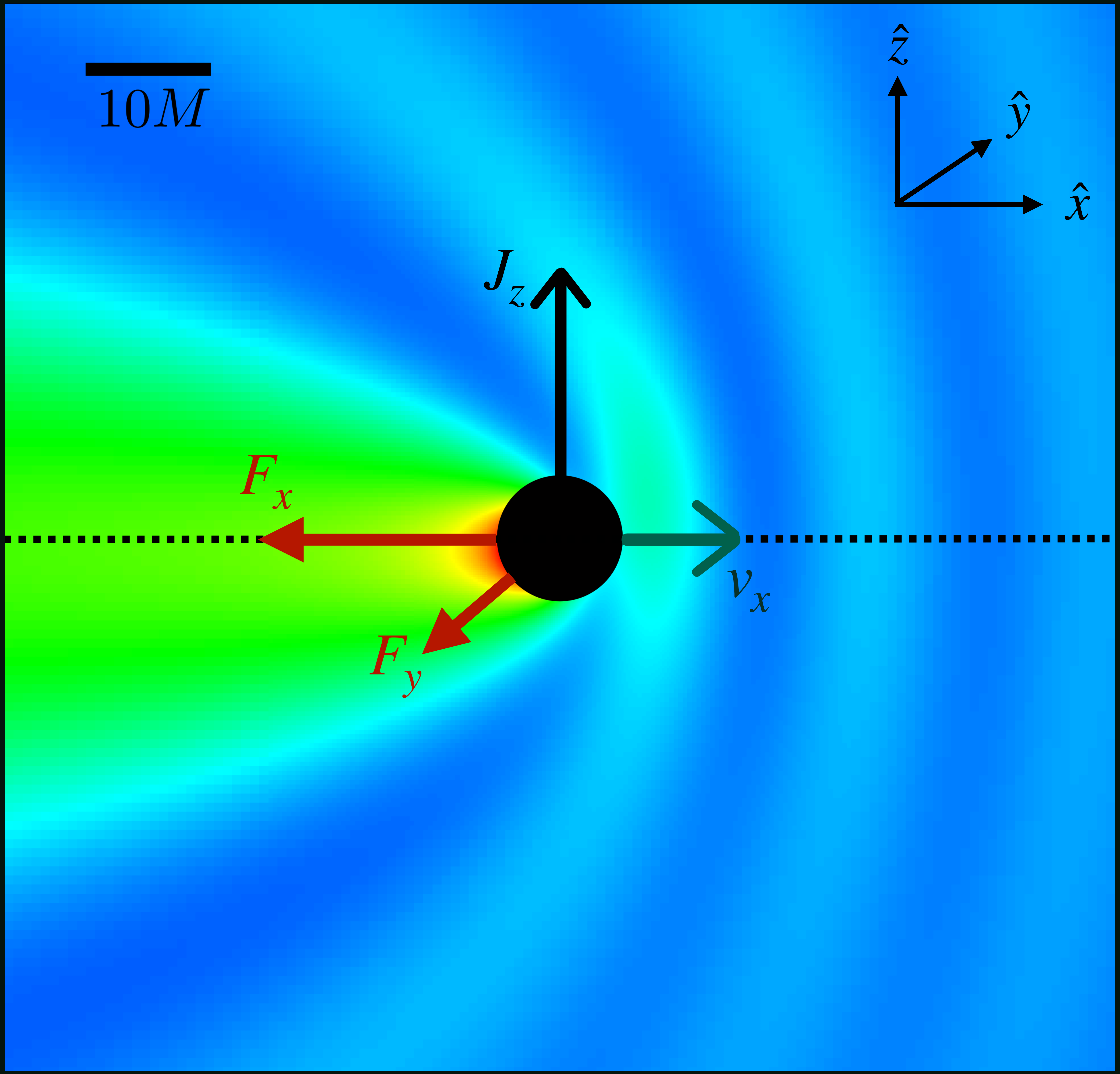
R.Vicente and V.Cardoso PRD 105, 083008 (2022)

DT, R.Vicente, K.Clough, T.Helfer, E.Berti, P.Ferreira, L.Hui PRD 108, L121502 (2023)

Bend it like dark matter

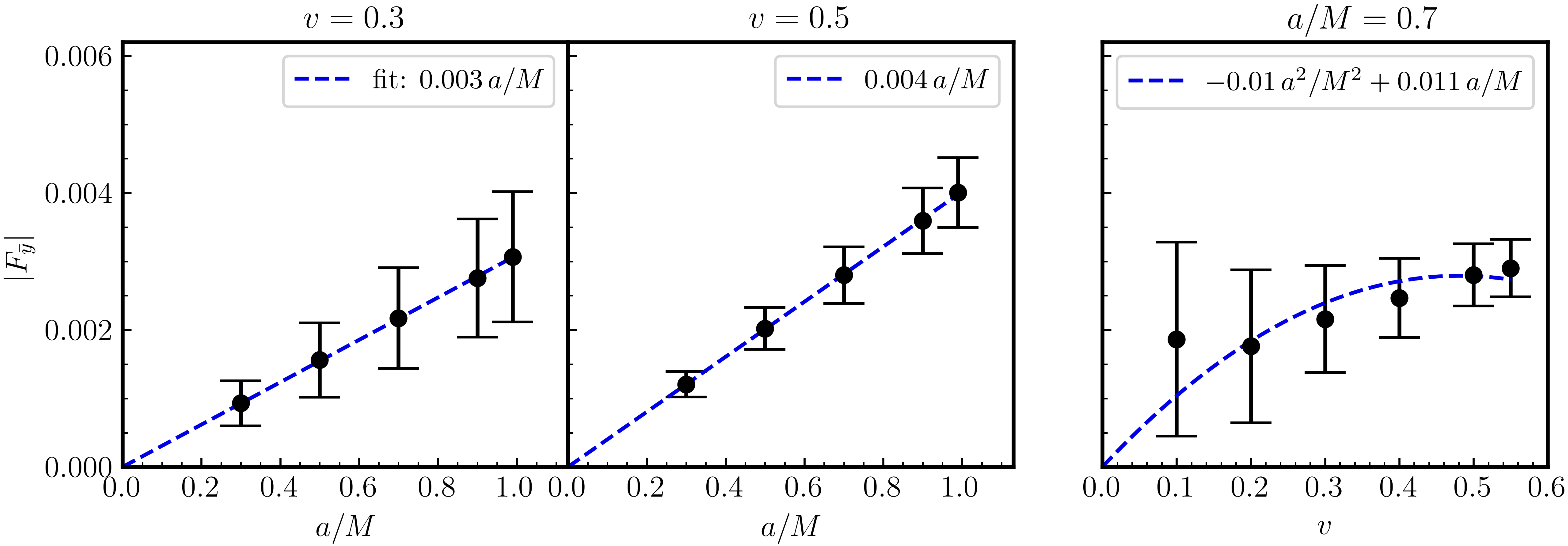
Spin-curvature effect

- Analogous effect for BHs
- Due to gravitational interactions
- Curvature displaces matter
- Asymmetry in flow pattern
- Preferential attraction to one side
- Numerical set-up as before



[Image credit: Zipeng Wang]

Spin-curvature effect

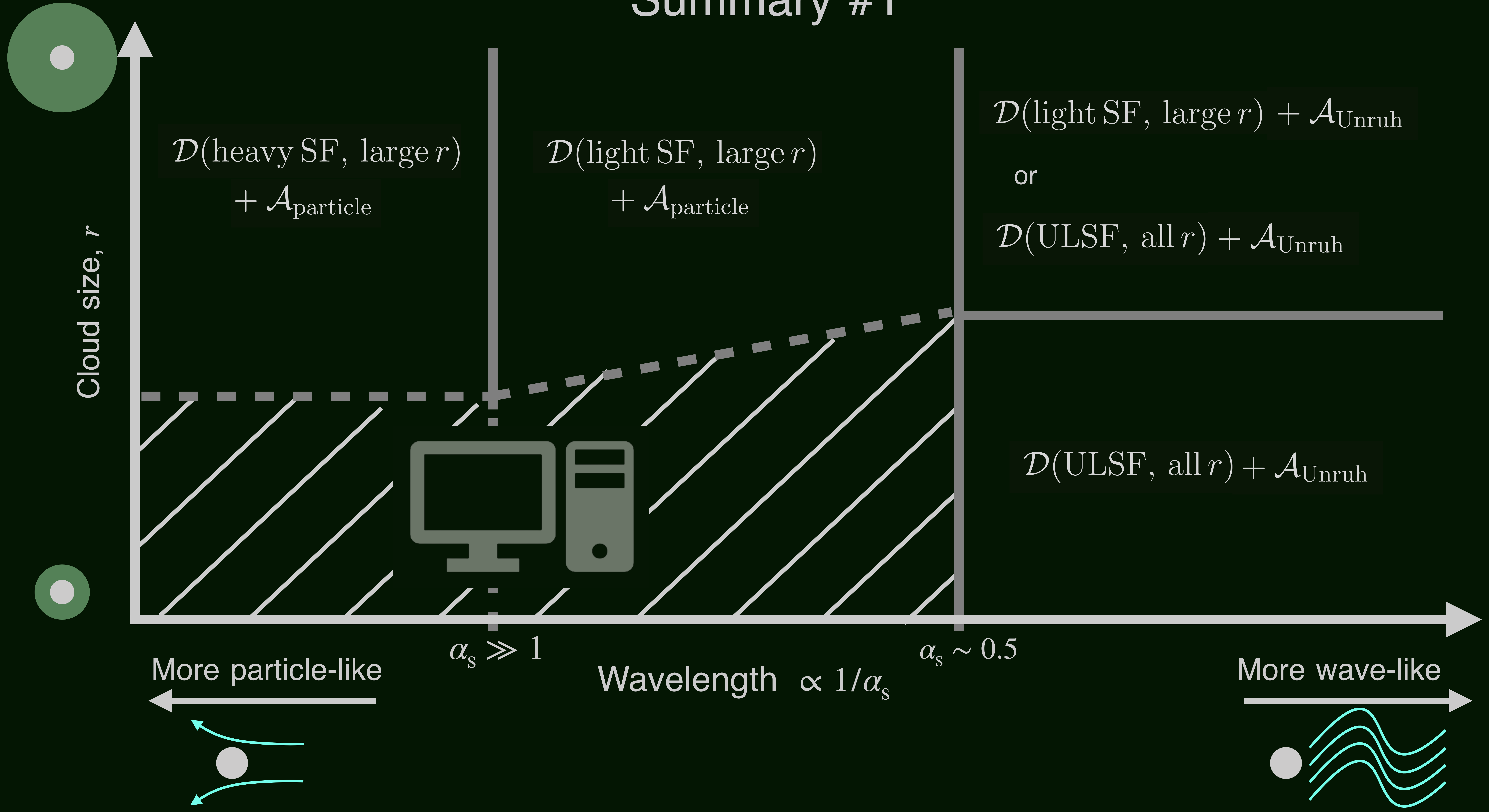


Z.Wang, T.Helfer, **DT**, K.Clough, E.Berti, arXiv: 2402.07977

In agreement with:

C.Dyson, J.Redondo-Yuste, M. van de Meent, V.Cardoso Rev. D 109, 104038 (2024)

Summary #1



DT, R.Vicente, K.Clough, T.Helfer, E.Berti, P.Ferreira, L.Hui PRD 108, L121502 (2023)

GRDzhadzha (aka the code that cannot be named)

J.Aurrekoetxea, J.Bamber, S.Brady, K.Clough, T.Helfer, J.Marsden, M.Radia, **DT** and Z.Wang JOSS 9, 5956 (2024)

- Get it here! https://github.com/GRTLCollaboration/GRDzhadzha.git
- Evolution of a matter field on a static BH background
- Calculates a range of diagnostic quantities for the matter field

  - energy density

  - linear and angular momenta

  - energy and momentum fluxes
- Good for problems with negligible backreaction (e.g. single BHs with matter environments)

Can we ignore back-reaction?

- Typical DM halo densities: $\rho_{\mathrm{DM},0} \sim 1\, M_\odot\, \mathrm{pc}^{-3}$
- In code units: $\frac{\rho_{\mathrm{DM}}}{R_s^{-2}} \sim 10^{-30} \times \left(\frac{M_{\mathrm{BH}}}{10^6 M_\odot}\right)^2 \times \left(\frac{\rho}{\rho_0}\right)$
- Ratio $\ll 1$
- With no back-reaction:

  - can treat matter field as perturbation on a fixed background

  - accretion rates/fluxes calculated to first order in the density

Measuring forces

- Total drag force ≡ change in the total ADM momentum

$$F_x \equiv \partial_t P_x^{ADM} = -\int_{\partial\Sigma_o} \alpha N_j T^j_x \,\mathrm{d}S$$

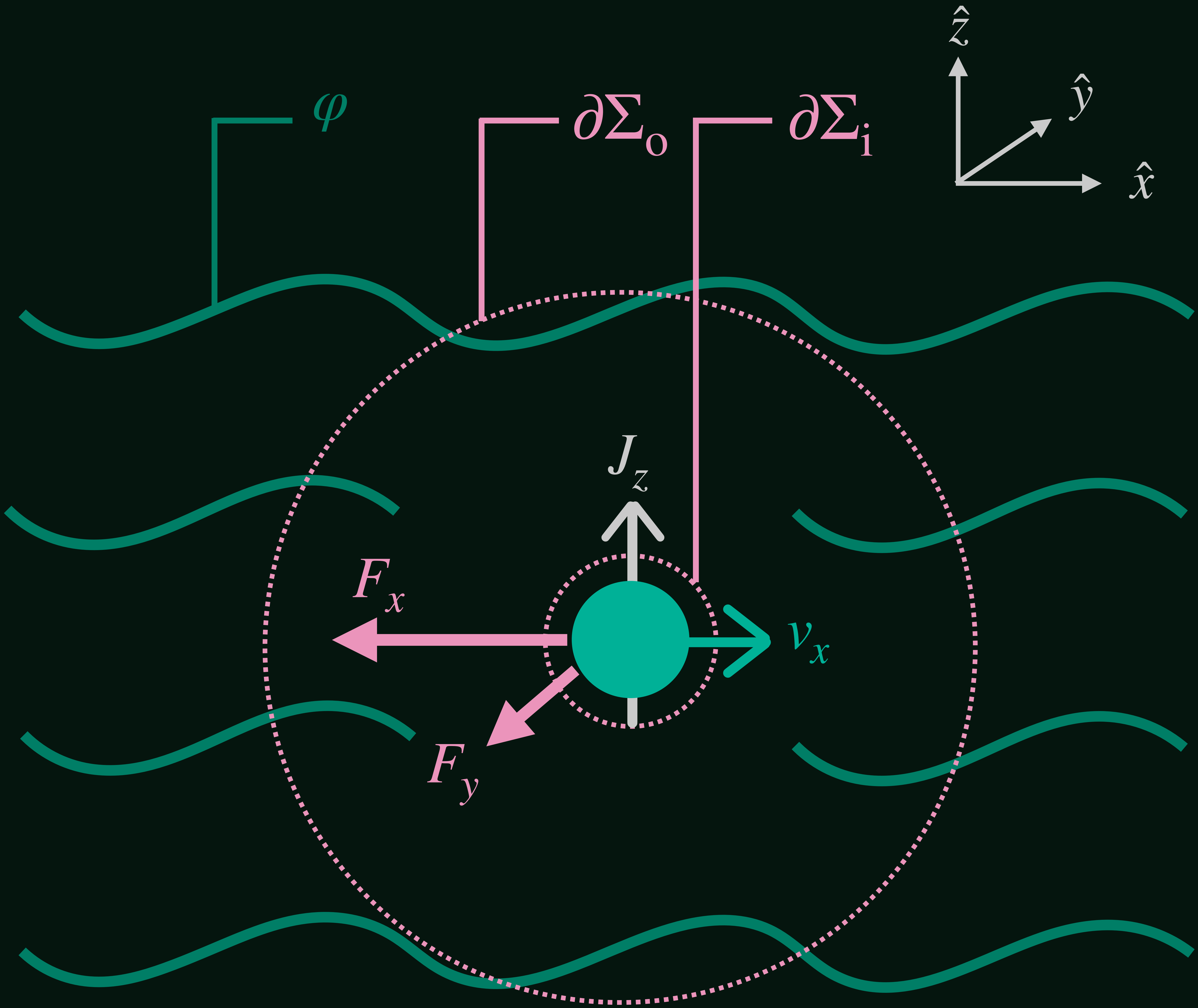
- Dynamical friction

$$F_{\mathrm{DF}} \approx \int_{\Sigma_o - \Sigma_i} \mathrm{d}^3x \sqrt{-g}\, T^\mu_\nu \, {}^{(4)}\Gamma^\nu_{\mu x}$$

- Momentum accretion

$$F_{\mathrm{MA}} \approx -\int_{\partial\Sigma_i} \alpha N_j T^j_x \,\mathrm{d}S$$

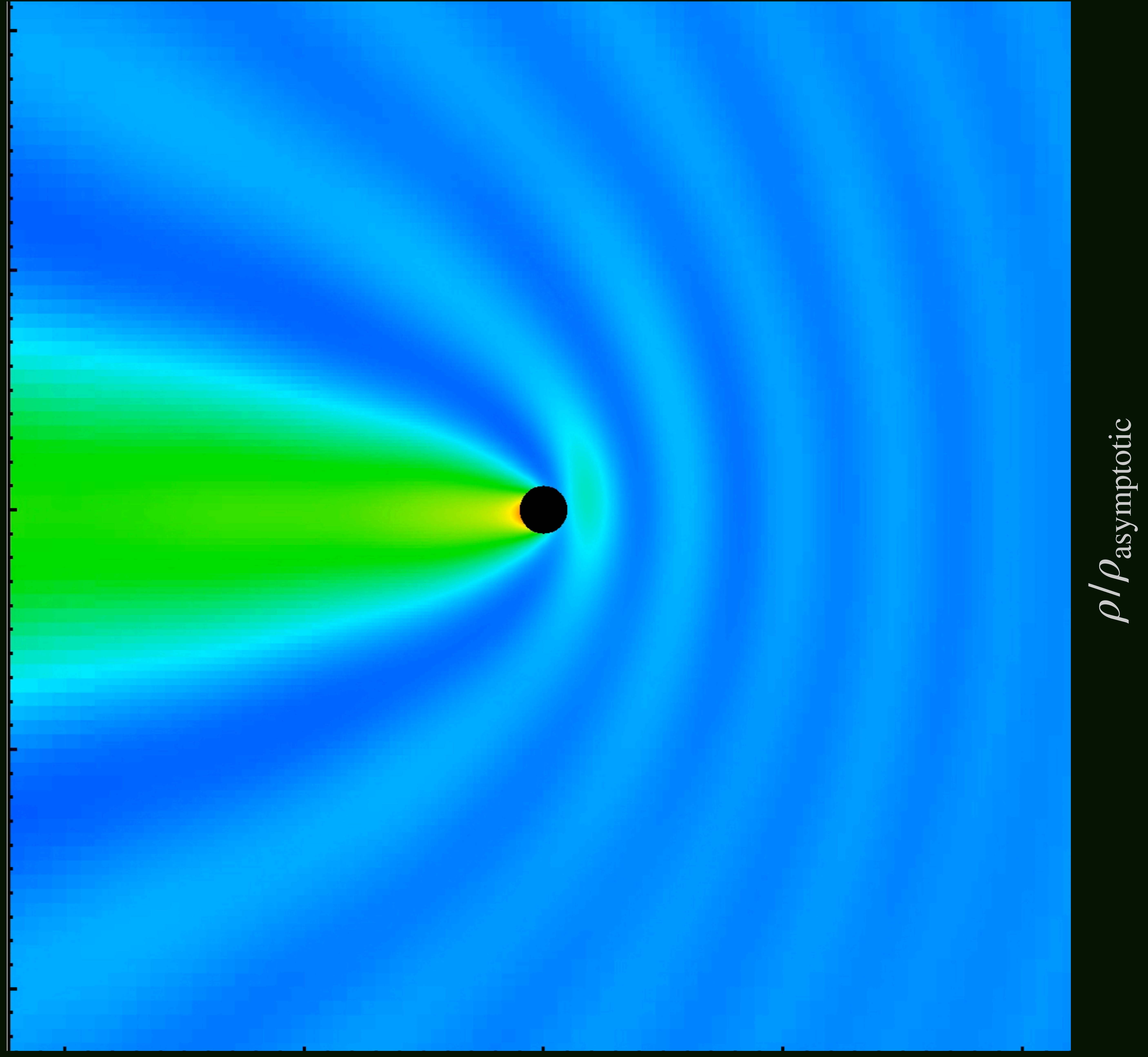
- Spin-curvature force, F_y



[K. Clough CQG 38, 167001 (2021); (case specific: DT+ PRD 108, L121502 (2023), and Z.Wang+, arXiv: 2402.07977)

Advantages of fixed background?

- Can attach coordinates to the BH
- Would need a huge simulation box if BH was moving
- Can do much longer time evolution
- It is many times lighter/faster!



[Movie credit: Zipeng Wang]

Resources - Core hours

CPUh per sim:

```
level 8 at time 19.9813 (304.246 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9844 (304.272 M/hr). Boxes on this rank: 1 / 64
level 6 at time 19.9875 (304.297 M/hr). Boxes on this rank: 1 / 64
level 7 at time 19.9875 (304.274 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9875 (304.25 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9906 (304.275 M/hr). Boxes on this rank: 1 / 64
level 7 at time 19.9938 (304.301 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9938 (304.278 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9969 (304.303 M/hr). Boxes on this rank: 1 / 64
```

Fixed Background

~ 16.4 hrs/sim

```
level 8 at time 19.9813 (18.3777 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9844 (18.3798 M/hr). Boxes on this rank: 1 / 64
level 6 at time 19.9875 (18.382 M/hr). Boxes on this rank: 1 / 64
level 7 at time 19.9875 (18.3813 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9875 (18.3805 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9906 (18.3827 M/hr). Boxes on this rank: 1 / 64
level 7 at time 19.9938 (18.3849 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9938 (18.3842 M/hr). Boxes on this rank: 1 / 64
level 8 at time 19.9969 (18.3864 M/hr). Boxes on this rank: 1 / 64
```

NR

~ 274 hrs/sim

CO_2 footprint for 1 sim of $T \sim 5000\,M$ using GRDzhadhza

http://www.green-algorithms.org

Green Algorithms

How green are your computations?

Details about your algorithm

To understand how each parameter impacts your carbon footprint, check out the formula below and the methods article

Runtime (HH:MM)	16 : 25
Type of cores	CPU
Number of cores	80
Model	Xeon Gold 6248
Memory available (in GB)	376

6.87 kg CO2e
Carbon footprint

20.29 kWh
Energy needed

7.50 tree-months
Carbon sequestration

39.26 km
in a passenger car

1.2%
of a flight NYC-London

[Lannelongue et. al, Advanced science, 8(12), 2100707; arXiv:2007.07610]

CO_2 footprint for 1 sim of $T \sim 5000\,M$ using NR

http://www.green-algorithms.org

Green Algorithms

How green are your computations?

Details about your algorithm

To understand how each parameter impacts your carbon footprint, check out the formula below and the methods article

Runtime (HH:MM): 274

Type of cores: CPU

Number of cores: 80

Model: Xeon Gold 6248

Memory available (in GB): 376

CO_2

114.68 kg CO2e
Carbon footprint

338.64 kWh
Energy needed

10.43 tree-years
Carbon sequestration

655.33 km
in a passenger car

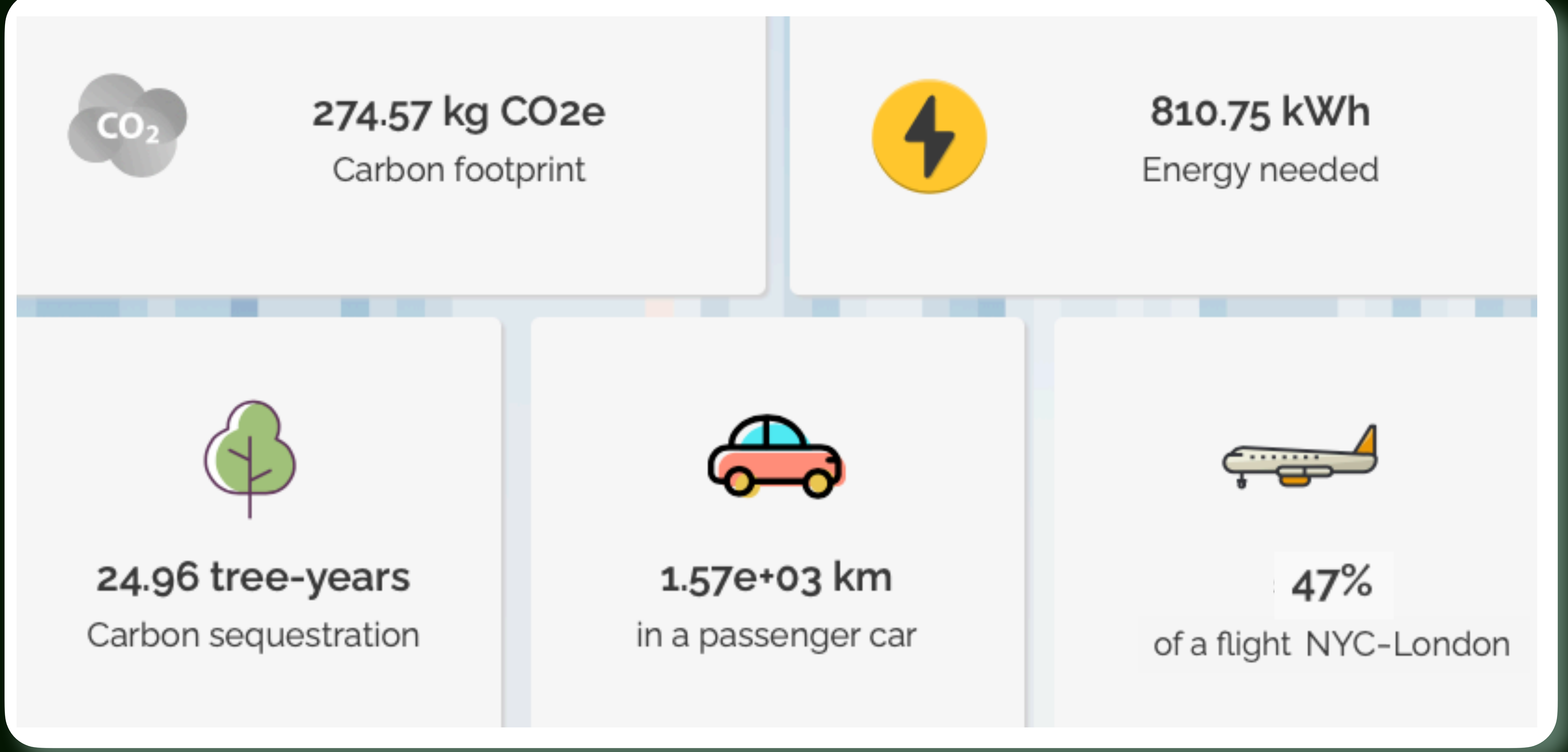
20%
of a flight NYC-London

[Lannelongue et. al, Advanced science, 8(12), 2100707; arXiv:2007.07610]

Dynamical friction work

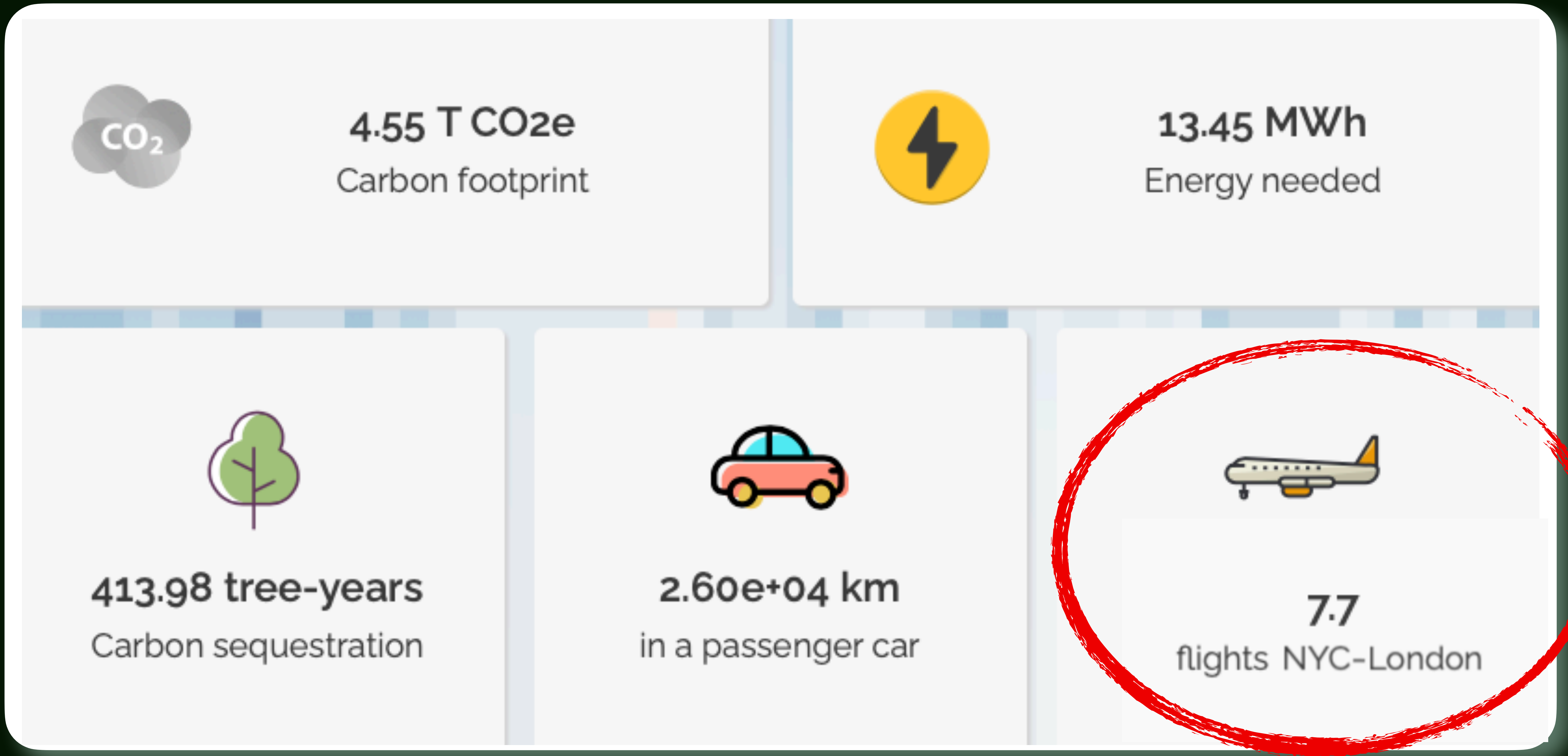
with

Джа-джа



If instead I used NR

~ 16 x more emissions!!



Summary #2

Numerically calculating drag forces

- Can be a useful tool for validating analytic approximations
- Covering regions of parameter space unaccessible to analytics
- Can make nice videos!

Use fixed background (e.g. GRDzhadzha) when possible

- Faster
- Cheaper
- Less harm to the planet?

Some other things to keep in mind

- Storage

- $\gtrsim 40$ simulations - inc double/half resolution
- Mb/Gb per checkpoint file:

Fixed Background

NR

```
dtraykova@sakura01:~/test_NR-vs-Fixed> du -sh Fixed/
209M    Fixed/hdf5/FixedBG_ScalarField_000000.3d.hdf
209M    Fixed/hdf5/FixedBG_ScalarField_000007.3d.hdf
dtraykova@sakura01:~/test_NR-vs-Fixed> du -sh NR/hdf
2.8G    NR/hdf5/ScalarField_000000.3d.hdf5
2.8G    NR/hdf5/ScalarField_000007.3d.hdf5
```

- ~50 checkpoints + ~300 plot files per sim:

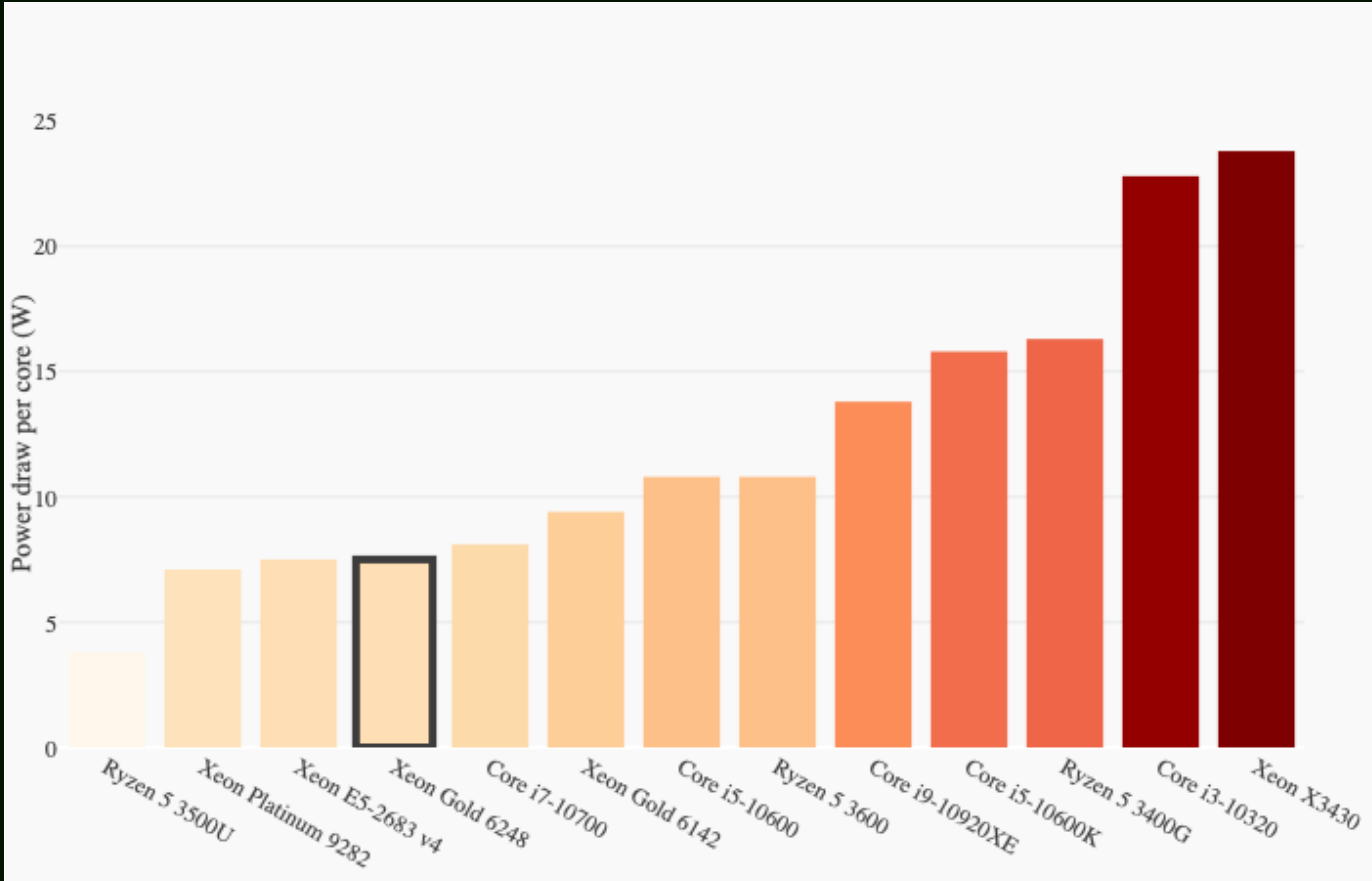
2.7 TB for fixed background

vs

7.8 TB for NR

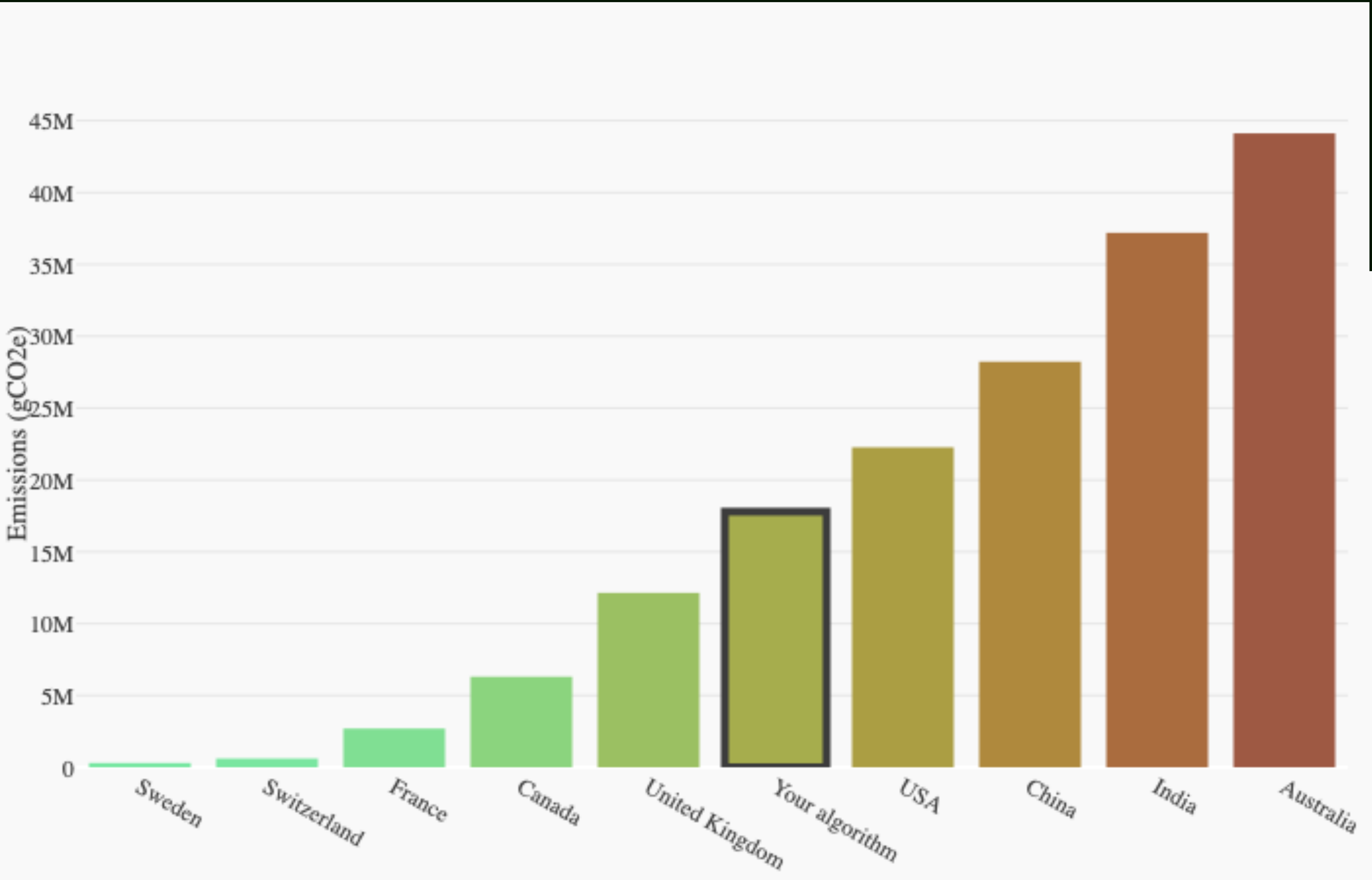
Some other things to keep in mind

- Storage
- CPU efficiency



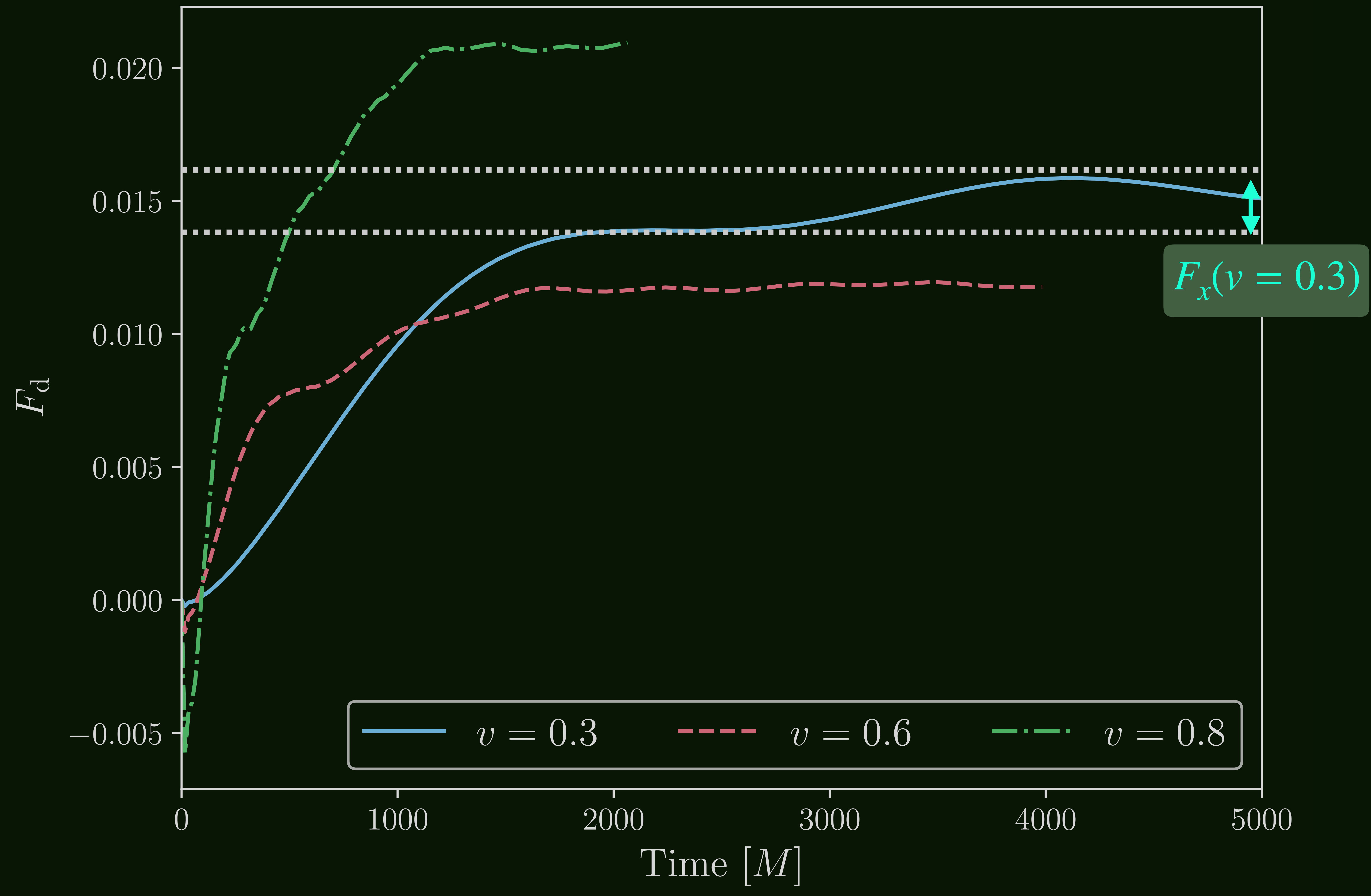
Some other things to keep in mind

- Storage
- CPU efficiency
- Cluster location



Estimating the numerical force

- Oscillations due to matter being wave-like
- Oscillation amplitude and error bars (↓ with time)
- Amplitude - larger and longer to settle at small v and μ
- → avg. run time $t \sim 5000\,M$



Джа-джа

Background

- The metric in 3+1: $\mathrm{d}s^2 = -\alpha^2\,\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\,\mathrm{d}t)(\mathrm{d}x^j + \beta^j\,\mathrm{d}t)$
- $\alpha\,,\beta^i$ and γ_{ij} - fixed; diff. depending on form of the metric, implemented:

  - Boosted isotropic Schwarzschild

  - Kerr-Schild

  - Boosted Kerr

- Components calculated analytically at each point -> no need for them or their derivatives to be stored on the grid!

Джа-джа

Field evolution

- E.g. single scalar field:

  - Only 2 evolution parameters (rather than 10+)

$$\partial_t \varphi = \alpha \Pi + \beta^i \partial_i \varphi$$

$$\partial_t \Pi = \alpha \gamma^{ij} \partial_i \partial_j \varphi + \alpha \left(K\Pi - \gamma^{ij} \Gamma^k_{ij} \partial_k \varphi - m^2 \varphi \right) + \partial_i \varphi \partial^i \alpha + \beta^i \partial_i \Pi$$

$K_{ij} = \frac{1}{2\alpha}\left(-\partial_t \gamma_{ij} + D_i \beta_j + D_j \beta_i \right)$, as background fixed $\partial_t \gamma_{ij} = 0$

- Available matter fields:

  - real/complex scalar field with quadratic and self-interacting potential

  - real/complex vector field

  - classical fluid (in progress)

Джа-джа

Diagnostics

- Energy density
- Angular momentum
- Linear momentum
- Energy and momentum flux
- Can extract their integrals over a surface/volume