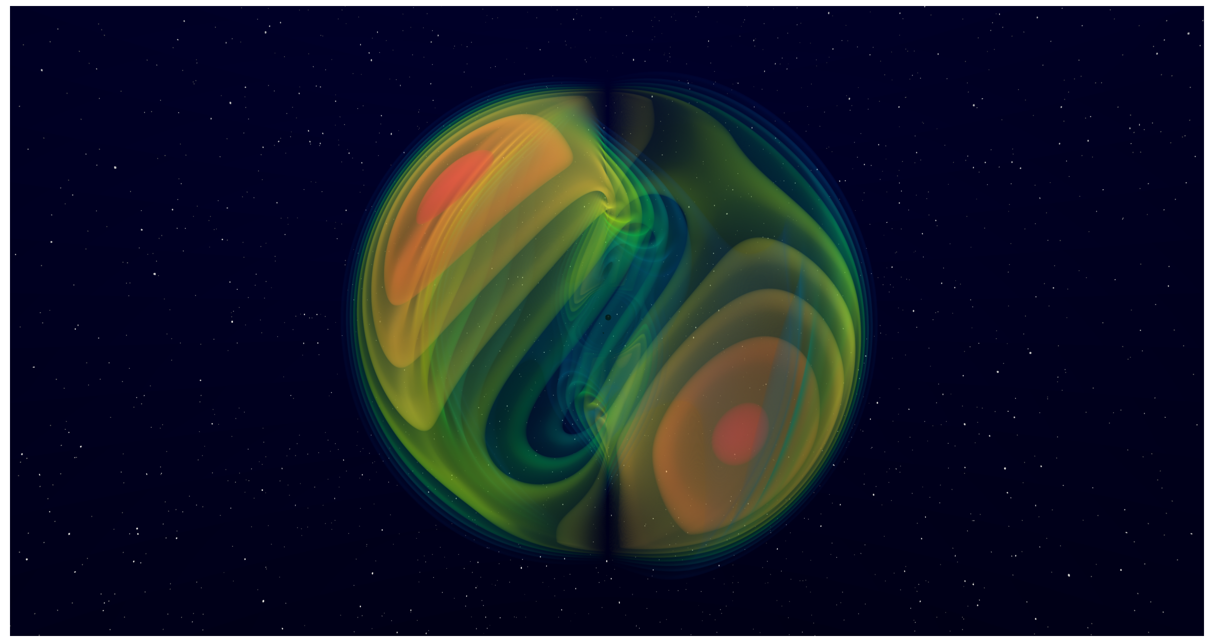
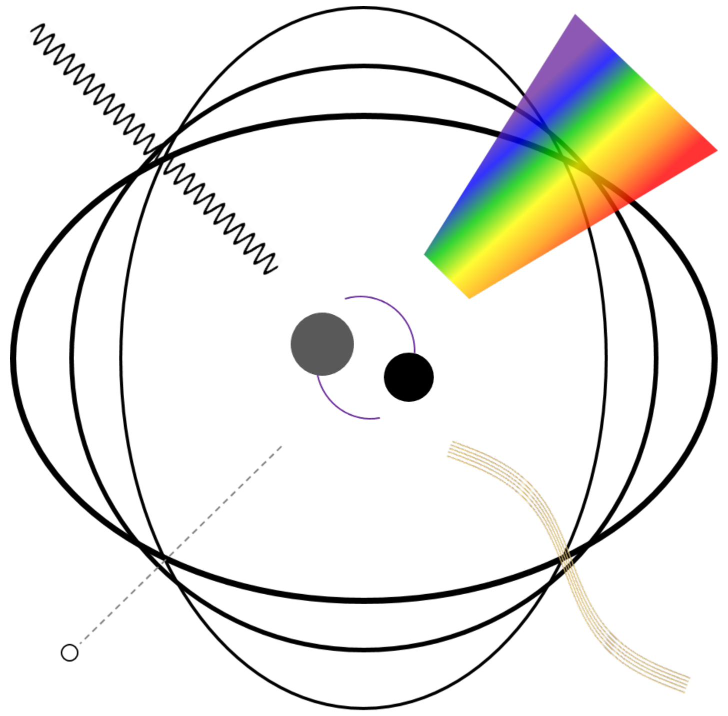
GW190412

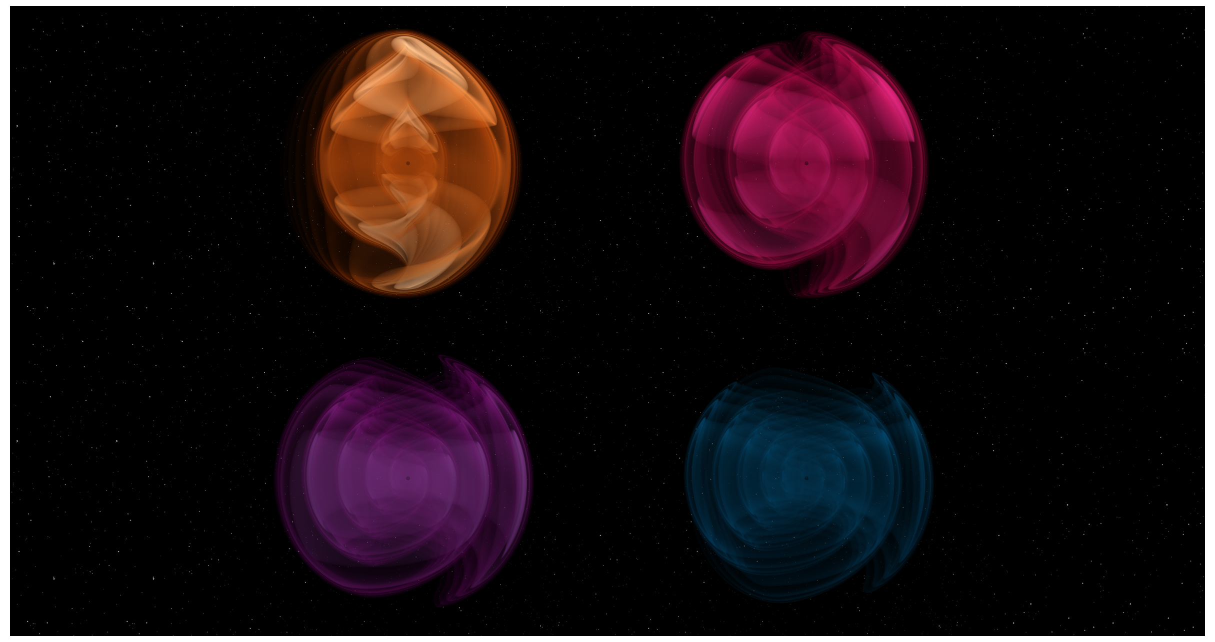


(credit:Vitor Cardoso & Paolo Pani)



(credit: Peter Shawhan)

GW190814



Advances and Challenges in Solving the Relativistic Two-Body Problem

Alessandra Buonanno

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Deutsche Forschungsgemeinschaft
German Research Foundation

International Balzan Foundation

"Black Holes Inside and Out", Niels Bohr Institute, Copenhagen

August 26, 2024

Motivations/Outline

- **Gravitational waves** have become a **groundbreaking tool to explore** the Universe.

- Inferring **astrophysical and cosmological information** from GW observations, detecting **possible deviations from GR** and **distinguishing** them **from astrophysical environmental** and **cosmological effects, rely on accurate predictions** of **two-body dynamics** and **gravitational radiation.**

- What **role waveform models have played in the detection of** GW signals **from binary systems and the extraction of unique scientific insights** with the **LIGO-Virgo-KAGRA** detectors? What have been the main **theoretical advances in this field?**

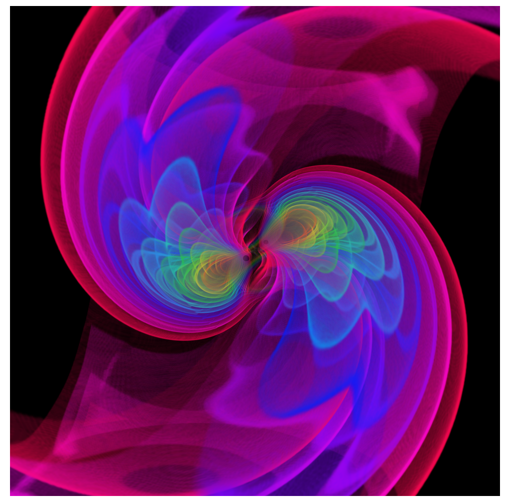
- **Upcoming runs with current and future detectors** in space and on the ground, **require ever more accurate and precise** waveform models, which **include *all* physical effects** (spins, tides, eccentricity, beyond-GR effects, non-vacuum GR's effects, etc.).

- What **theoretical challenges must be addressed** to **achieve these results**?

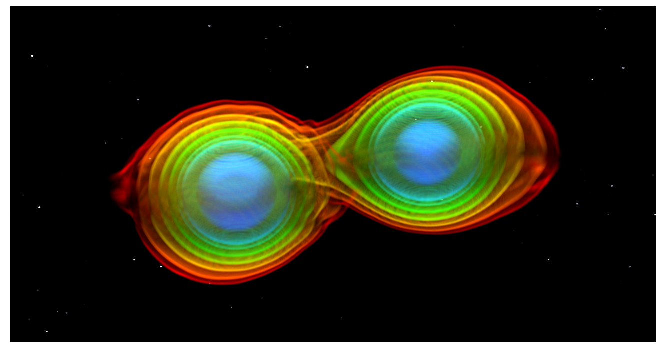
Discovering/Characterizing Black Holes & Neutron Stars in the Universe

- As today, GWs were observed by **LIGO-Virgo detectors** from **90 coalescences, plus tens of events** pulled out from public data **with independent analysis.** *(Abbott+ PRX 13 (2023) 4, 041039)* *(Nitz+23, Mehta+23, Wadekar+23)*

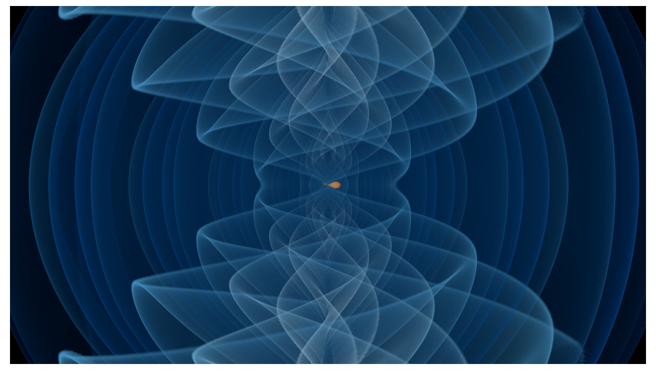
GW150914

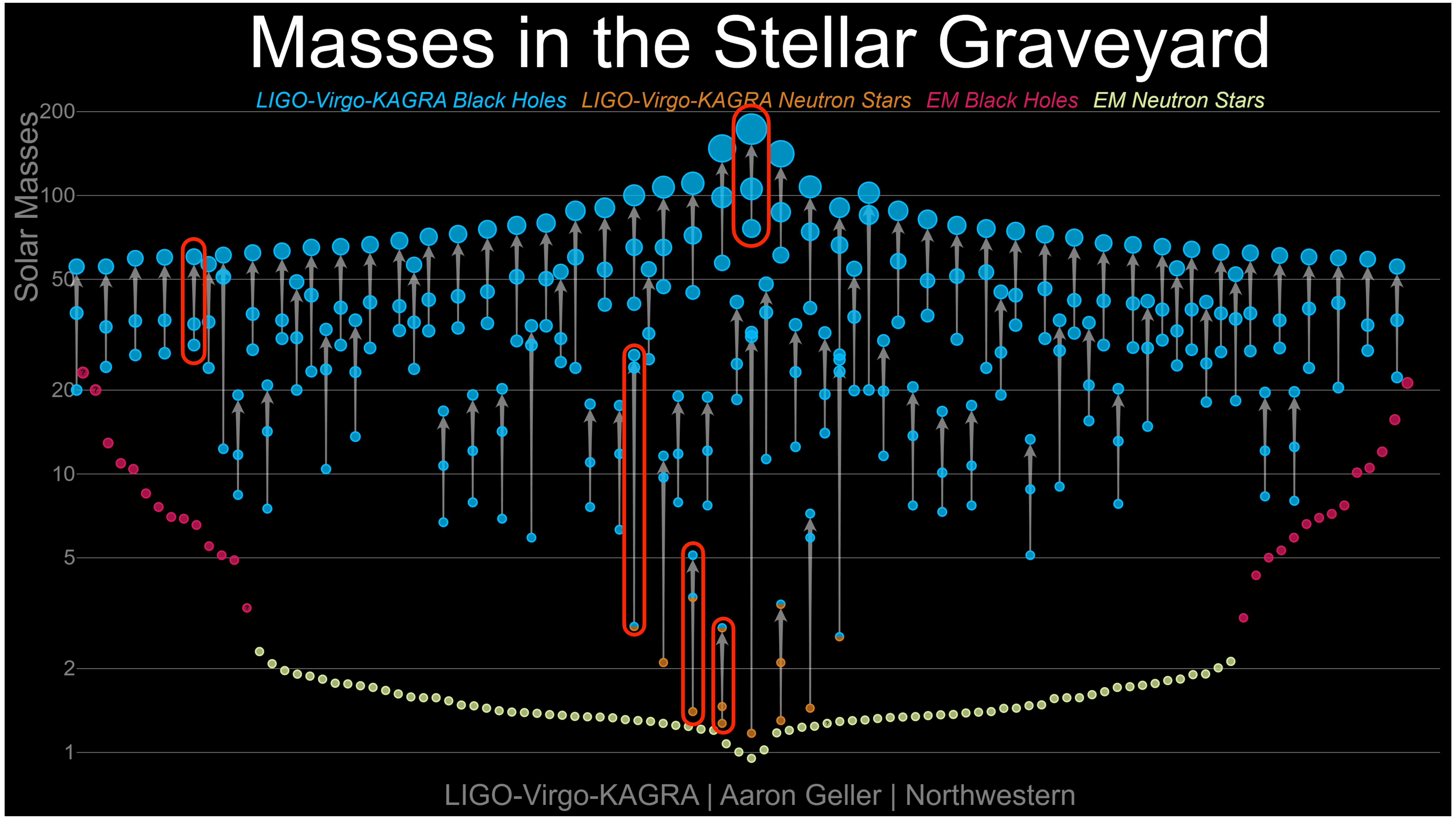


GW170817

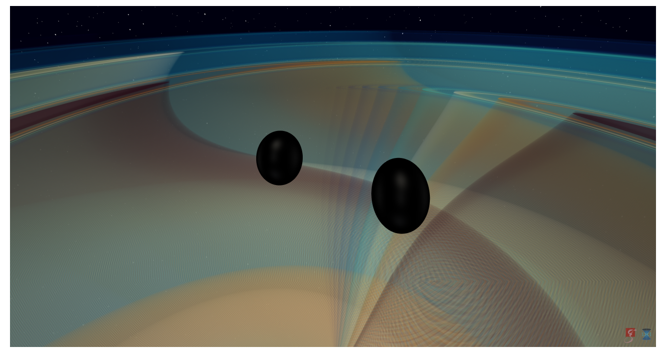


GW230529

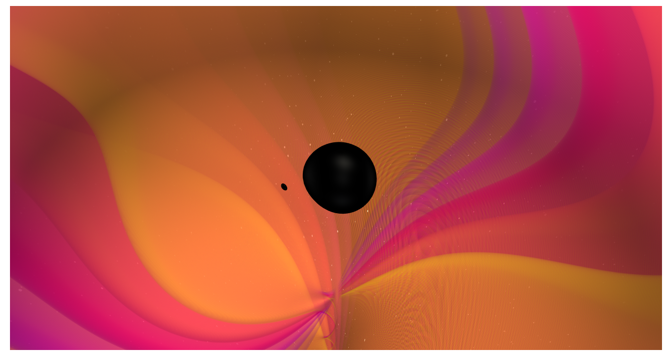


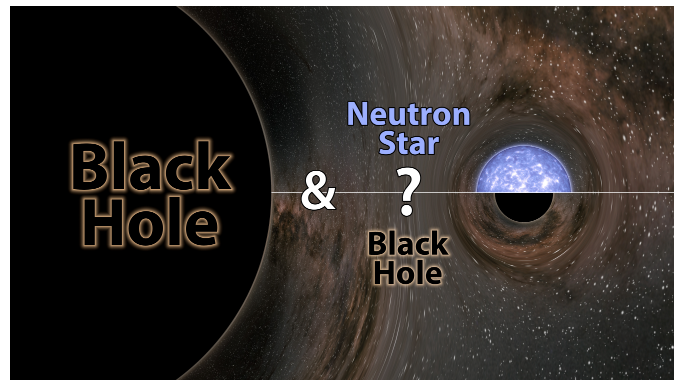


GW190521



GW190814





- Ongoing LIGO-Virgo-KAGRA observing run **(O4) has already announced 122 signal candidates**.

Gravitational Waves: Signatures of Dynamical Spacetime

- First paper by **Einstein on gravitational waves in 1916**.

(Collected papers of Albert Einstein)

DOC. 32 INTEGRATION OF FIELD EQUATIONS 201

Doc. 32

Session of the physical-mathematical class on June 22, 1916 [p. 688]

Approximative Integration of the Field Equations of Gravitation

by A. Einstein

For the treatment of the special (not basic) problems in gravitational theory one can be satisfied with a first approximation of the $g_{\mu\nu}$. The same reasons as in the special theory of relativity make it advantageous to use the imaginary time variable $x_4 = it$. By "first approximation" we mean that the quantities $\gamma_{\mu\nu}$, defined by the equation

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu}, \quad (1)$$

are small compared to 1, such that their squares and products are negligible compared with first powers; furthermore, they have a tensorial character under linear, orthogonal transformations. In addition, $\delta_{\mu\nu} = 1$ or $\delta_{\mu\nu} = 0$ resp. depending upon $\mu = \nu$ or $\mu \neq \nu$.

- Second paper by **Einstein on gravitational waves in 1918**.

(Collected papers of Albert Einstein)

DOC. 1 GRAVITATIONAL WAVES 23

It is clear that S is the density of the radially flowing gravitational radiation toward the "outside" in the direction $(\alpha_1, \alpha_2, \alpha_3)$, provided one puts

$$A_{\mu\nu} = \frac{\sqrt{\kappa}}{8\pi R}\dddot{\mathfrak{J}}_{\mu\nu}. \quad (29)$$

If one forms the mean value of S over all directions of space for a fixed value of $A_{\mu\nu}$, one obtains the mean density $\bar{S}$ of the radiation. Finally, $\bar{S}$ multiplied by $4\pi R^2$ is the energy loss (per time unit) of the mechanical system due to gravitational waves. The calculation finds

$$4\pi R^2\bar{S} = \frac{\kappa}{80\pi}\left[\sum_{\mu\nu}\dddot{\mathfrak{J}}^2_{\mu\nu} - \frac{1}{3}\left(\sum_{\mu}\dddot{\mathfrak{J}}_{\mu\mu}\right)^2\right]. \quad (30)$$ [31]

wrong by a factor 2!

This result shows that a mechanical system which permanently retains spherical symmetry cannot radiate; this is in contrast to the result of the previous paper, marred by an error in calculation. [32]

- **Linearization** of Einstein's equations (weak field), and **wave equation for perturbations**:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Quadrupole formula **for the energy flux of gravitational waves.**

- **GW sources** are objects like a "rotating dumbbell", e. g., realized by a **binary star system**.

The Black-Hole Solution of Einstein's Equations

Über das Gravitationsfeld eines Massenpunktes nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. EINSTEIN hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\delta \int ds = 0, \quad \text{wobei} \quad ds = \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \quad \mu, \nu = 1, 2, 3, 4 \qquad (1)$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltigkeit.

Die Ausführung der Variation ergibt die Bewegungsgleichungen des Punktes

$$\frac{d^2 x_\alpha}{ds^2} = \sum_{\mu,\nu} \Gamma^\alpha_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds}, \quad \alpha, \beta = 1, 2, 3, 4 \qquad (2)$$

wobei

$$\Gamma^\alpha_{\mu\nu} = -\frac{1}{2} \sum_\beta g^{\alpha\beta} \left(\frac{\partial g_{\mu\beta}}{\partial x_\nu} + \frac{\partial g_{\nu\beta}}{\partial x_\mu} - \frac{\partial g_{\mu\nu}}{\partial x_\beta} \right) \qquad (3)$$

ist und $g^{\alpha\beta}$ die zu $g_{\alpha\beta}$ koordinierte und normierte Subdeterminante in der Determinante $|g_{\mu\nu}|$ bedeutet.

Dies ist nun nach der EINSTEINschen Theorie dann die Bewegung eines masselosen Punktes in dem Gravitationsfeld einer im Punkt $x_1 = x_2 = x_3 = 0$ befindlichen Masse, wenn die »Komponenten des Gravitationsfeldes« Γ überall, mit Ausnahme des Punktes $x_1 = x_2 = x_3 = 0$, den »Feldgleichungen«

GRAVITATIONAL FIELD OF A SPINNING MASS AS AN EXAMPLE OF ALGEBRAICALLY SPECIAL METRICS

Roy P. Kerr*
University of Texas, Austin, Texas and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio
(Received 26 July 1963)

Goldberg and Sachs[1] have proved that the algebraically special solutions of Einstein's empty-space field equations are characterized by the existence of a geodesic and shear-free ray congruence, k_μ. Among these spaces are the plane-fronted waves and the Robinson-Trautman metrics[2] for which the congruence has nonvanishing divergence, but is hypersurface orthogonal.

In this note we shall present the class of solutions for which the congruence is diverging, and is not necessarily hypersurface orthogonal. The only previously known example of the general case is the Newman, Unti, and Tamburino metrics,[3] which is of Petrov Type D, and possesses a four-dimensional group of isometries.

If we introduce a complex null tetrad (t^* is the complex conjugate of t), with

$$ds^2 = 2tt^* + 2mk,$$

then the coordinate system may be chosen so that

$$t = P(r + i\Delta)d\zeta,$$

$$k = du + 2\,\mathrm{Re}(\Omega d\zeta),$$

$$m = dr - 2\,\mathrm{Re}\{[(r - i\Delta)\dot{\Omega} + iD\Delta]d\zeta\} + \Big\{r\dot{P}/P + \mathrm{Re}[P^{-2}D(D^*\ln P + \dot{\Omega}^*)] + \frac{m_1 r - m_2\Delta}{r^2 + \Delta^2}\Big\}k; \qquad (1)$$

where ζ is a complex coordinate, a dot denotes differentiation with respect to u, and the operator D is defined by

$$D = \partial/\partial\zeta - \Omega\partial/\partial u.$$

P is real, whereas Ω and m (which is defined to be $m_1 + im_2$) are complex. They are all independent of the coordinate r. Δ is defined by

$$\Delta = \mathrm{Im}(P^{-2}D^*\Omega).$$

There are two natural choices that can be made for the coordinate system. Either (A) P can be chosen to be unity, in which case Ω is complex, or (B) Ω can be taken pure imaginary, with P different from unity. In case (A), the field equations are

$$(m - D^*D^*D\Omega) = |\partial_u D\Omega|^2, \qquad (2)$$

$$\mathrm{Im}(m - D^*D^*D\Omega) = 0, \qquad (3)$$

$$D^*m = 3m\dot{\Omega}. \qquad (4)$$

The second coordinate system is probably better, but it gives more complicated field equations.

It will be observed that if m is zero then the field equations are integrable. These spaces correspond to the Type-III and null spaces with

- The **Schwarzschild/Kerr solution** to Einstein's gravitational equations turned out to **describe the curvature of space-time around every astrophysical black hole**, so far.
- **Via *simple mappings*** that involve the **binary's mass ratio**, the **dynamics in the probe limit (test-body around BH)** can **also inform us about the two-body dynamics** and radiation of comparable-mass BHs.

What is the Strength of Gravitational Waves from Binaries?

- Typical **GW strength** (or strain):

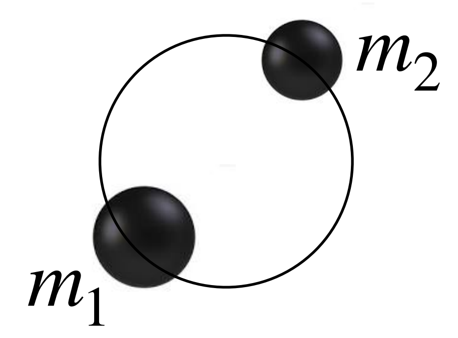
$$h \sim \frac{G}{c^4}\frac{\ddot{Q}}{D} \qquad h \sim \nu \frac{GM}{c^2 D}\left(\frac{\mathrm{v}}{c}\right)^2$$

↑ dimensionless

circular orbits: $\mathrm{v}^2 = r^2\omega^2 = \frac{GM}{r}$

$$\frac{\mathrm{v}}{c} = \left(\frac{GM\omega}{c^3}\right)^{1/3}$$ Kepler law

(binary)



$\mathrm{v} \rightarrow$ velocity

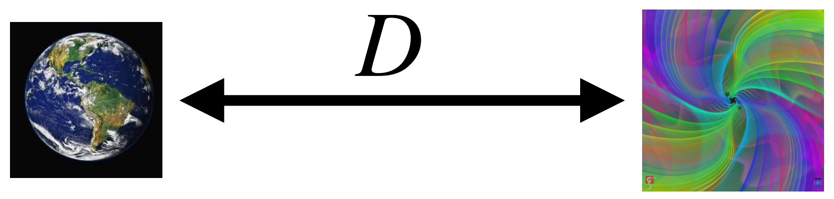
$M \rightarrow$ total mass

$Q \rightarrow$ binary's quadrupole moment

$\mu = \frac{m_1 m_2}{M} \rightarrow$ reduced mass

$\nu = \frac{\mu}{M} \rightarrow$ symmetric mass ratio $\qquad 0 \leq \nu \leq \frac{1}{4}$

- The **farther** the source, the **weaker** the signal on Earth



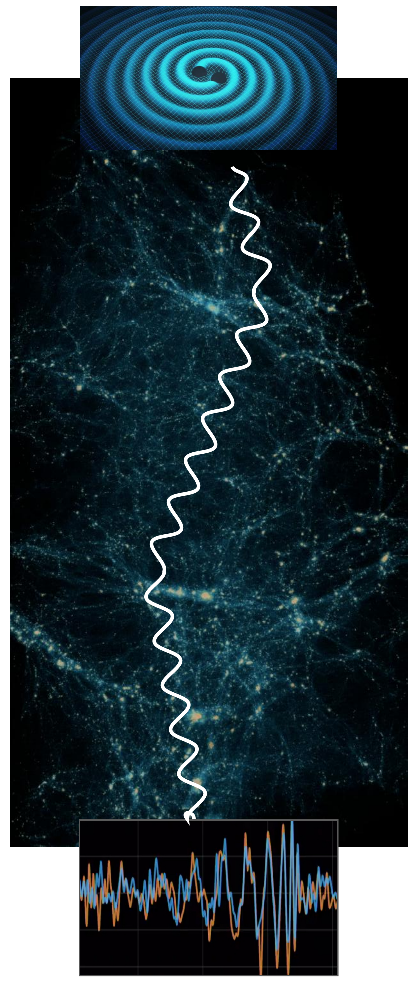
- For a **binary neutron star in Virgo Cluster**: $h \sim 10^{-21}$

- Typical **GW luminosity** (or power): $\mathscr{L}_{\rm GW} \sim \nu^2 \frac{c^5}{G}\left(\frac{\mathrm{v}}{c}\right)^{10}$

$$\mathscr{L}_{\rm GW} \sim 10^{23}\, \mathscr{L}^{\rm EM}_{\rm Sun} \qquad \frac{c^5}{G} \sim 10^{59}\,\frac{\rm erg}{\rm sec}$$

GW power can be similar or larger than the one of **whole visible Universe.**

- GW propagation is (almost) unaffected by matter/energy: **pristine probes.**



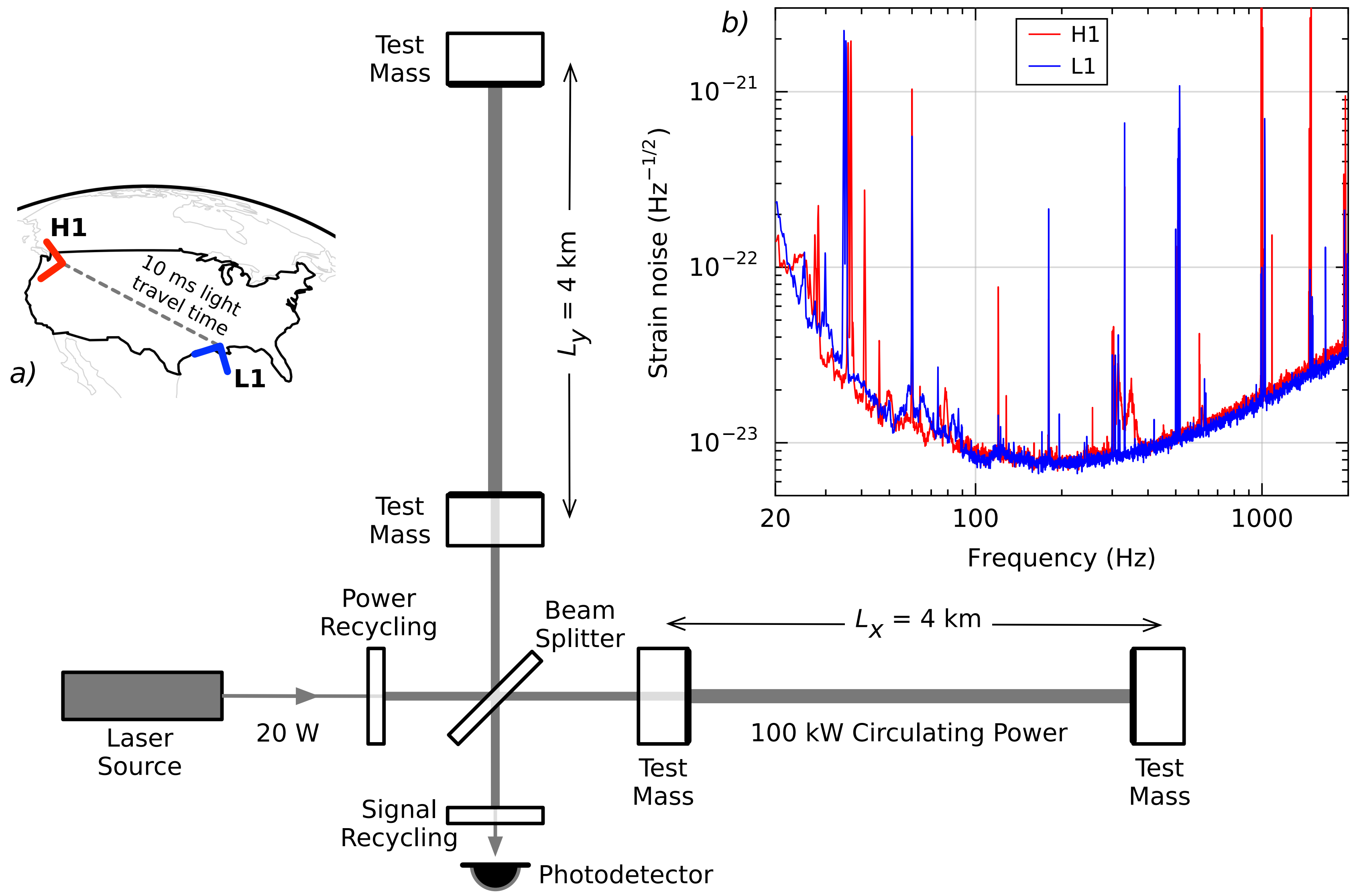
(credit: Zumalacarregui)

How we Detect Gravitational Waves from Binaries

LIGO in Hanford, WA

Virgo in Pisa, Italy

KAGRA, Japan

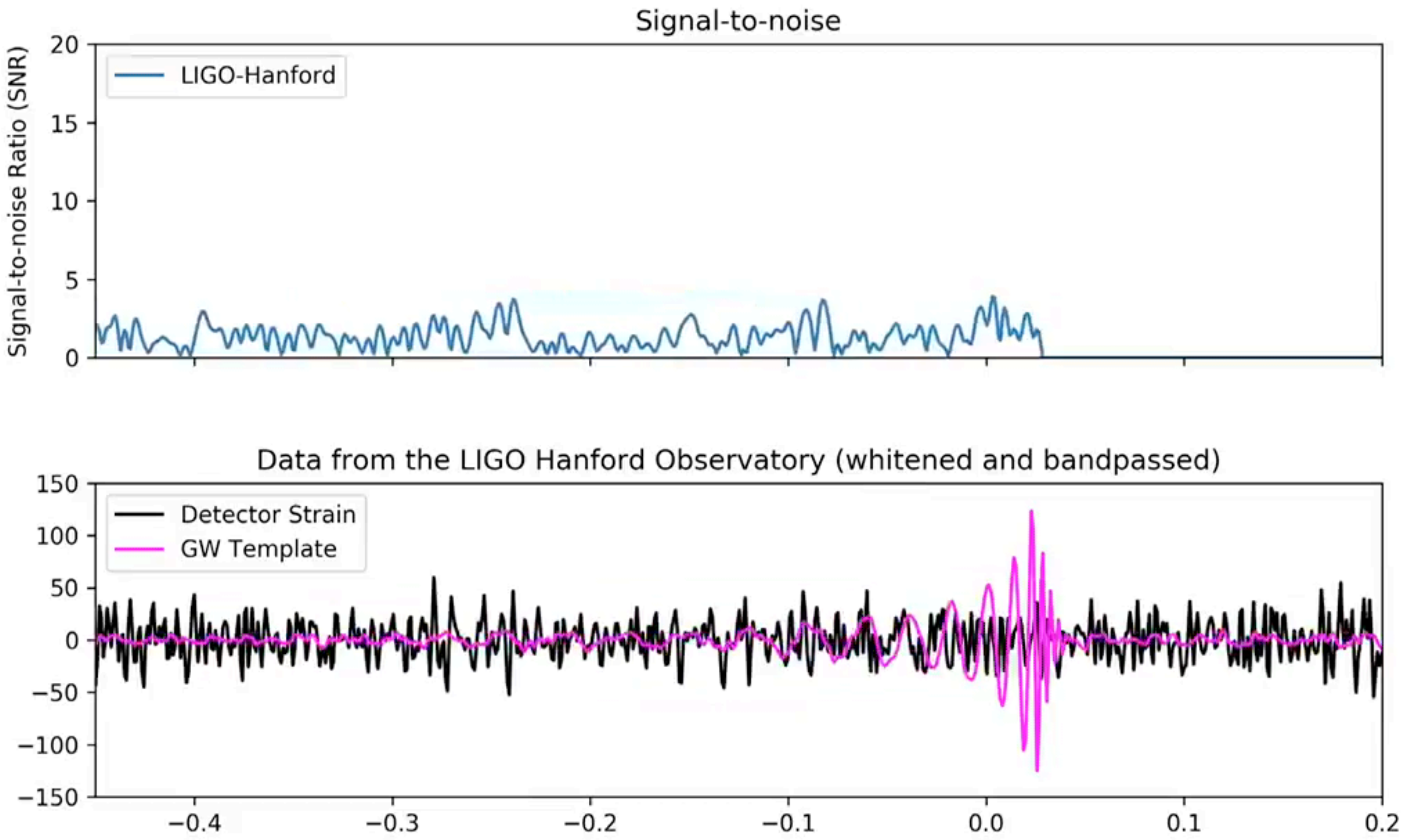


$$\Delta L = L\, h \sim 10^{-16}\,\mathrm{cm}$$

$$L = 4\,\mathrm{km} \Rightarrow h \sim 10^{-21}$$

- LIGO/Virgo measure **displacements of mirrors** at about a **ten-thousandth** of a **proton's diameter.**

- **Matched filtering** (or signal processing) is used to **detect signals from coalescing binaries** composed of black holes and neutron stars.



(credit: Alex Nitz)

- **Bank of templates** contains several hundred thousands of signals; **inference analyses** upon detection to extract source properties use **millions of waveform models**.

What determines the Gravitational-Wave Phase?

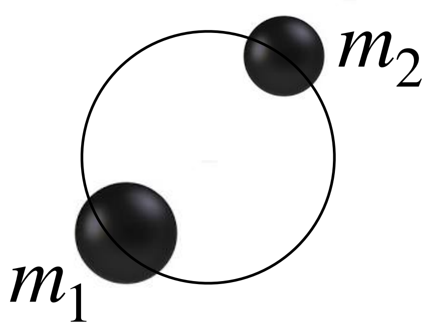
$$h(t) \sim \nu \frac{GM}{c^2 D} \left(\frac{\mathrm{v}}{c}\right)^2 \cos 2\Phi(t)$$

$$\frac{\mathrm{v}}{c} = \left(\frac{GM\omega}{c^3}\right)^{1/3}$$

Kepler law

circular orbits: $\mathrm{v}^2 = r^2\omega^2 = \frac{GM}{r}$

(binary)



$\mathrm{v} \to$ velocity

$M \to$ total mass

$Q \to$ binary's quadrupole moment

$\mu = \frac{m_1 m_2}{M} \to$ reduced mass

$\nu = \frac{\mu}{M} \to$ symmetric mass ratio $\quad 0 \leq \nu \leq \frac{1}{4}$

- How do we determine the **fast-varying GW phase** $\Phi_{\mathrm{GW}}(t) = 2\Phi(t)$?

  - Binding **energy**:

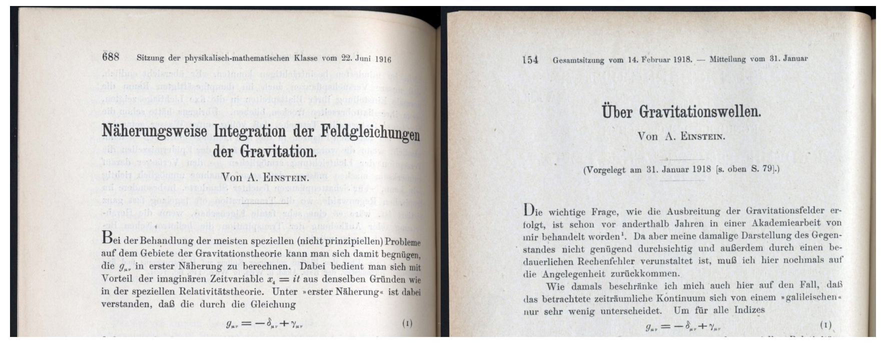
$$E(\mathrm{v}) = -\frac{\mu}{2}\mathrm{v}^2 + \cdots$$

Newtonian gravity

  - **GW luminosity**:

$$\mathscr{L}_{\mathrm{GW}}(\mathrm{v}) = \frac{32}{5}\nu^2 \frac{c^5}{G}\left(\frac{\mathrm{v}}{c}\right)^{10} + \cdots$$

Einstein 1916-1918



- **Energy-balance** equation:

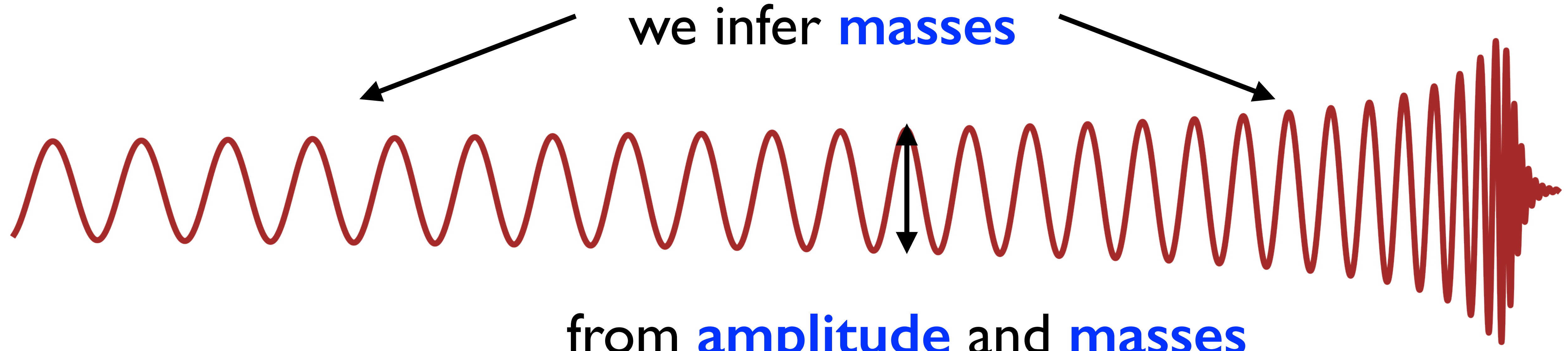
$$\frac{dE(\omega)}{dt} = -\mathscr{L}_{\mathrm{GW}}(\omega) \longrightarrow \dot{\omega}(t) = -\frac{\mathscr{L}_{\mathrm{GW}}(\omega)}{dE(\omega)/d\omega}$$

- **Gravitational-wave phase**:

$$\Phi_{\mathrm{GW}}(t) = 2\Phi(t) = \frac{1}{\pi}\int^t \omega(t')\,dt'$$

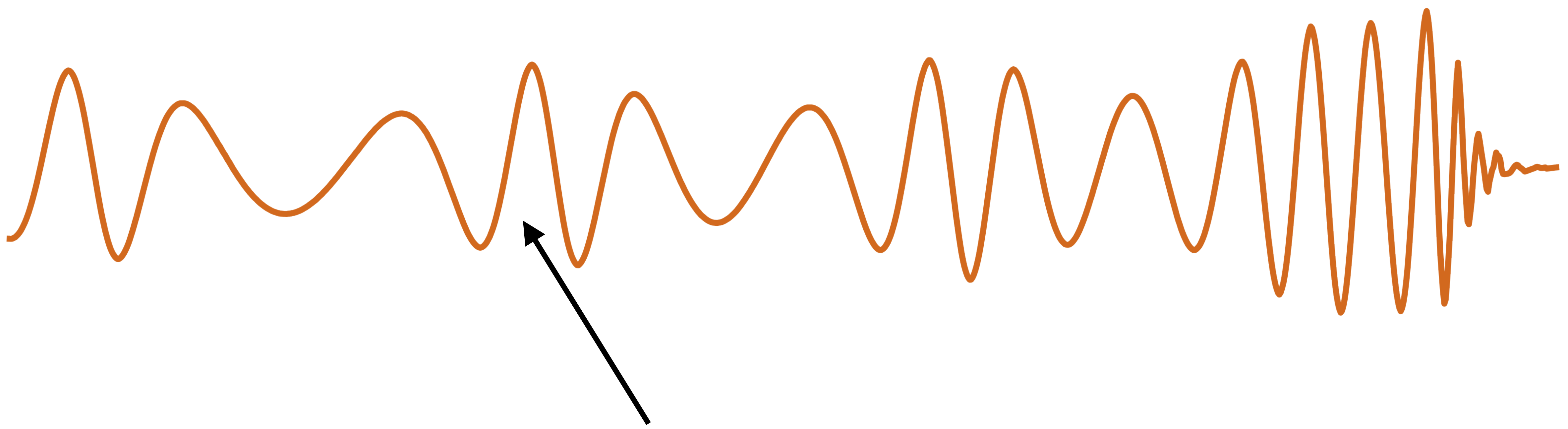
Properties of Astrophysical Sources via Gravitational Waves

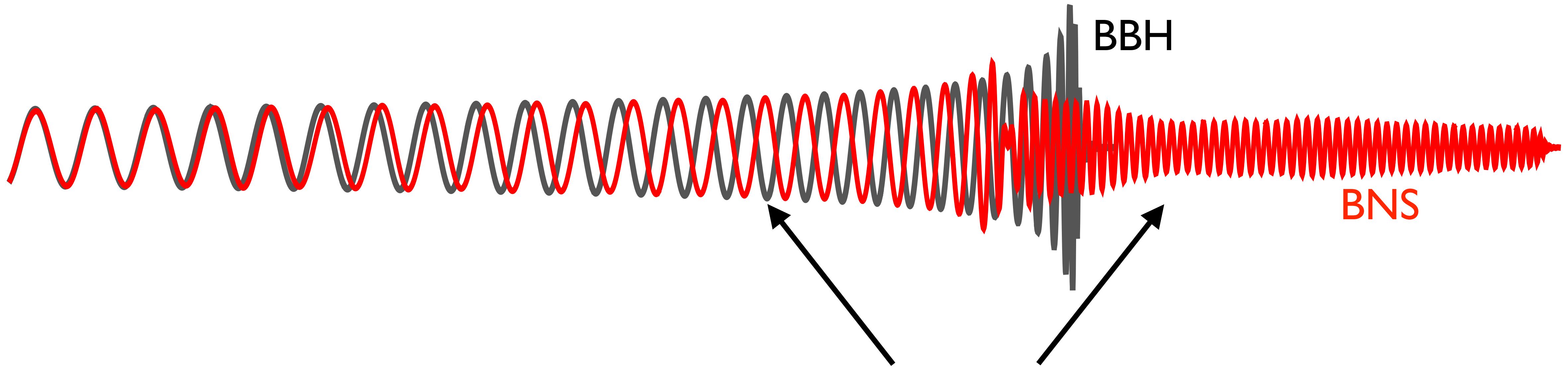
from **frequency evolution**
we infer **masses**



from **amplitude** and **masses**
we infer **distance**

from **time of arrival**, **amplitude and phase** at detectors we infer **sky location**





from **modulations** of **amplitude and phase**
we infer **spins and eccentricity**

from **differences in** late **inspiral and merger of BBHs**
we infer **tidal deformation**, and **NS composition**

By **comparing to waveforms with deviations from GR,** we can **probe** the theory of **gravity**

Solving Two-Body Problem in General Relativity

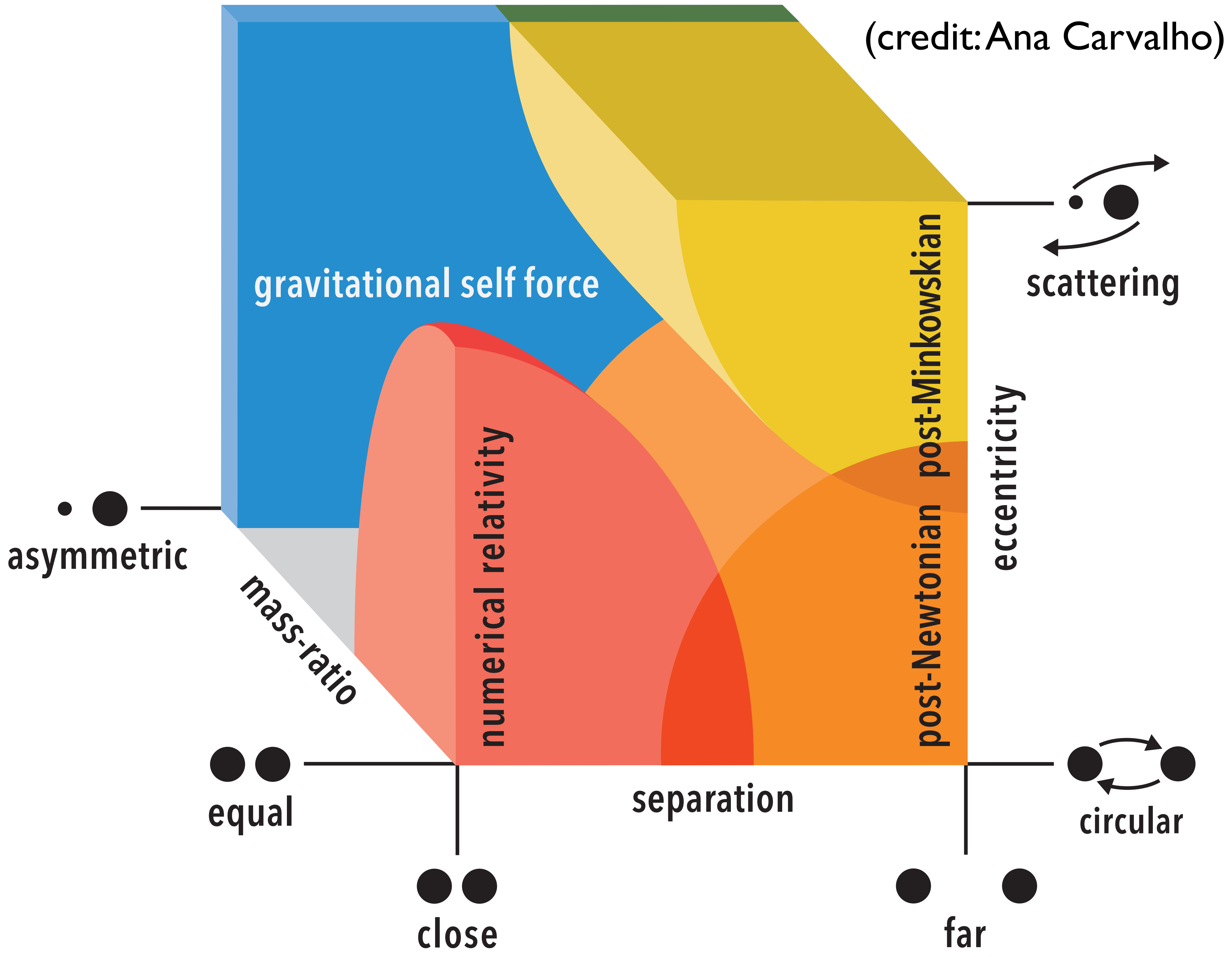
- **GR** is **non-linear theory**.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

-**accurately**, but **numerically** on supercomputers (**slow** way)

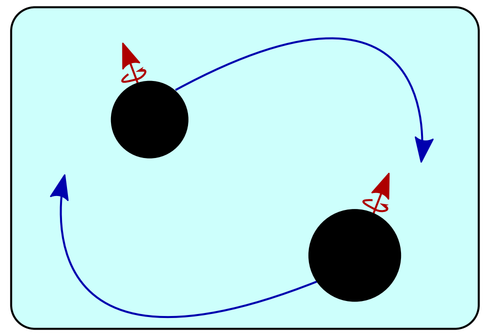


(credit: Ana Carvalho)

- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Post-Newtonian** (large separation, and slow motion)

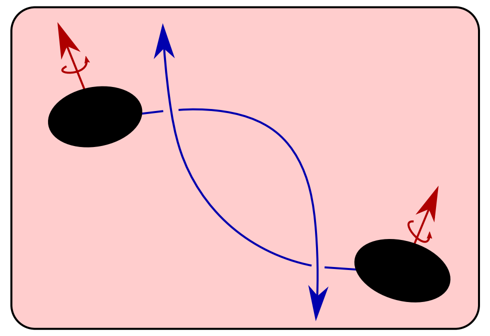
expansion in $\mathrm{v}^2/c^2 \sim GM/rc^2$



(Droste, Lorentz, Einstein, Infeld, Hoffmann, … Blanchet, Damour, Iyer, Jaranowski, Schäfer, Will, … Goldberger, Porto, Rothstein, …)

- **Post-Minkowskian** (large separation, and fast motion)

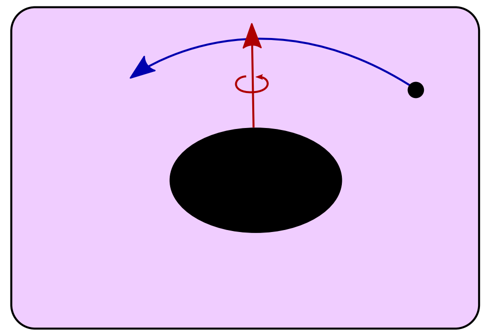
expansion in G



(Westpfahl, … Bern, Cheung, Hermmann, Parra-Martinez, Roiban, Rothstein, Solon, Shen, Zeng … Khälin, Porto, … Mogull, Jakobsen, Plefka, Steinhoff … Damgaard, Vanhove … Brandhuber, Travaglini …)

- **Gravitational self-force** (strong field)

expansion in m_2/m_1



(Barack, Deitweiler, Mino, Poisson, Pound, Quinn, Sasaki, Tanaka, van de Meent, Wald, Warburton, Wardell, Whiting, …)

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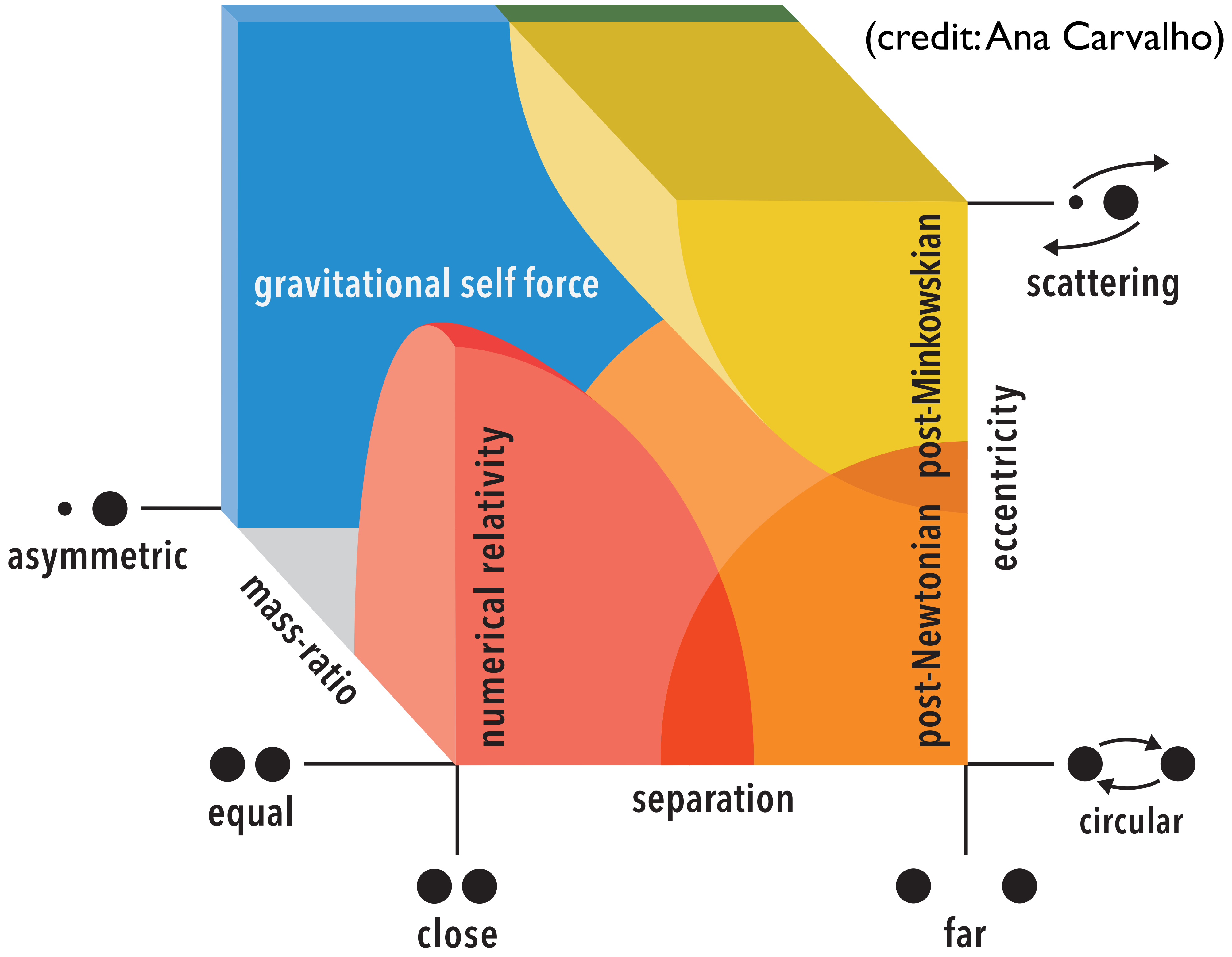
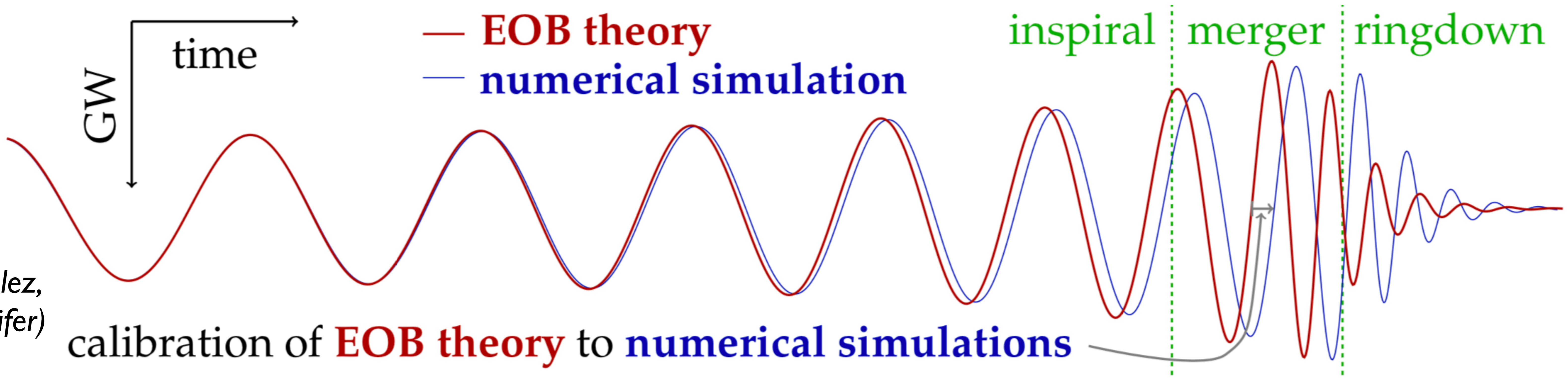
-accurately, but numerically on supercomputers (slow way)

- Synergy between analytical and numerical relativity is crucial to provide GW detectors with templates to use for searches and inference analyses.



- Effective-one-body (EOB) theory (combines results from all methods, i.e., for entire coalescence)

(AB, Damour, … Barausse, Bohé, Cotesta, Estellés, Khalil, Mihaylov, Ossokine, Pan, Pompili, Pürrer, Ramos-Buades, Shao, Taracchini, … Nagar, Bernuzzi, Agathos, Albanese, Albertini, Bonino, Gamba, Gonzalez, Messina, Placidi, Rettegno, Riemenschneider, …. Iyer, Jaranowski, Schäfer)



Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**. $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Einstein's field equations can be solved:

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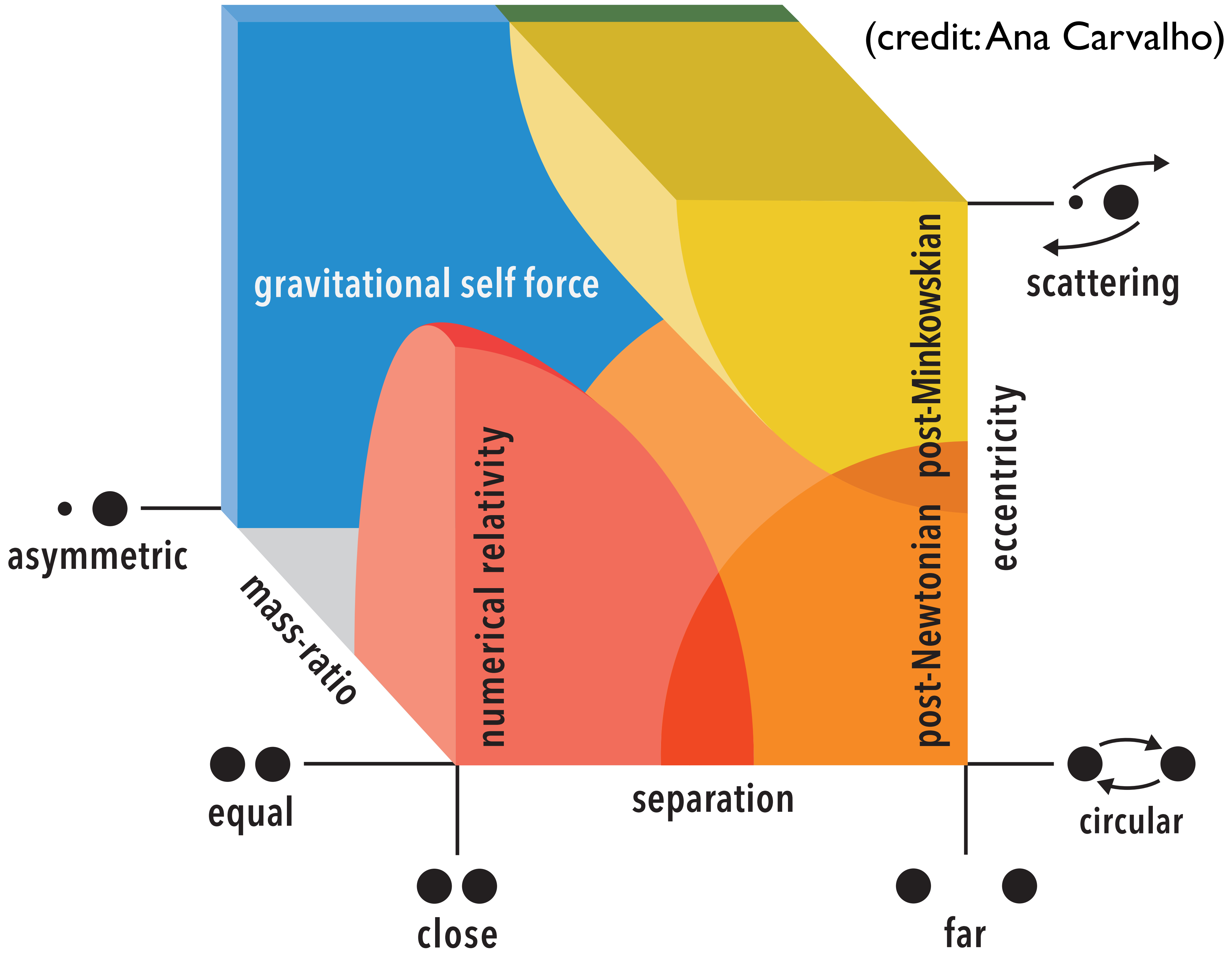
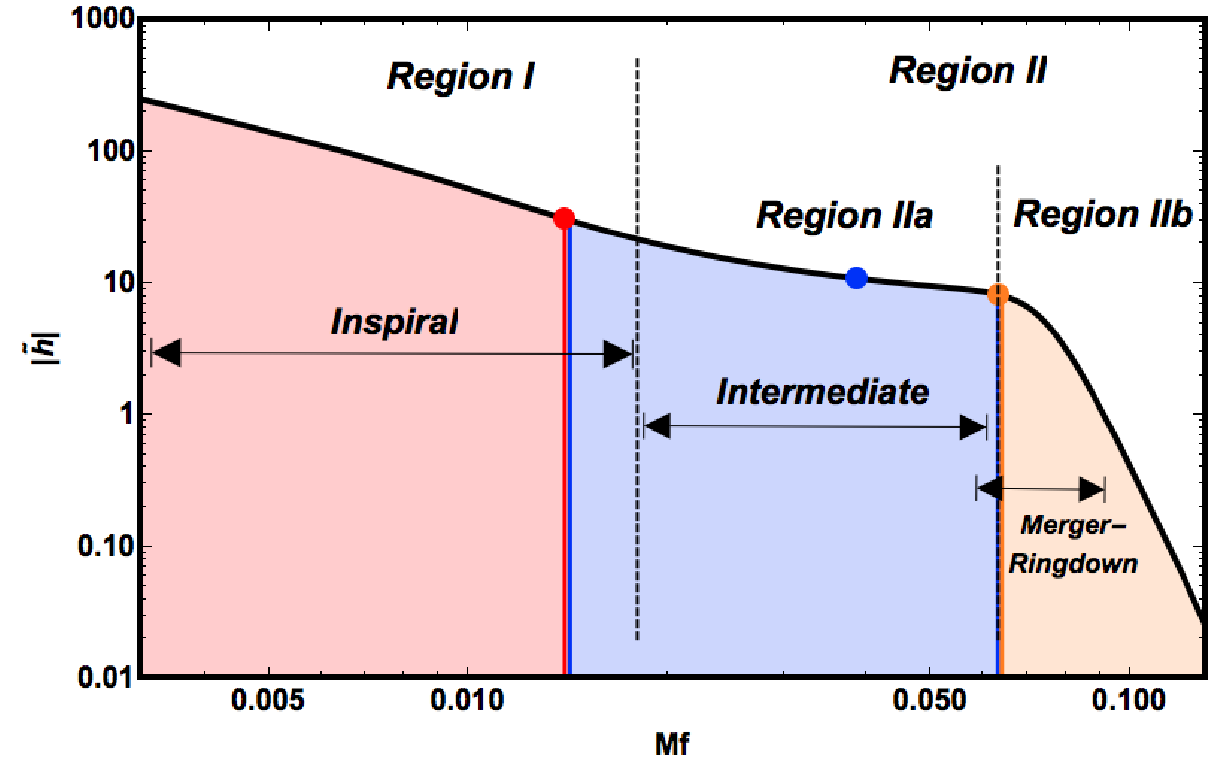


- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

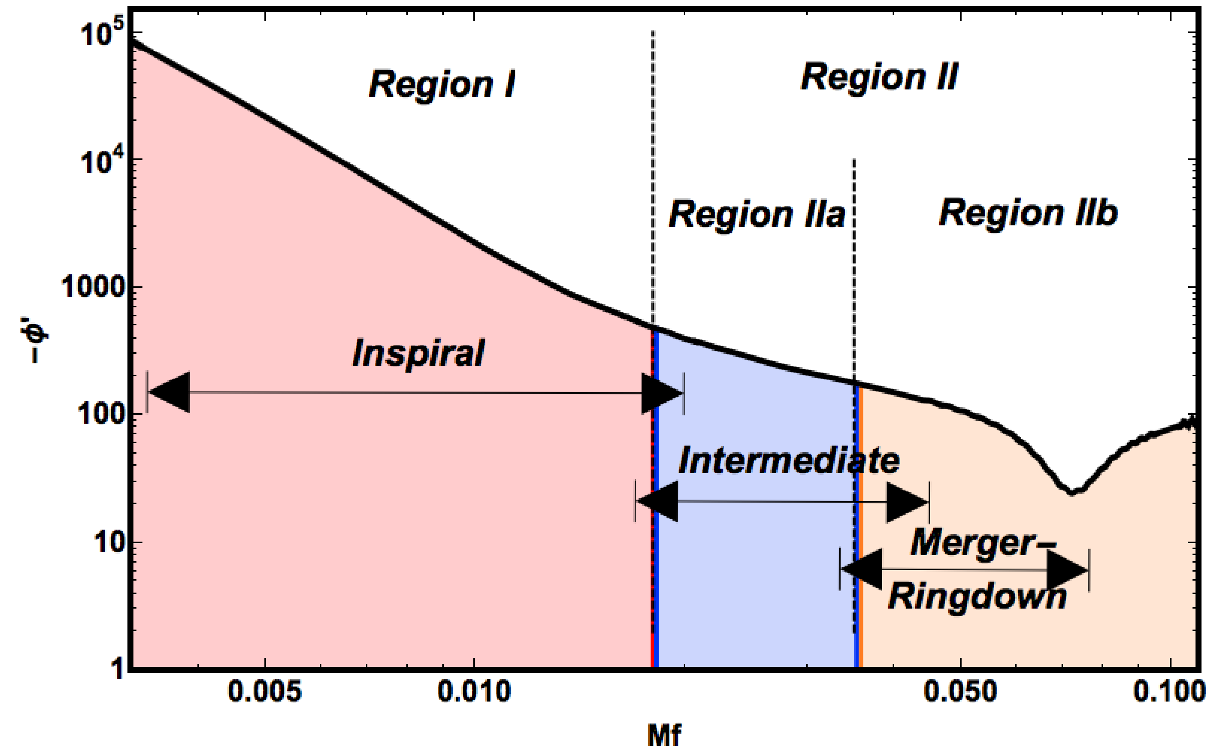
- **Phenomenological frequency-domain** waveforms (Phenom) built fitting to EOB, PN and NR.

(Ajith, Hannam, Husa, Ohme, … Bohé, Colleoni, García, Hamilton, Khan, London, Estellés, Pratten, Pürrer, Ramos-Buades, Quirós, Santamaria, Schmidt, Shrobana, Thompson, …)

Frequency-domain GW amplitude



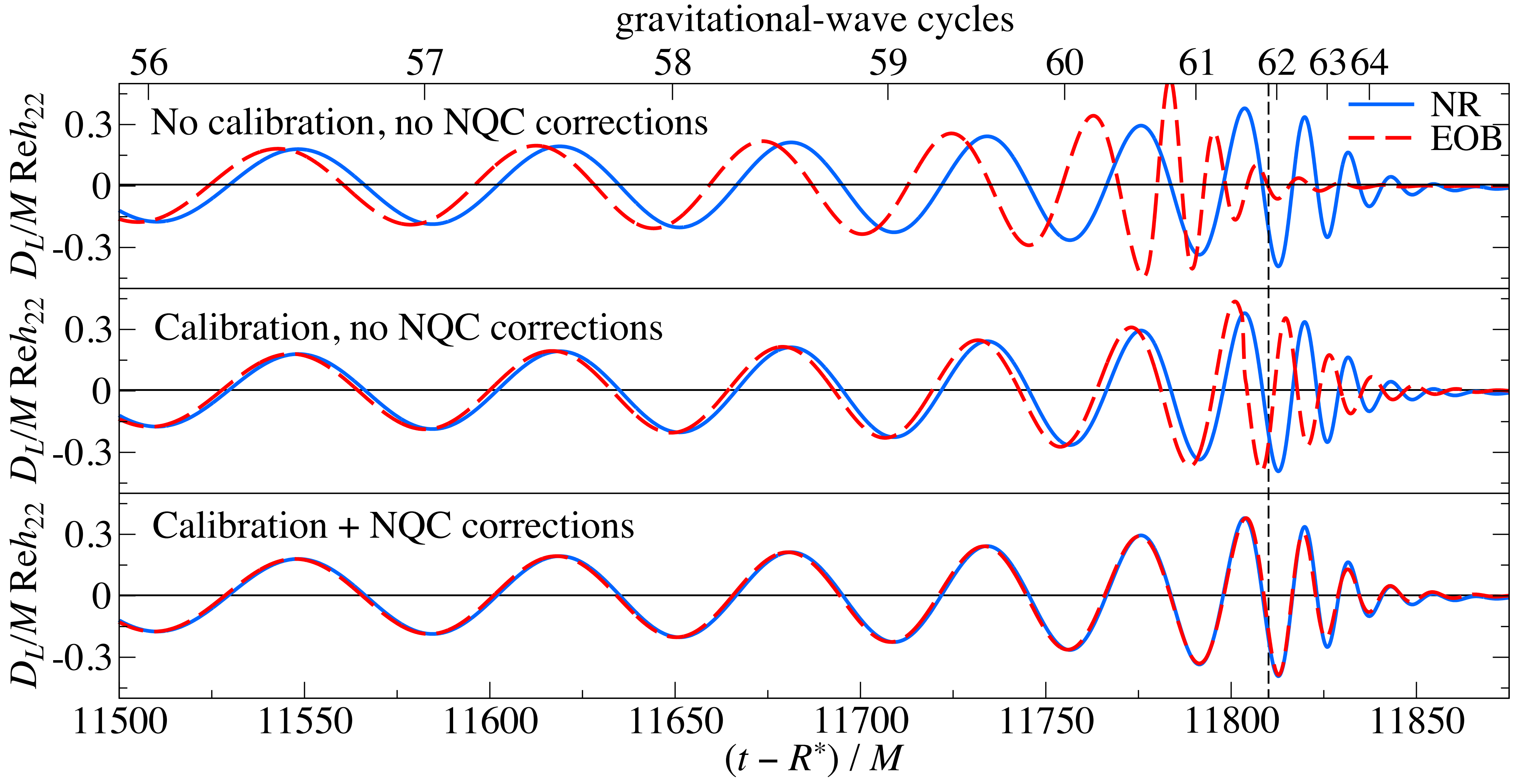
Frequency-domain GW phase derivative



(Khan+arXiv:1508.07253)

Completing Waveform Models with NR Information & Template Bank

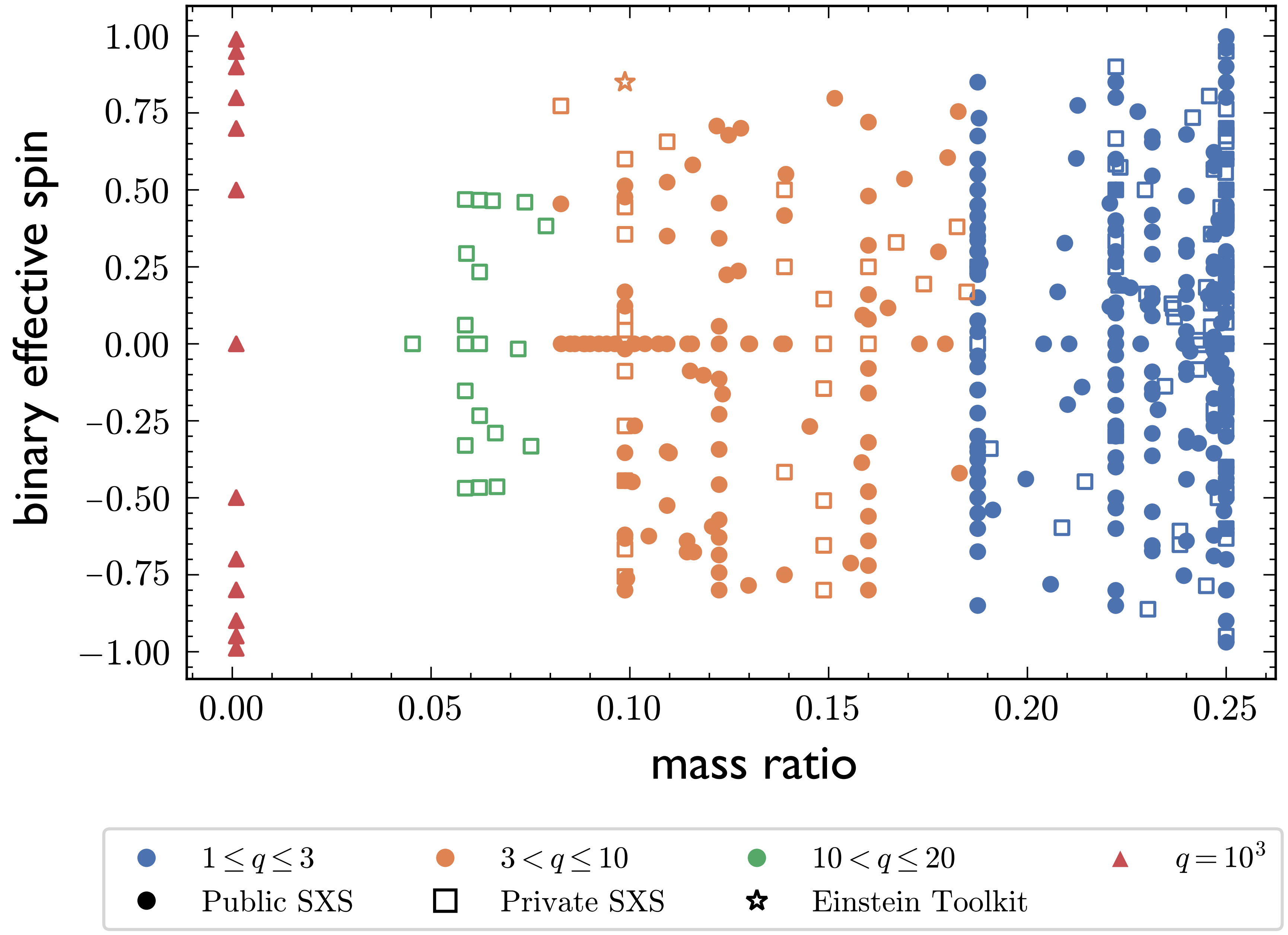
- We calibrate models to **inspiral-merger-ringdown NR** waveforms.



(credit: Andrea Taracchini)

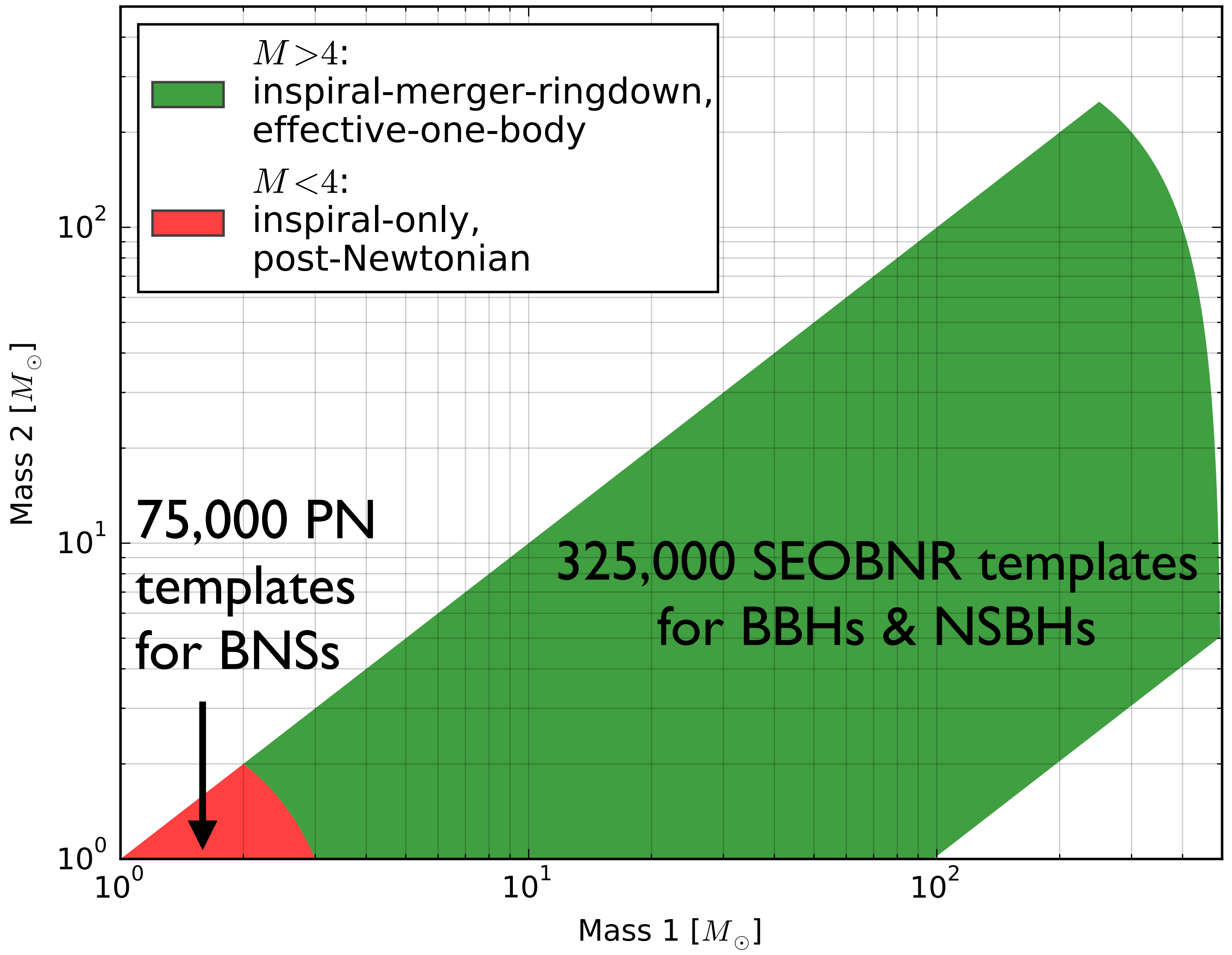
(NQC: non-quasi-circular corrections)

calibration using **441 NR waveforms**



(Pompili+arXiv:2303.18039)

- **Matched filtering** employed in LIGO/Virgo searches.



(Dal Canton & Harry arXiv:1705.01845)

(**SXS:** Simulating eXtreme Spacetime)

pySEOBNR

*(**SEOBNR:** Pompili+23, van de Meent+23, Ramos-Buades+23, Mihaylov+23, Khalil+23)*

*(**IMRPhenom:** Pratten+20, García-Quíros+20, Estélles+21, Thompson+23)*

*(**TEOBResumS:** Akcay+21, Gamba+22, Nagar+23)*

*(**NRSur:** Blackman+17, Varma+19, Yoo+23, Magaña Zertuche+24)*

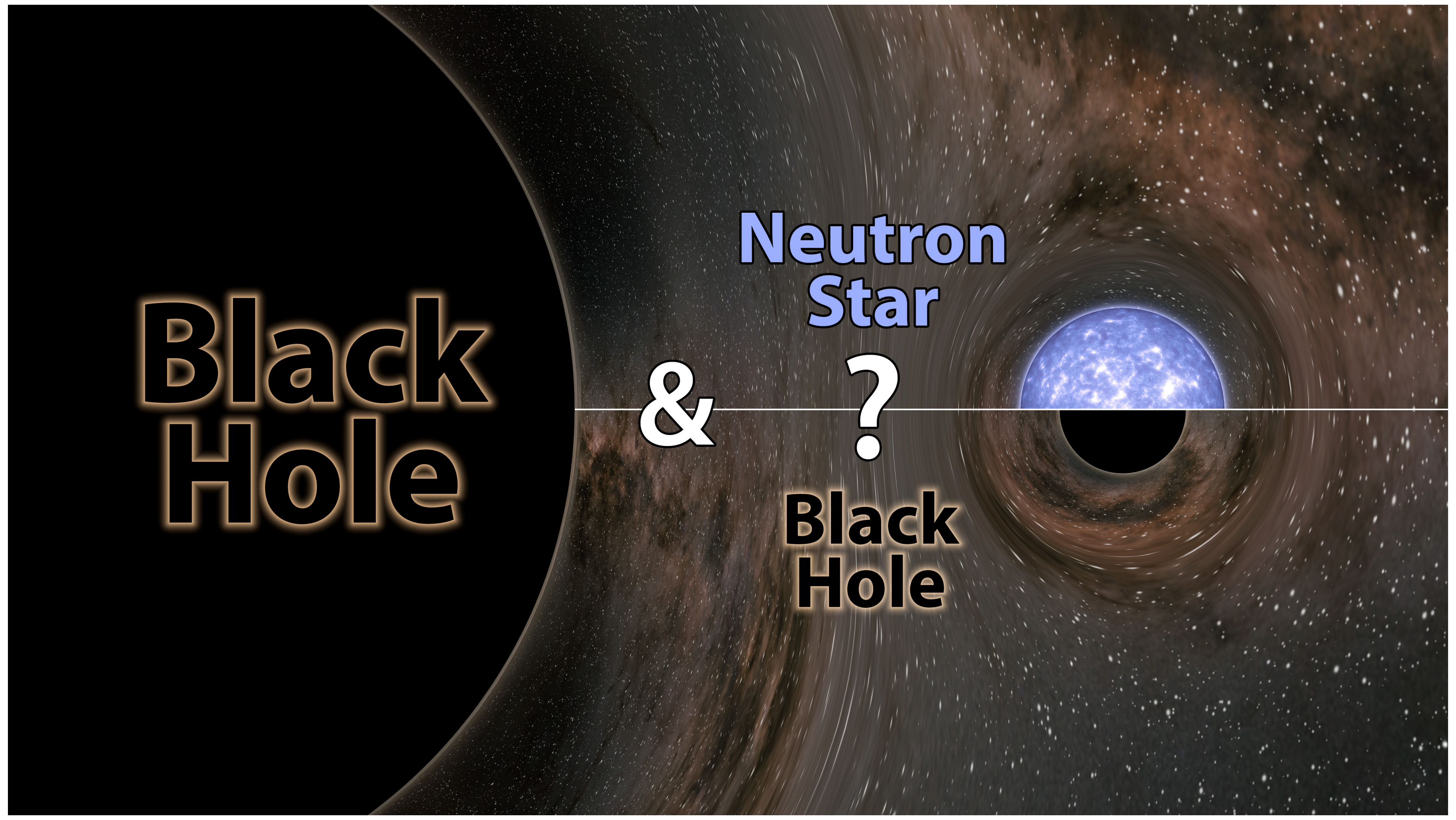
GW190814: a Binary with a Puzzling Companion

- Either the **largest neutron star** or the **smallest black hole.**

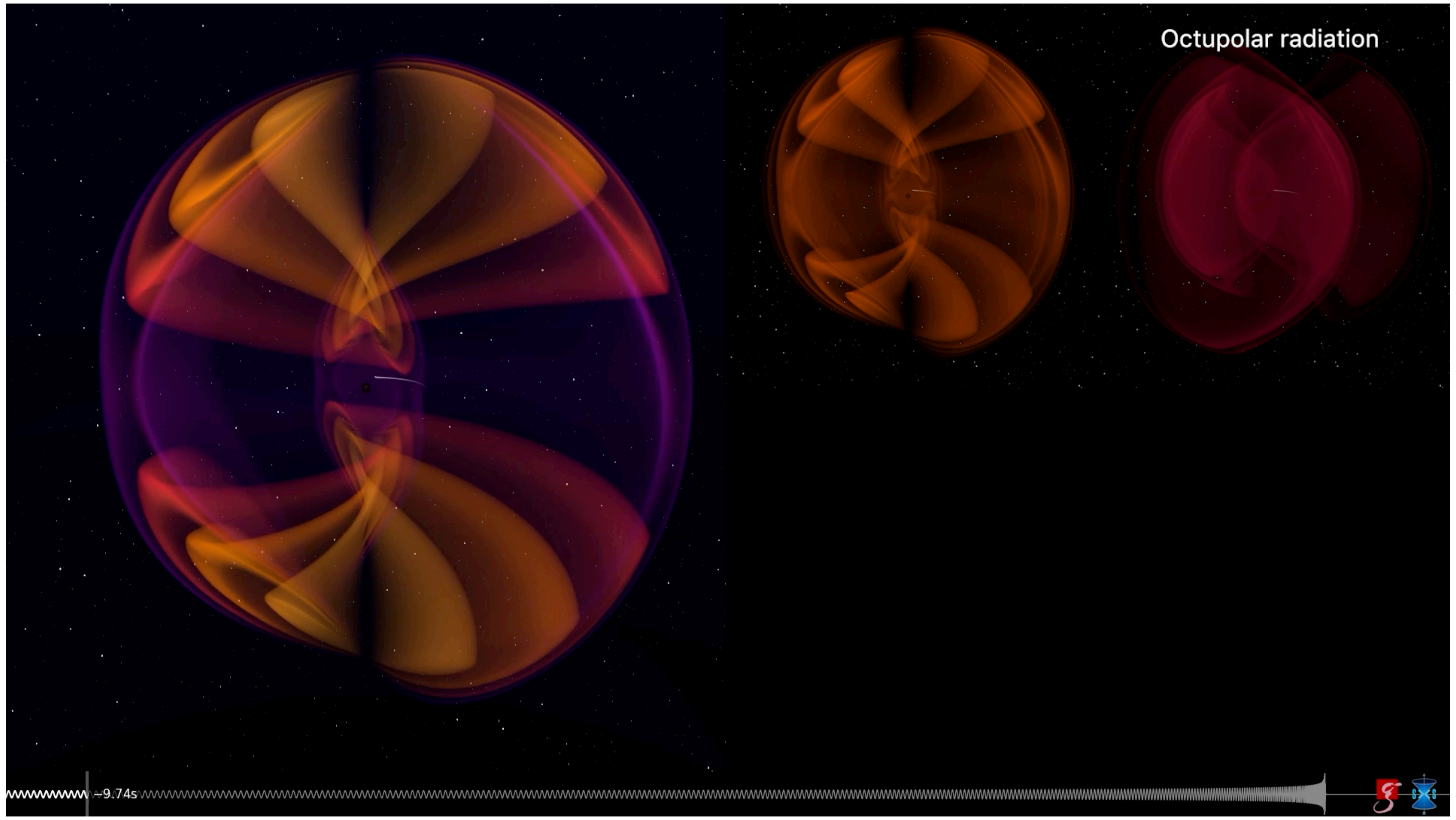
$$m_1 = 23.2^{+1.1}_{-1.0}\, M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\, M_\odot$$

- The **more substructure and complexity** the binary has (e.g., masses or spins of BHs are different) **the richer is the spectrum of radiation** emitted.

$$h_+ - i h_\times = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\varphi, \iota)\, h_{\ell m}(t)$$



(LIGO/Caltech/MIT/R. Hurt (IPAC))



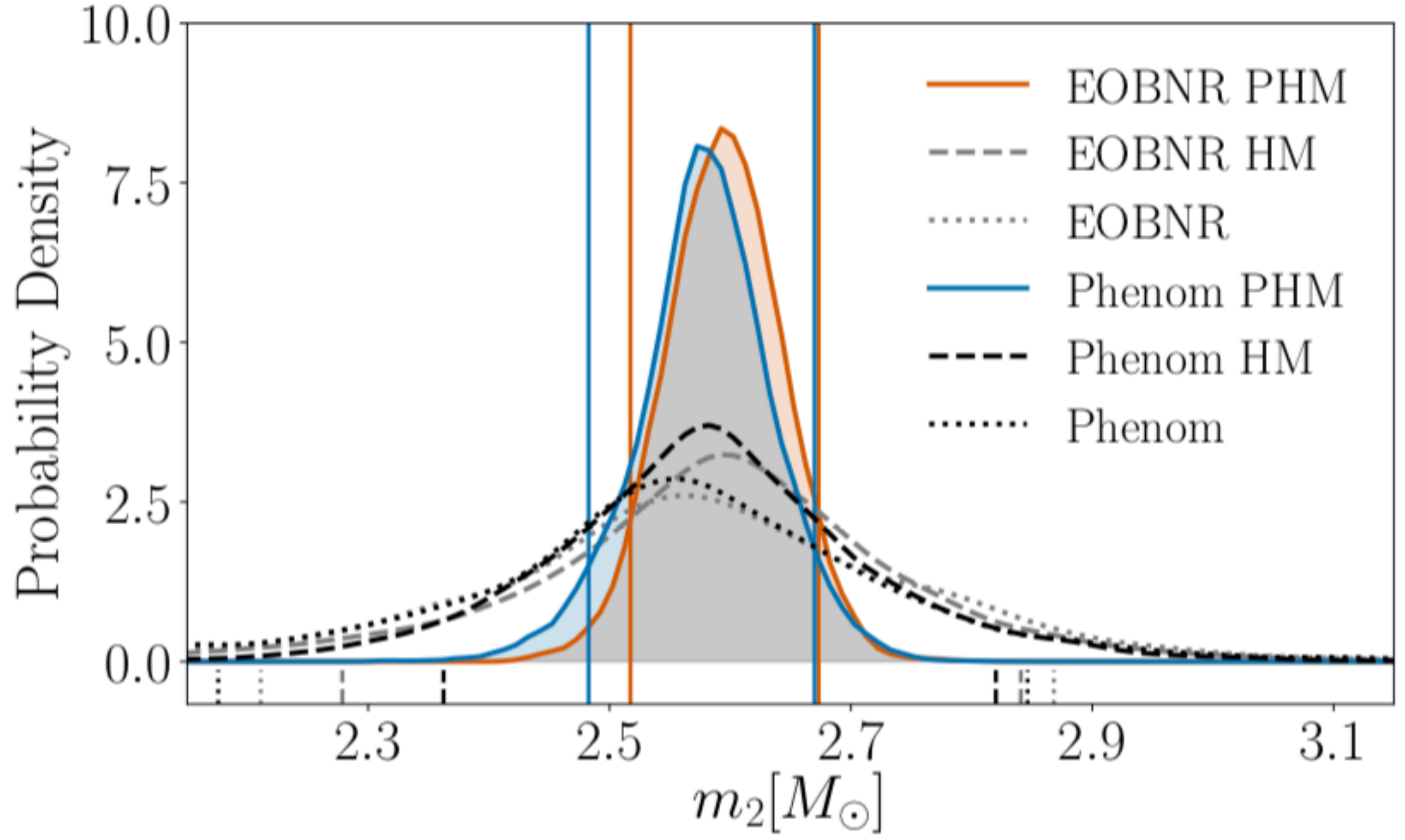
(credit: Fischer/Vu, Pfeiffer, Ossokine & AB; SXS Collaboration)

GW190814: a Binary with a Puzzling Companion (contd.)

- Either the **largest neutron star** or the **smallest black hole.**

$$m_1 = 23.2^{+1.1}_{-1.0}\, M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\, M_\odot$$

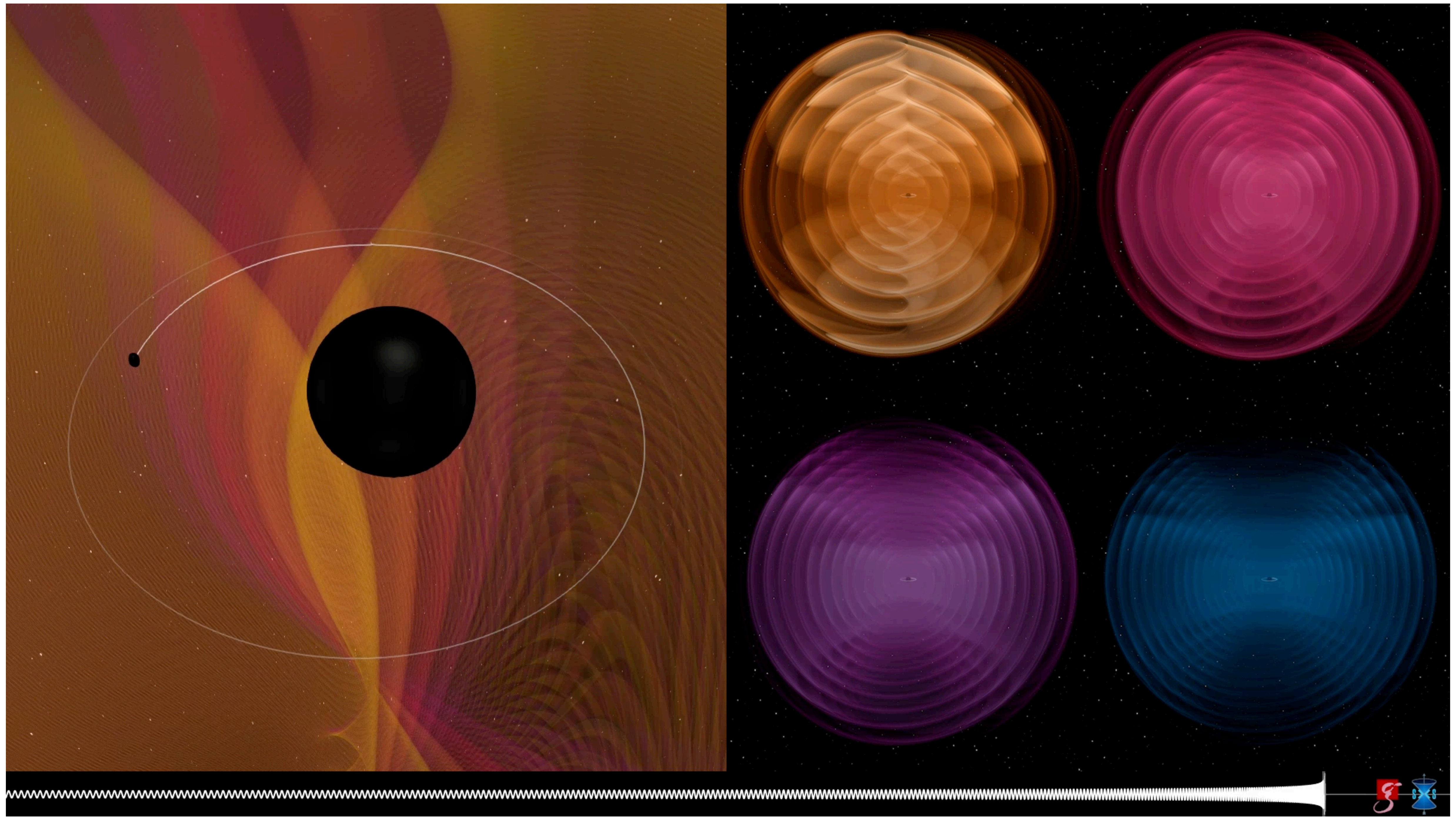
(Abbott et al. ApJ Lett 896(2020) L44)



- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

- The **more substructure and complexity** the binary has (e.g., masses or spins of BHs are different) **the richer is the spectrum of radiation** emitted.

$$h_+ - ih_\times = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\varphi, \iota)\, h_{\ell m}(t)$$



(credit: Fischer,/Vu Pfeiffer, Ossokine & AB; SXS Collaboration)

Accuracy of Spin-Precessing Waveform Models

quasi-circular, spin-precessing case

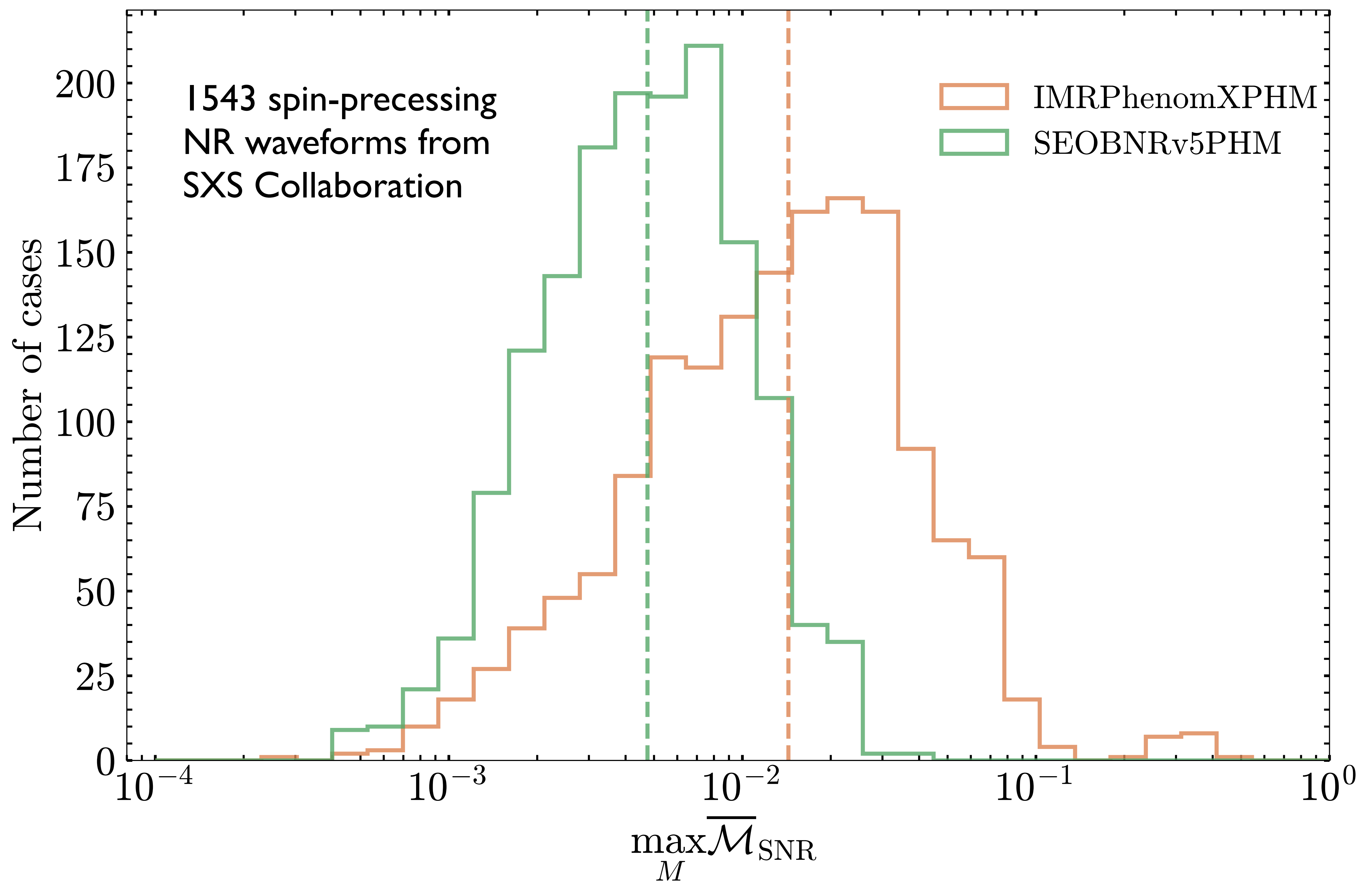
(Ramos-Buades, AB, Khalil, Estelles, Pompili & Ossokine, arXiv: 2303.18046)

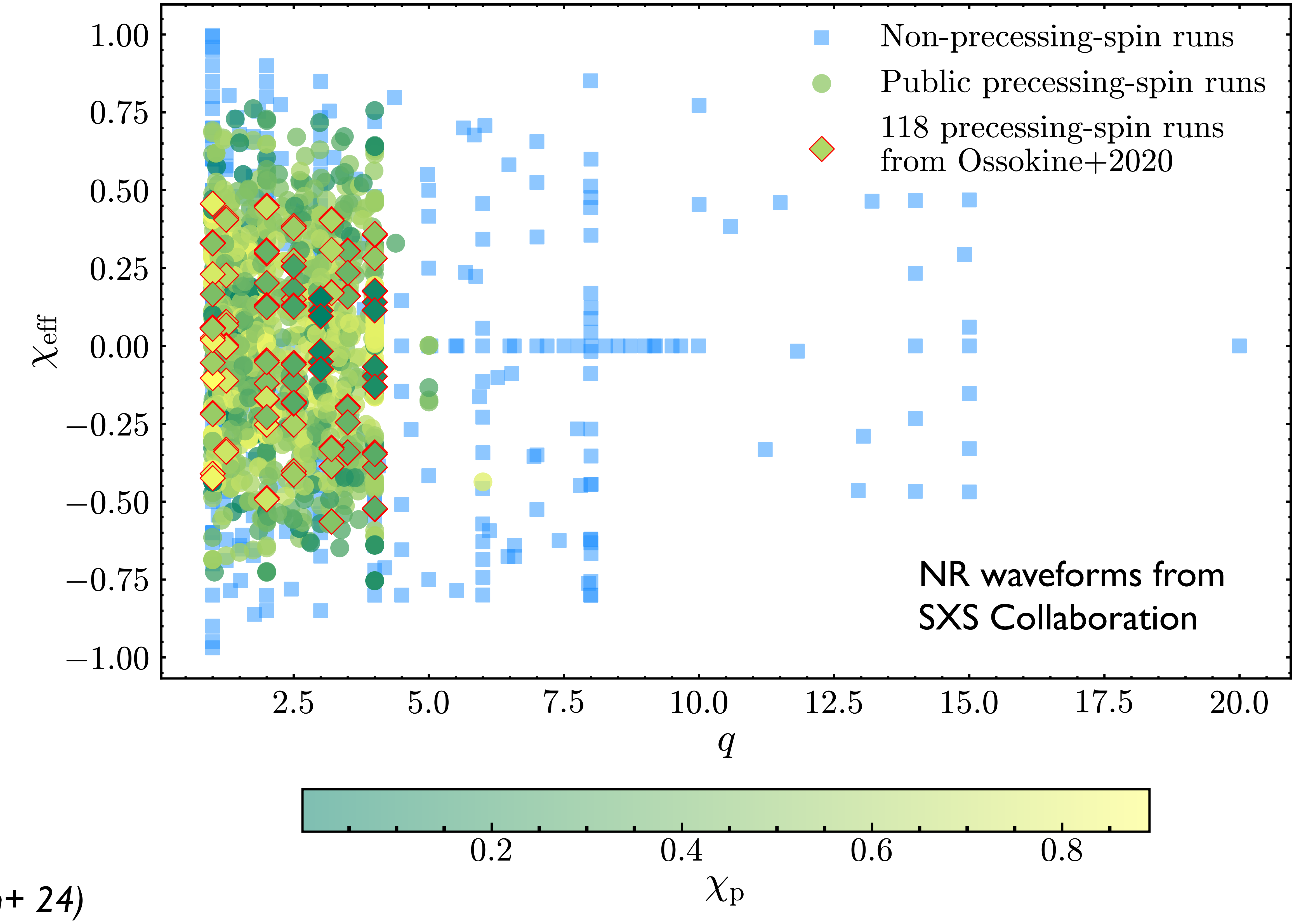
$$\mathcal{M} = 1 - \max_{t_0,\phi_0} \frac{(h_{\rm model}, h_{\rm NR})}{\sqrt{(h_{\rm model}, h_{\rm model})(h_{\rm NR}, h_{\rm NR})}} \qquad (h, g) = 4\,\mathrm{Re}\left[\int_{f_{\rm min}}^{f_{\rm max}} \frac{h(f)\, g^*(f)\, df}{S_n(f)}\right]$$

$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right) \cdot \hat{\mathbf{L}}$$

Mismatch $\mathcal{M} = 0$ implies models & NR match perfectly

χ_p measures the spin components on the orbital plane





(see recent improvements of IMRPhenomXPHM, e.g., Hamilton+21-23, Yu+ 23, Ghosh+ 23, Thompson+ 24)

Accuracy of Spin-Precessing Waveform Models

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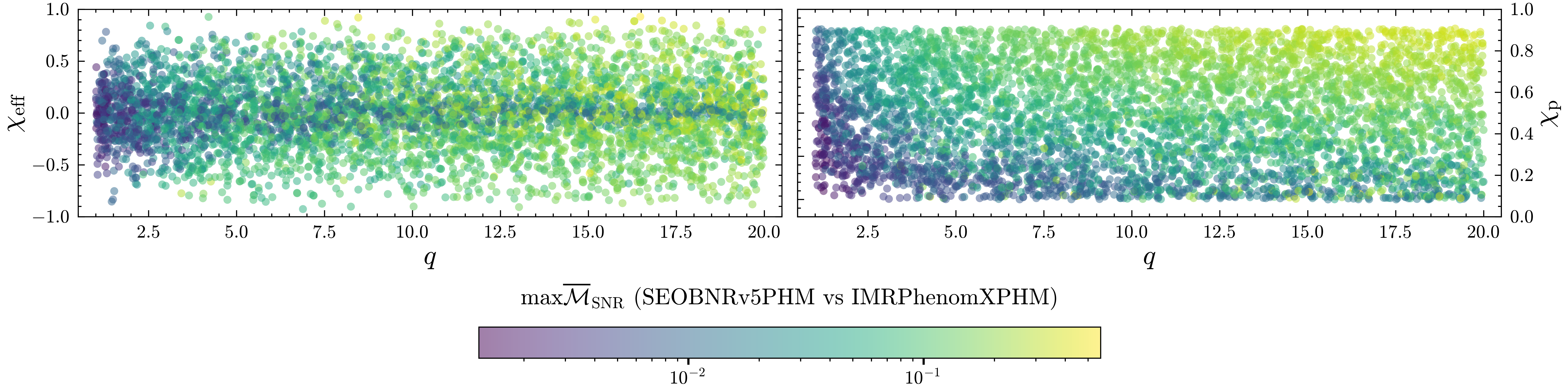
Mismatch $\mathcal{M} = 0$ implies models & NR match perfectly

χ_p measures the spin components on the orbital plane

mismatch against models



- We should **care about systematics.**

Systematics in the Spin-Precessing Quasi-Circular Sector

quasi-circular, spin-precessing case

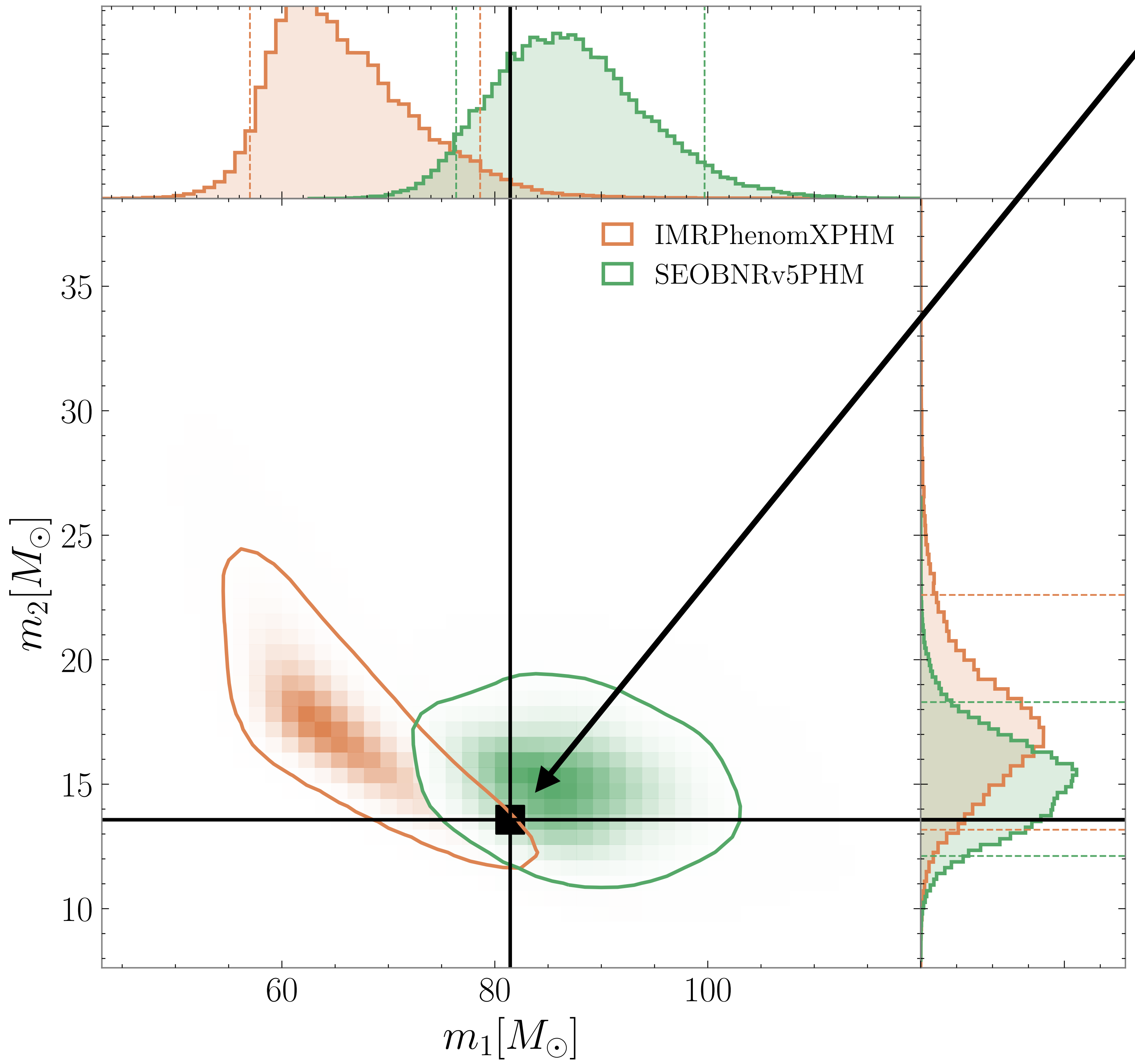
(Ramos-Buades, AB, Khalil, Estelles, Pompili & Ossokine, arXiv: 2303.18046)

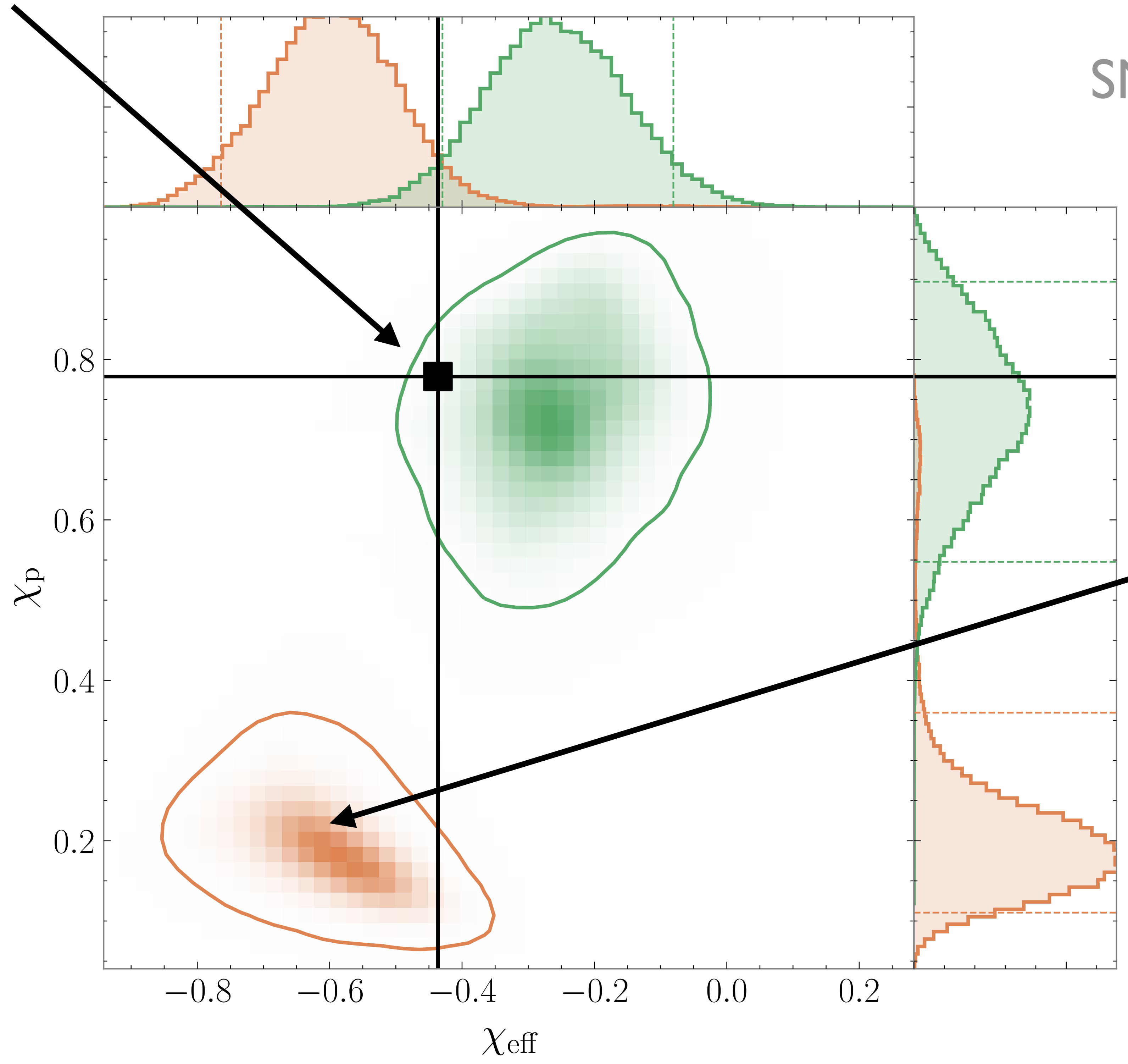
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$

χ_p measures the spin components on the orbital plane

$\mathcal{M}$(IMRPhenomXPHM | NR) = 12 % $\mathcal{M}$(SEOBNRv5PHM | NR) = 2 %

- Synthetic **NR signal is injected, and recovered with both models**





SNR = 20 with O5

- Due to larger systematics **model with $\mathcal{M} \sim 12\,\%$ erroneously measures low spin-precession**, and binary's formation scenario.

(see also Kolitsidou+ 24, MacUilliam+24)

Advances in Numerical Relativity: Spin Precession and Eccentricity

- Expanded parameter-space coverage of **quasi-circular spin-precessing**, and **eccentric spin-precessing** BBH simulations, which is also important for construction of NR **surrogate waveform models.**

- **NR simulations:**

quasi-circular

(**SXS:** *Boyle+19, Ossokine+ 20)*

(**UIB/ U of Cardiff:** *Husa+15)*

(**RIT:** *Healy+ 17, 19, 20)*

(**GT:** *Jani+ 17)*

(**U of Cardiff:** *Hamilton+ 23)*

eccentric (bound)

(**UIB:** *Ramos-Buades+ 19)*

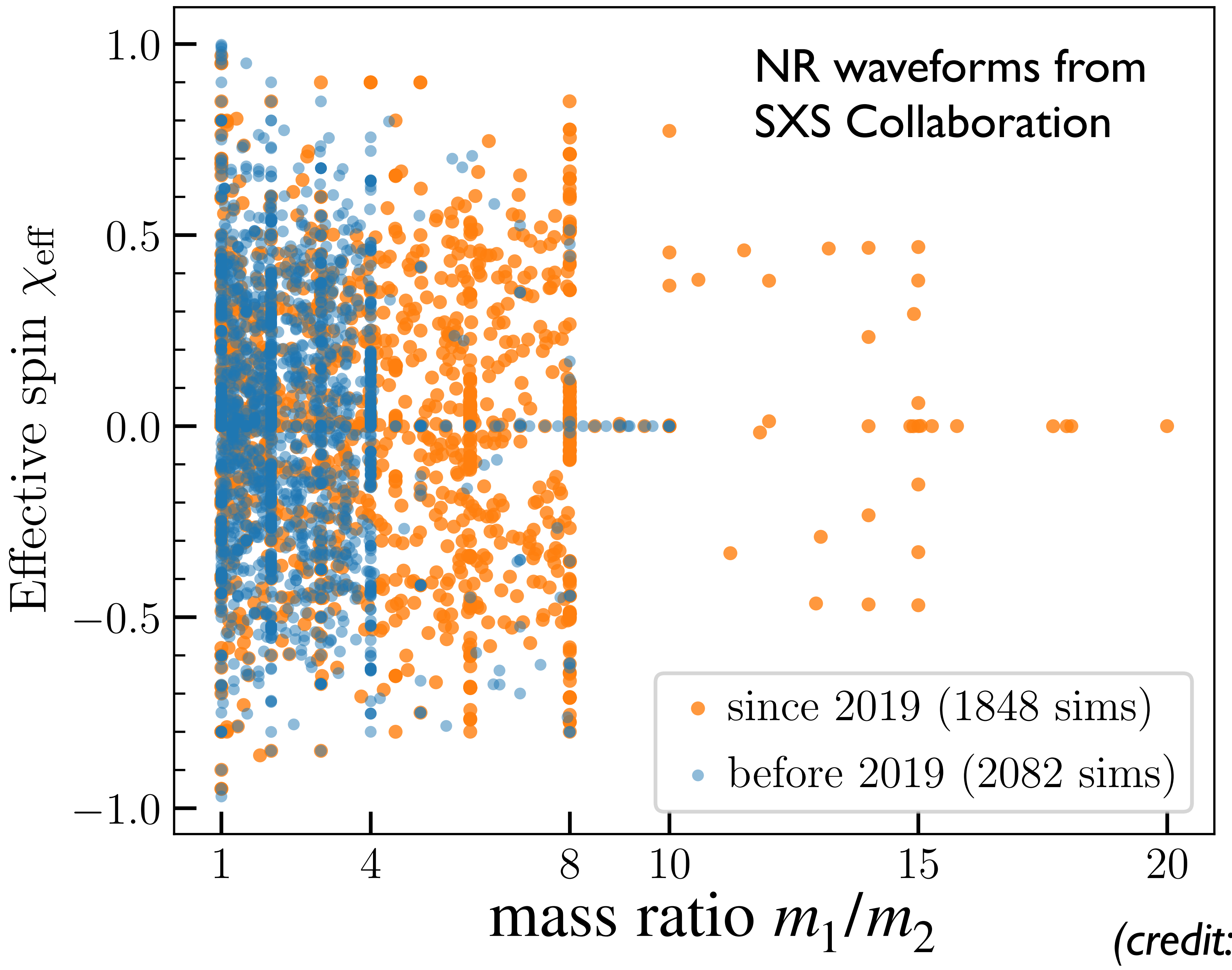
(**AEI:** *Ramos-Buades+ 22)*

(**UT, Austin:** *Ferguson+ 23)*

(**NCSA:** *Huerta+ 19)*

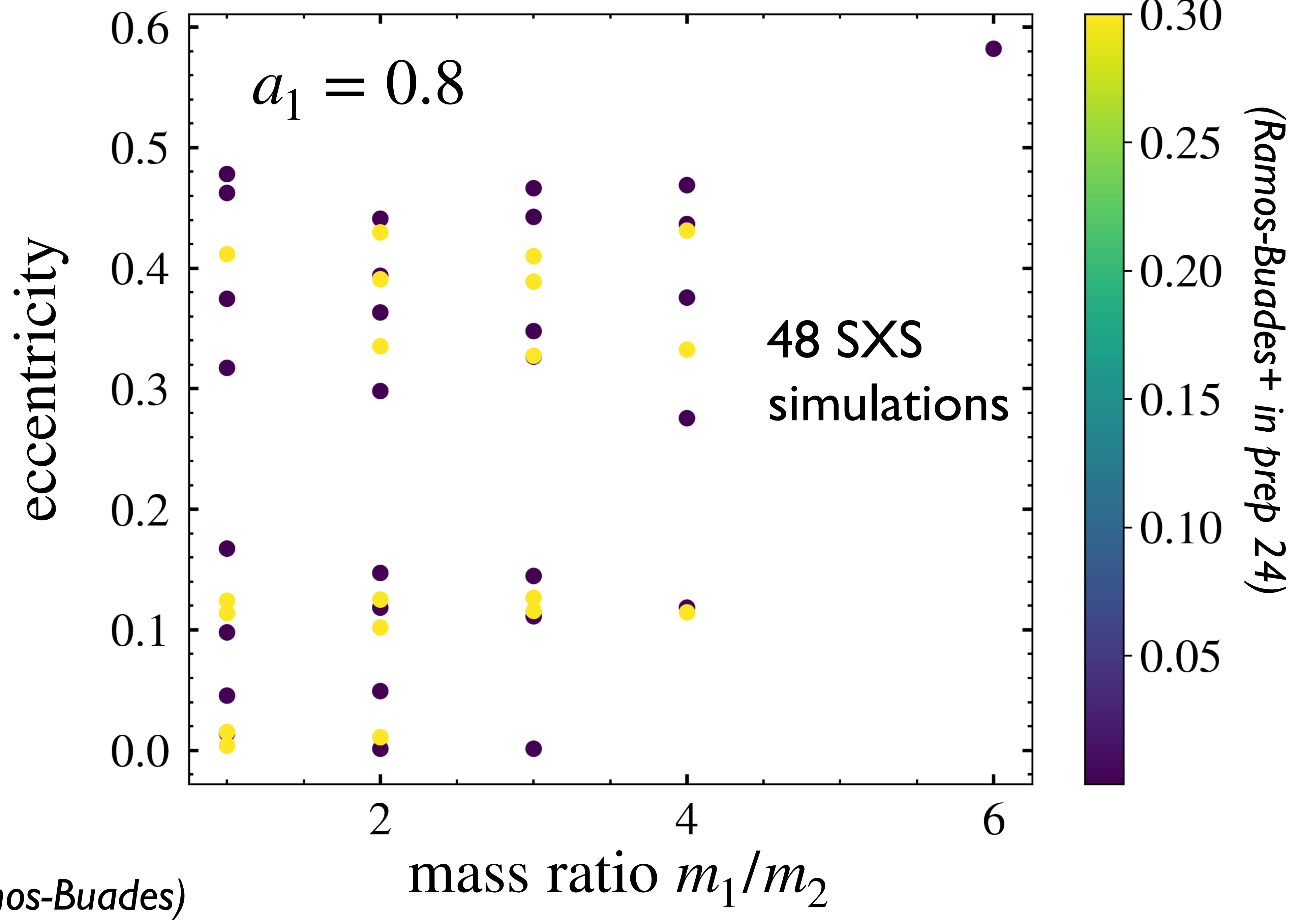
(**RIT:** *Healy+ 22)*

Spin-precessing, quasi-circular parameter-space

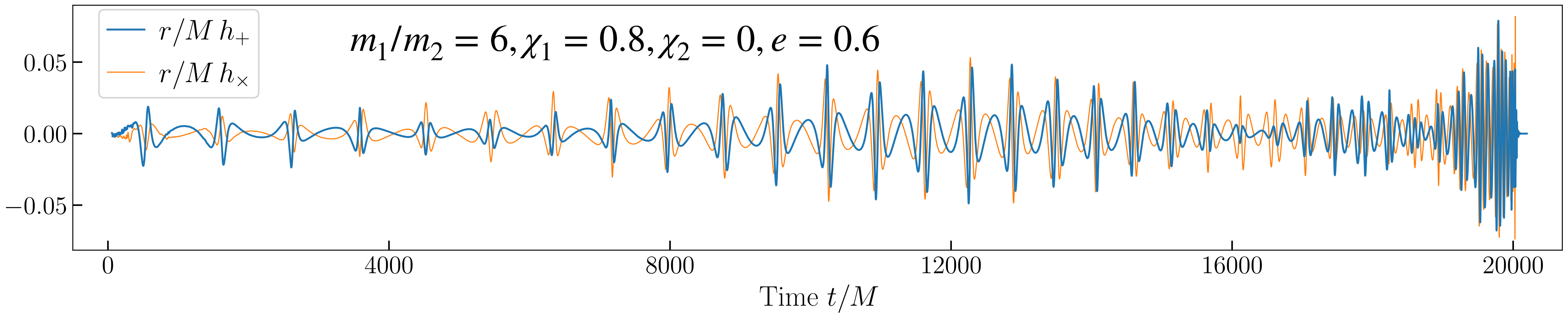


(credit: Harald Pfeiffer)

Spin-precessing eccentric parameter-space



(credit: Antoni Ramos-Buades)

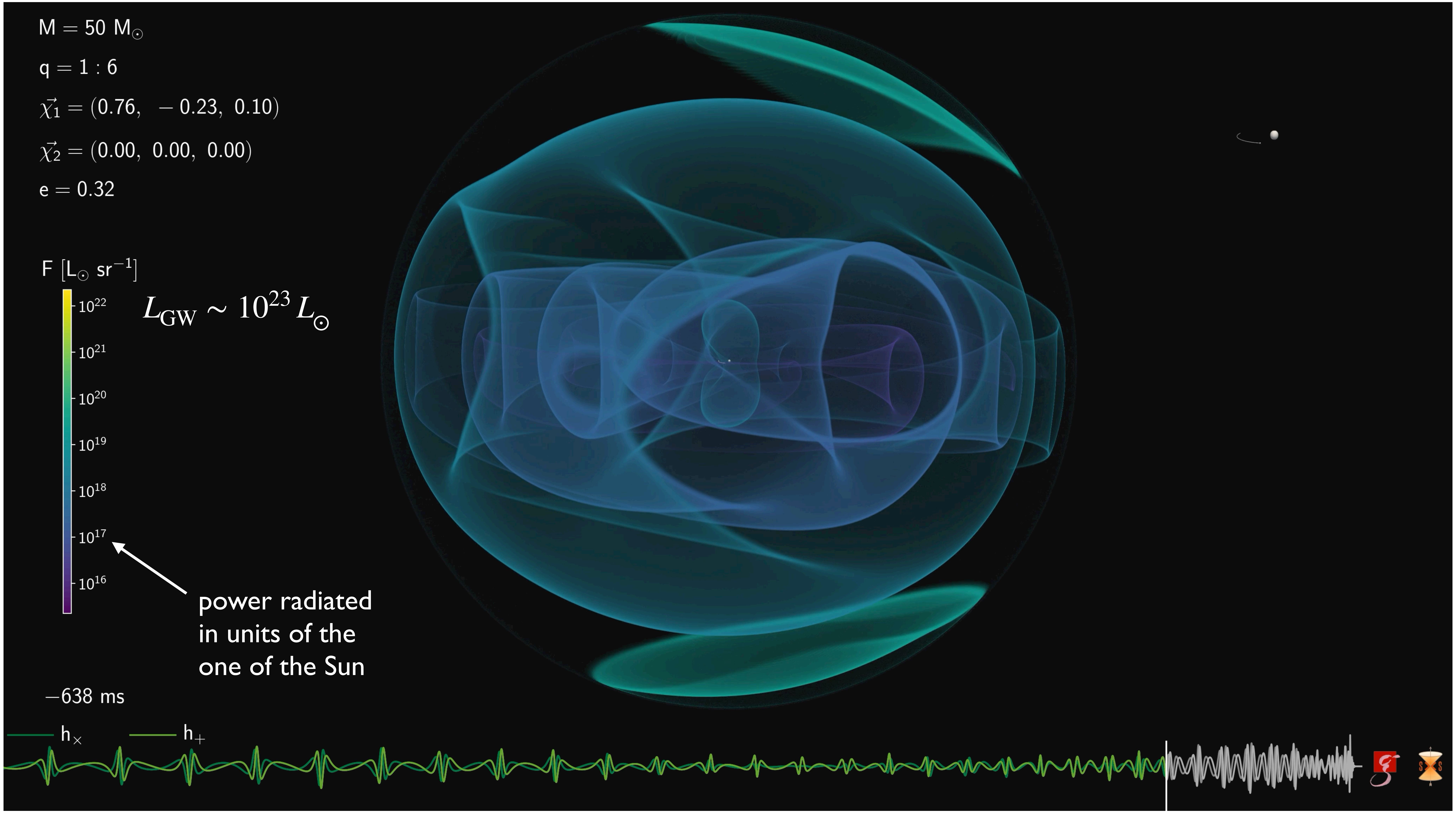


- **Other NR advances:**

memory effects, BMS frames

(Mitman+20-23)

Frontier of GW Modeling: Compact Objects on Generic Orbits



(credit: Ramos-Buades, Markin & Pfeiffer)

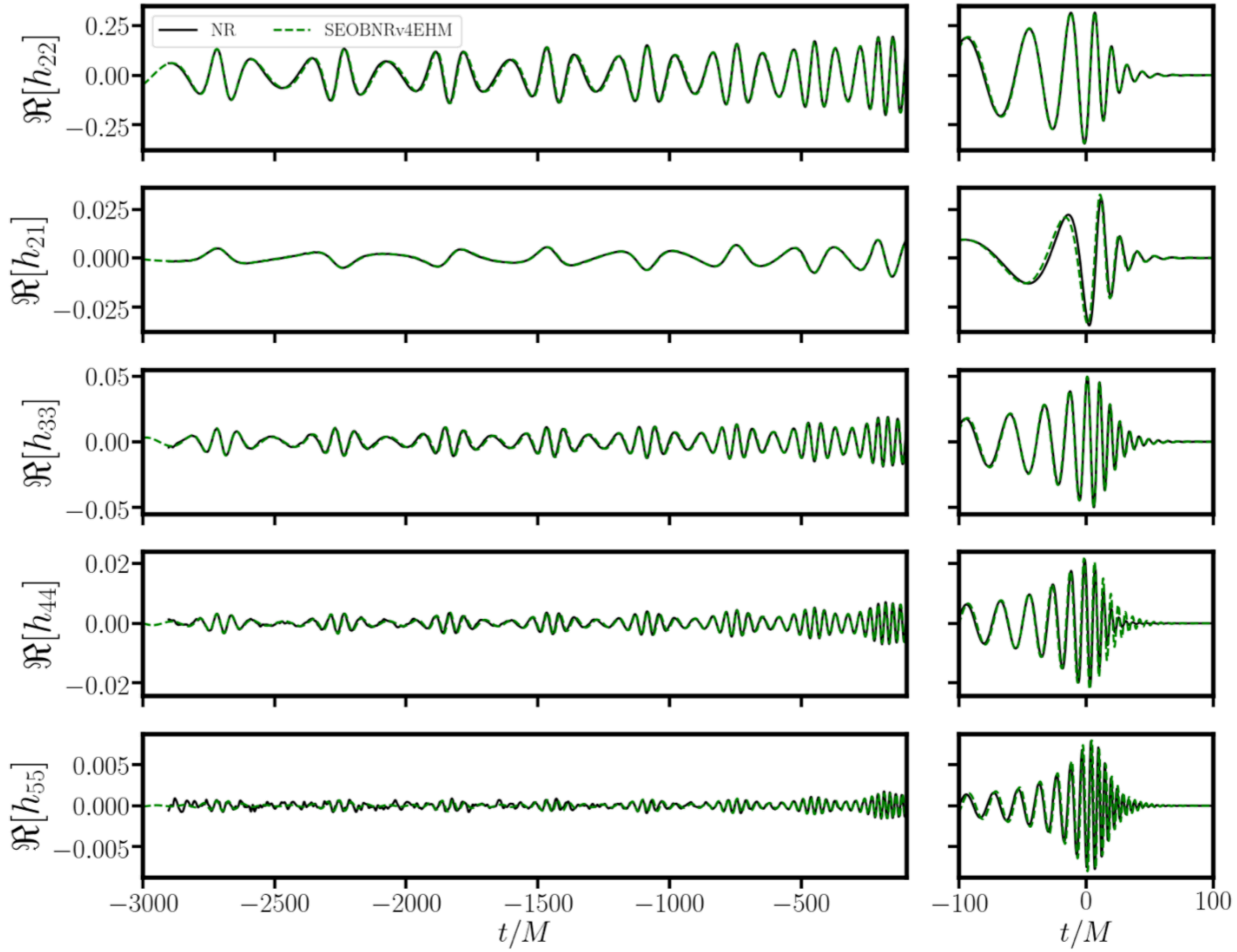
Advances in Modeling Generic Orbits: Non-Precessing Spins and Eccentricity

- **Eccentric, spinning non-precessing** IMR waveforms from EOB families are available, but **accuracy** against public NR waveforms has been **assessed only for low eccentricity (≤ 0.3) and small spins**.

bound orbits

SEOBNR

mass ratio = 2, non-spinning, e = 0.06

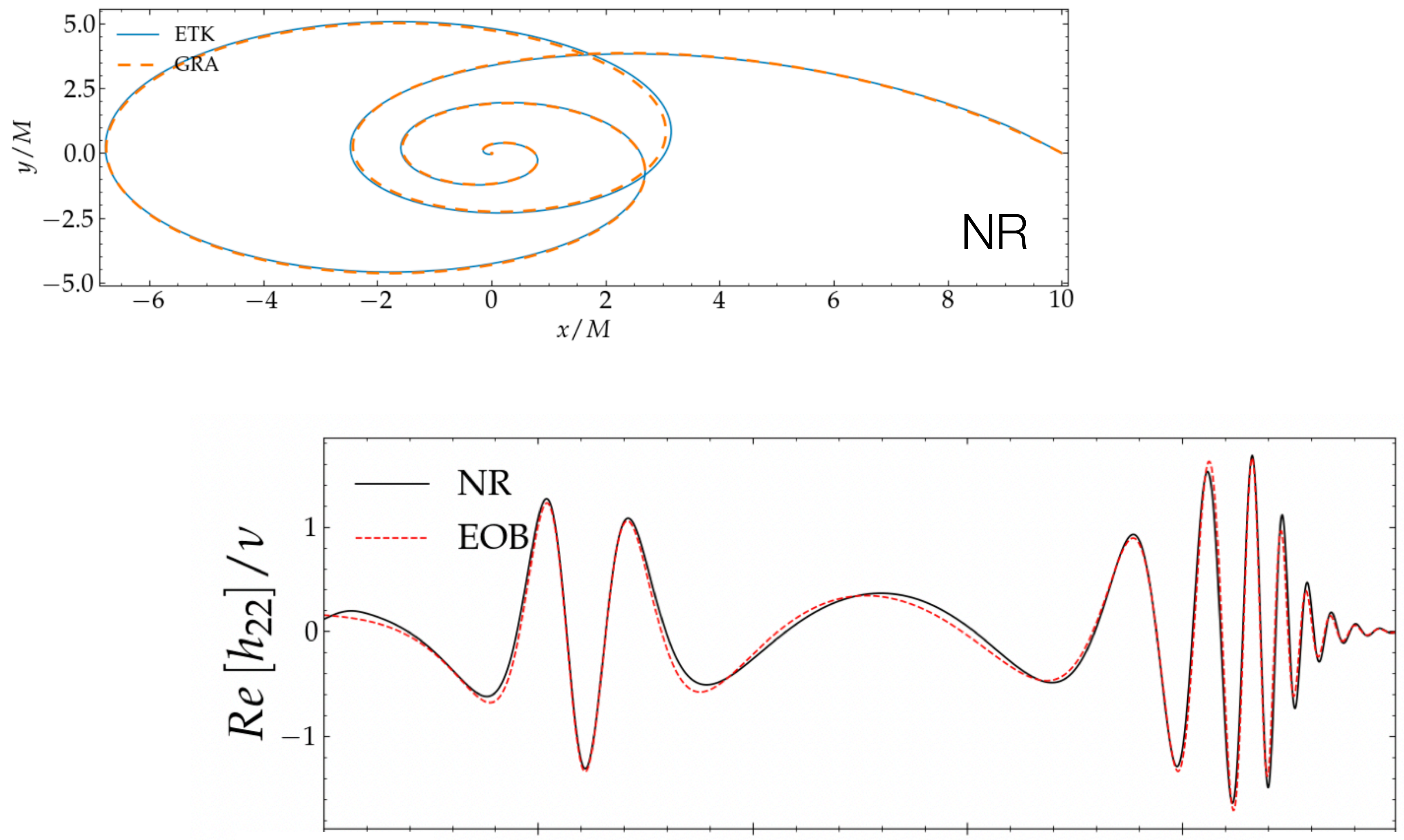


(Ramos-Buades+ arXiv: 2112.06952)

dynamical capture/zoom whirl

mass ratio = 1, non-spinning

TEOBResum-S



(Andrade+ arXiv: 2307.08697)

(Huerta+ 14-19, Hinder+ 17, Cao & Han 17; Loutrel & Yunes 16, 17, Ireland+ 19, Moore & Yunes 19, Tiwari+ 19, Chiaramello & Nagar 20, Ramos-Buades+ 20, Liu+ 21, Nagar+ 20, 21, Islam+ 21, Nagar & Rettegno 21, Khalil+ 21, Gamba+ 21, Placidi+ 21, Liu+ 21, 23, Nagar+ 24, Andrade+ 23 Gamba+ 24, Gamboa+ in prep 24)

- Several **studies to infer eccentricity in LVK population.**

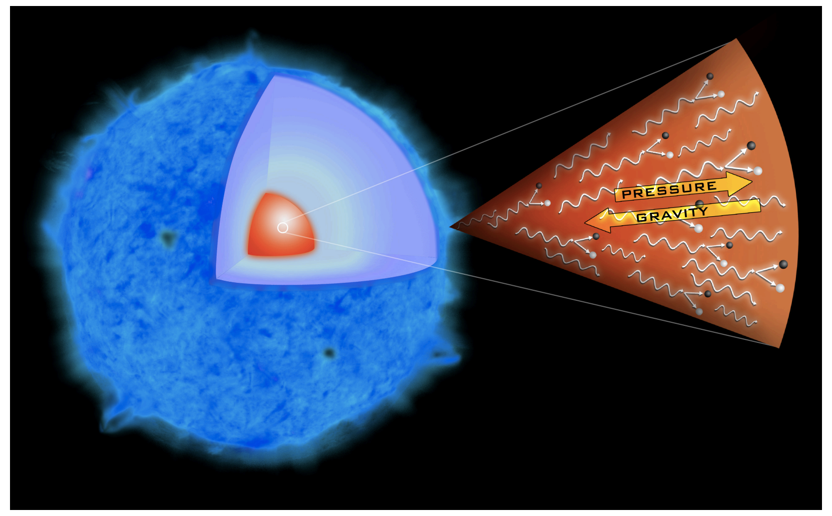
(Romero-Shaw+ 19-22, Gamba+ 21, Clarke+ 22, Knee+ 22, Iglesias+ 24, Ramos-Buades+ 22-23, Bonino+ 23, Gupte+24)

Impact on Identifying High Mass Gap BHs when Missing Physical Effects

- **Parameters** of synthetic **eccentric signal** that is **injected**:

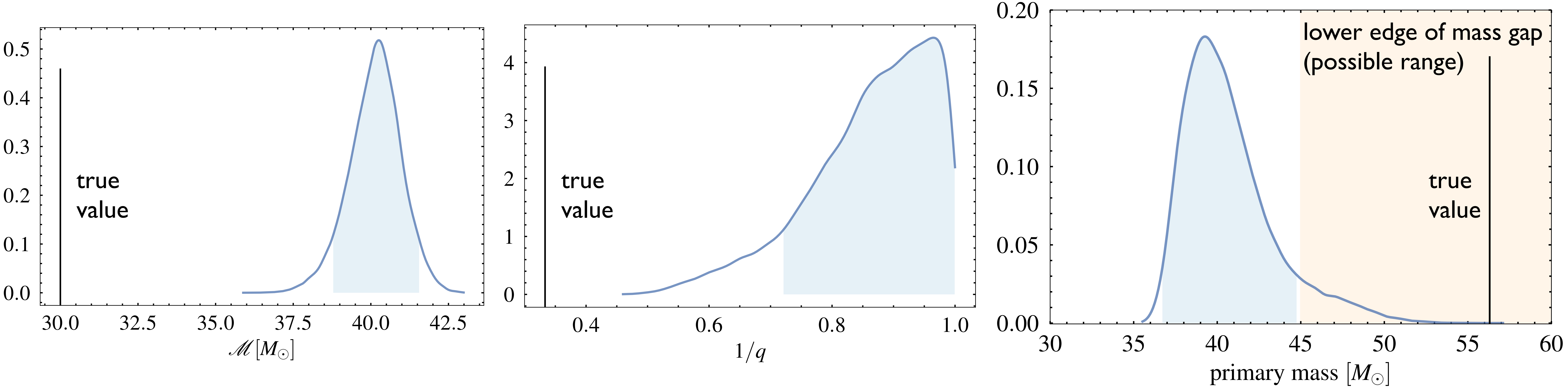
  eccentricity ~ 0.3

- **Signal is recovered with quasi-circular model** using Bayesian analysis.



(Kunz+ 02, De Boer+ 17, Farmer+ 20, Mehta+ 22)

SNR = 60 with O5



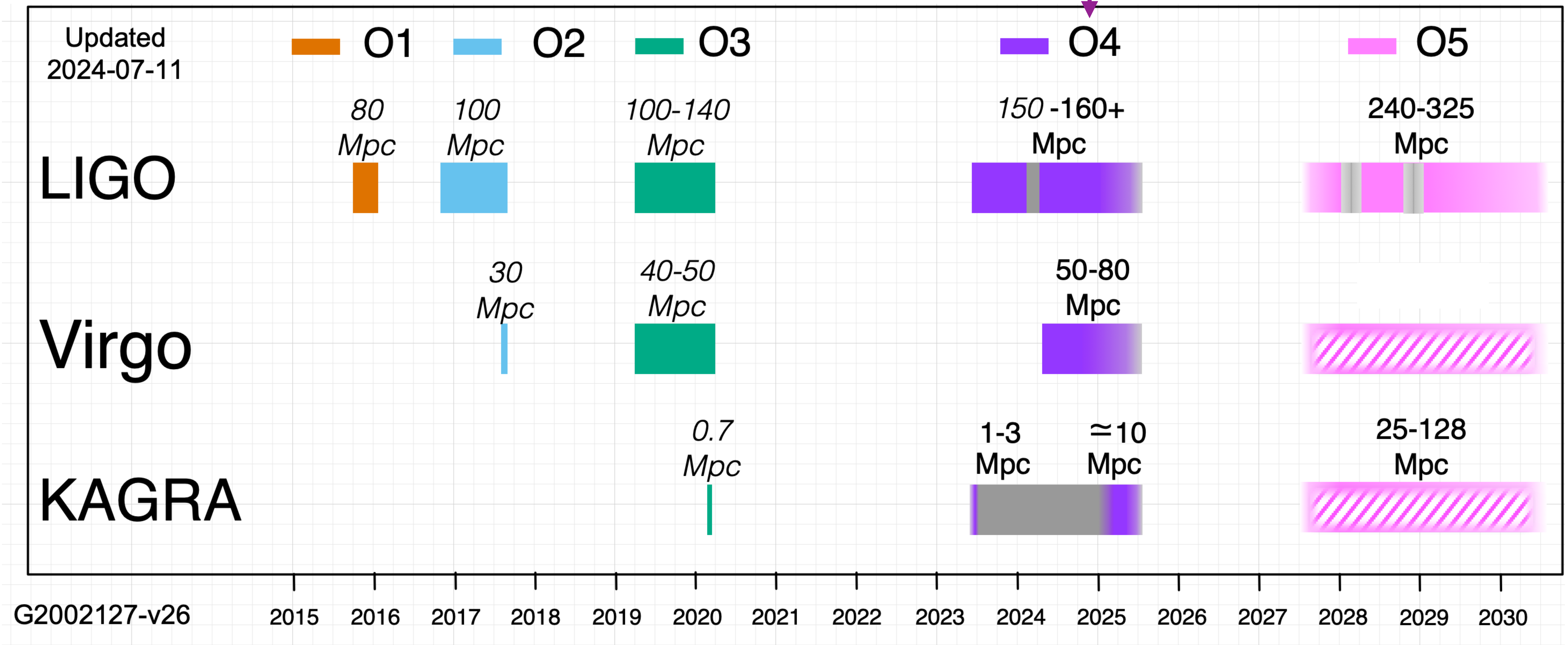
(credit: Antoni Ramos-Buades)

GW Astronomy on the Ground until 2030

(update of Aasi et al. Living Rev. Rel. 21, 2020)

Fourth observing (O4) run is ongoing.

O4a/b: 121 candidate signals observed by LIGOs/Virgo plus GW230529!

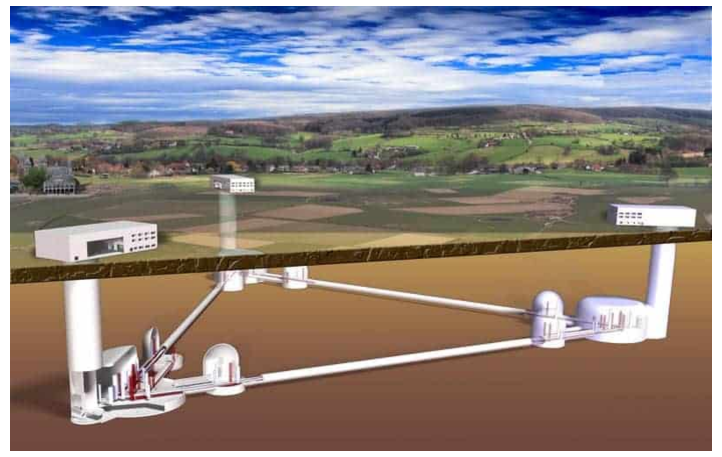


- **LIGO India in late 2020s.**
- **Further upgrades in early 2030:** $A^{\#}$ & $\mathrm{Virgo}_{\mathrm{next}}$

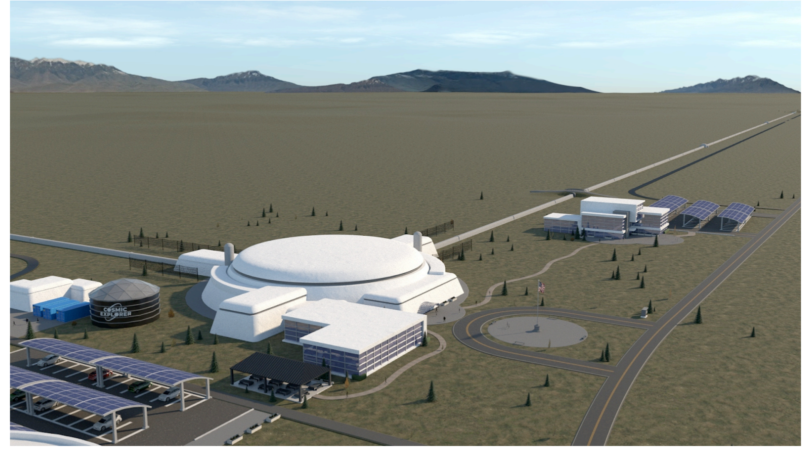
- Inference of **astrophysical properties** of BBHs, NSBHs and BNSs **in local Universe ($z \lesssim 1-2$).**

GW Astronomy on the Ground & Space in 2030s: from hectoHz to milli Hz

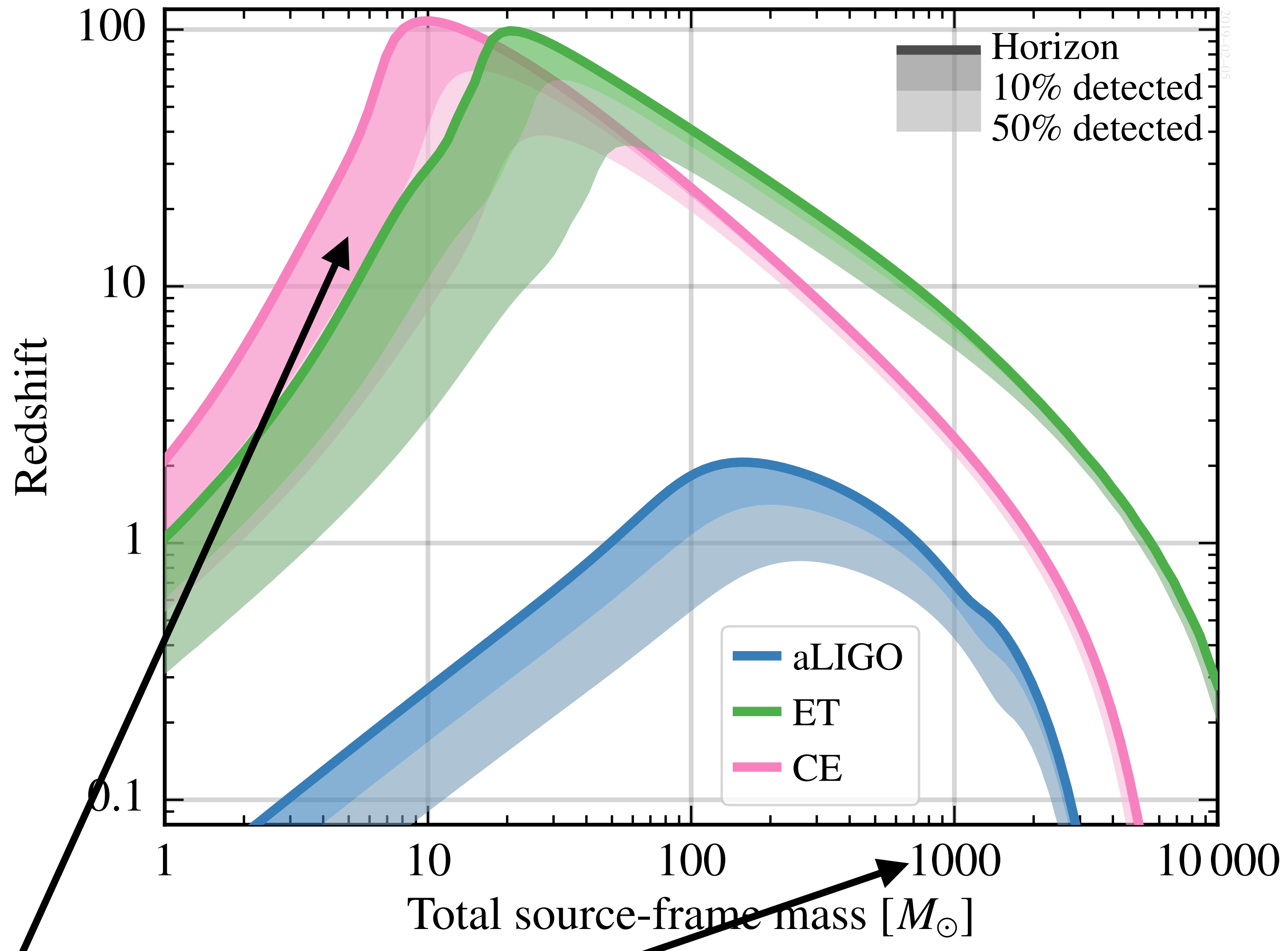
Einstein Telescope (ET)



Cosmic Explorer (CE)



(Kalogera+ arXiv:2111.0699)



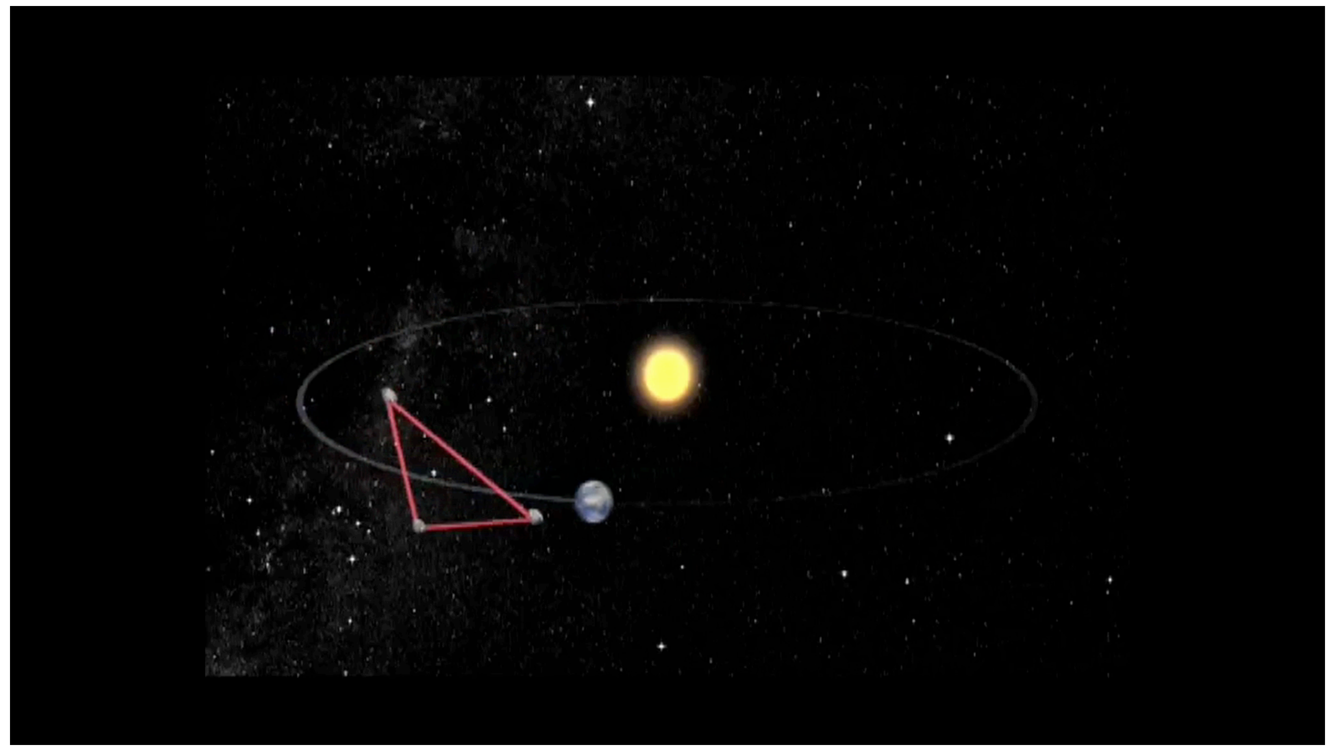
Observe BHs at much larger distance, when first stars formed, and more massive.

- Exquisite **characterization of binary BHs (NSs)**: the **number of events/yr** with **signal-to-noise ratio** > 100 will be $\sim 9{,}500\,(380)$.

(Borhanian & Sathyaprakash 22; Gupta et al. 23)

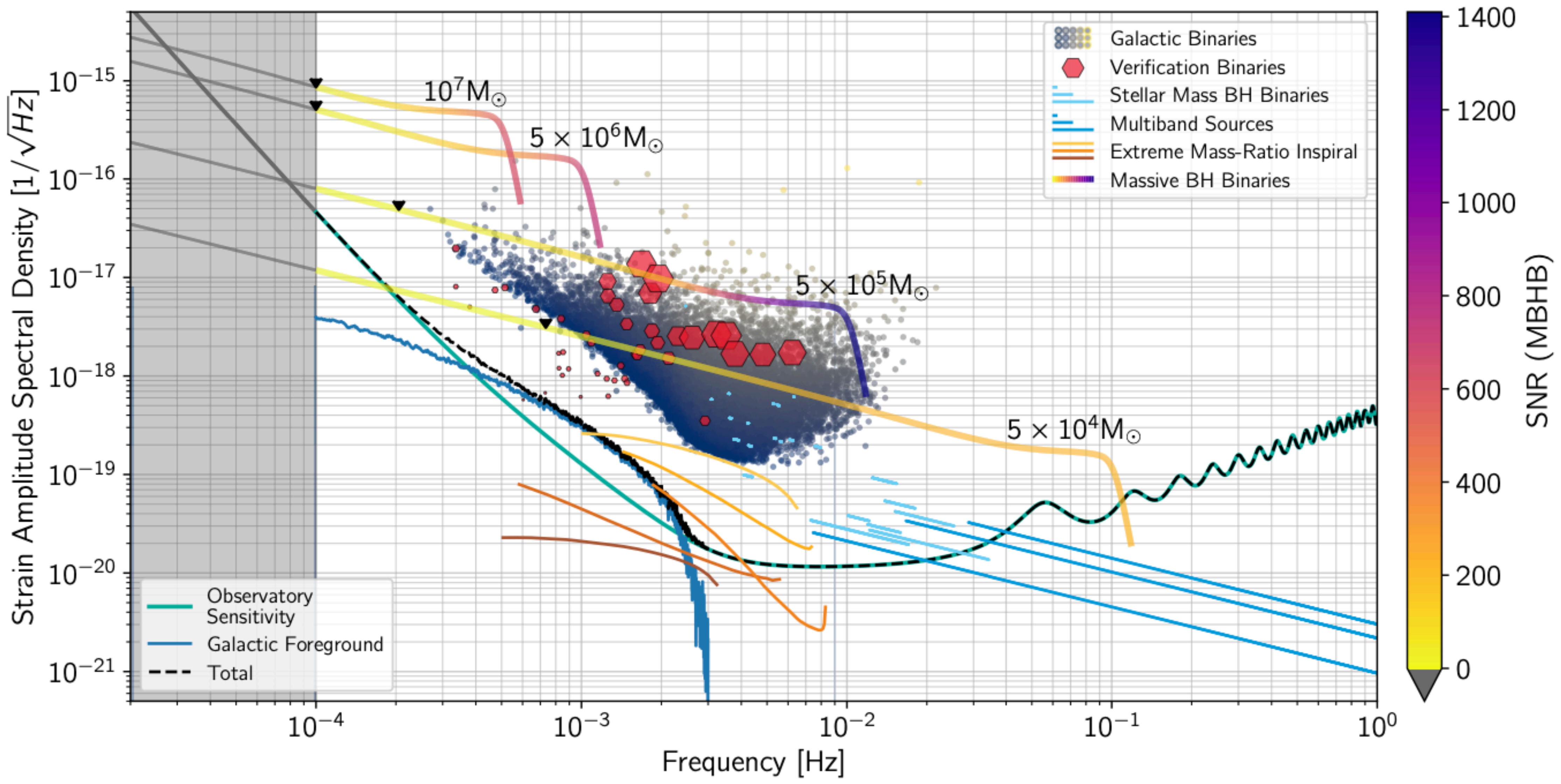
- **LISA adopted** as mission **by ESA** in Jan 2024; **launch** ~ 2035.

(credit: AEI/Milde Marketing/exozet)



- GW signals **will be loud and last for weeks/months.**

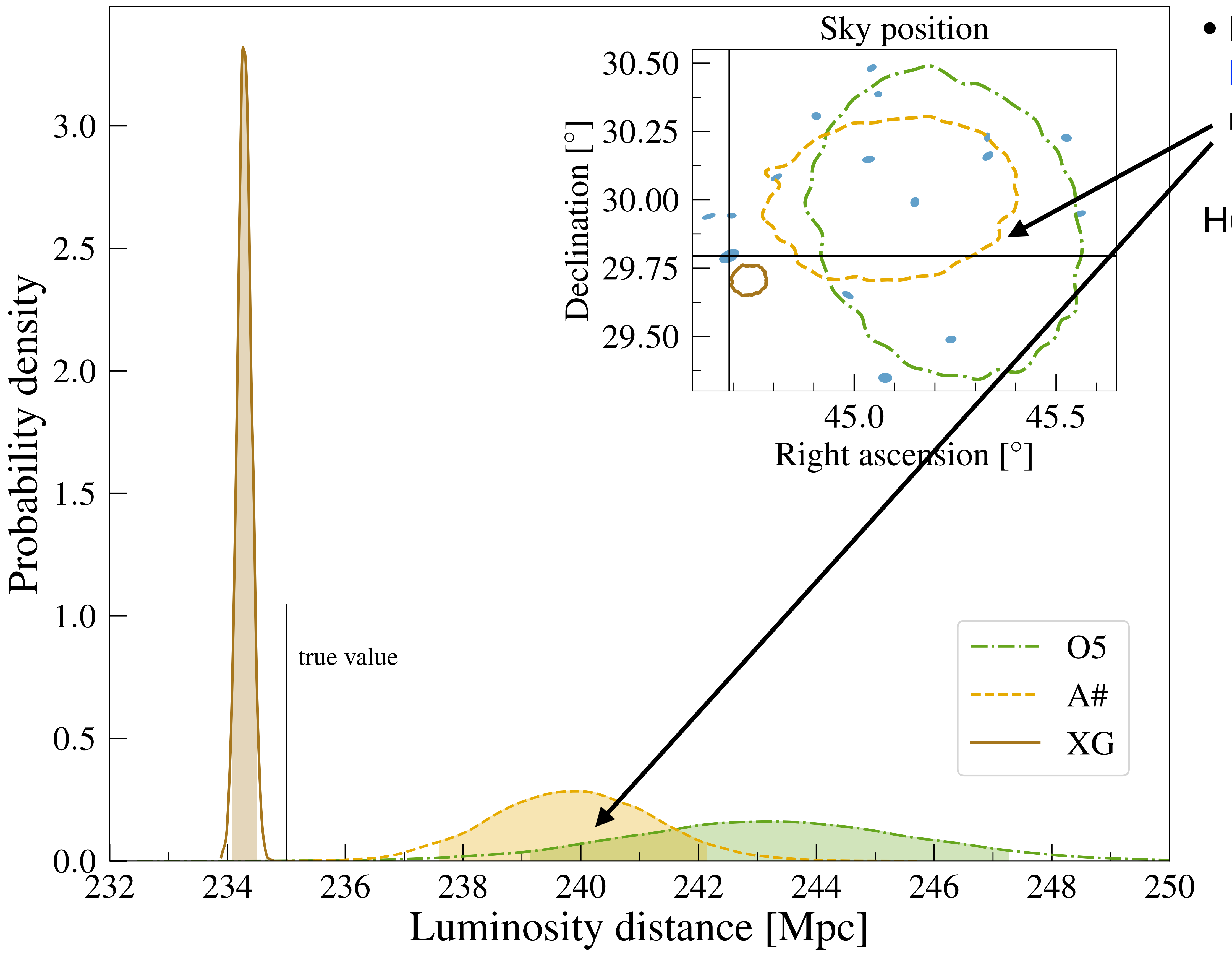
(LISA Red Book arXiv:2402.07571)



Precision GW Astronomy: The Accuracy Challenge

- BH binary **GW190814-like ($q \sim 10$), but highly precessing.**

$$\mathrm{SNR}_{\mathrm{O5}} = 119\,,\, \mathrm{SNR}_{\mathrm{A\#}} = 219\,,\, \mathrm{SNR}_{\mathrm{XG}} = 2490$$



- Due to systematics, **wrong Hubble-Lemaitre parameter** (expansion rate of the Universe) **is measured.**

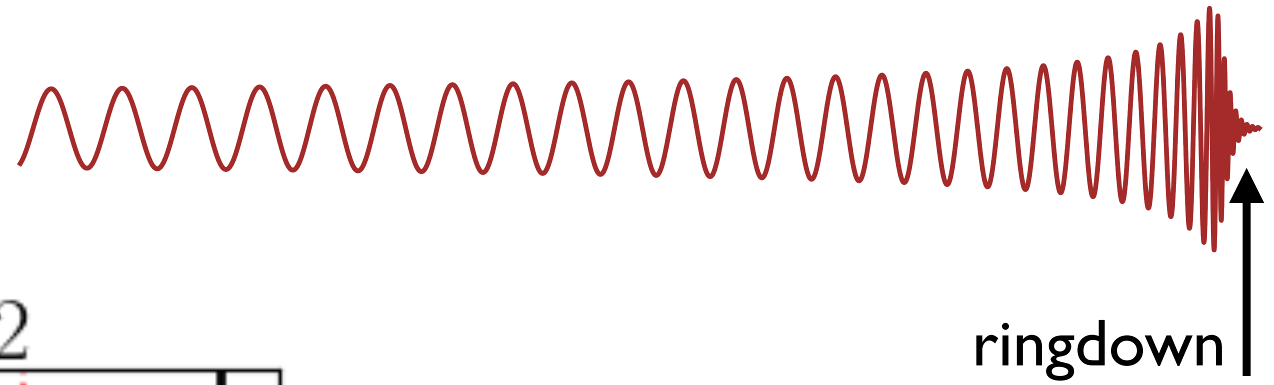
Hubble-Lemaitre flow velocity — luminosity distance

$$v_H = H_0\, d$$

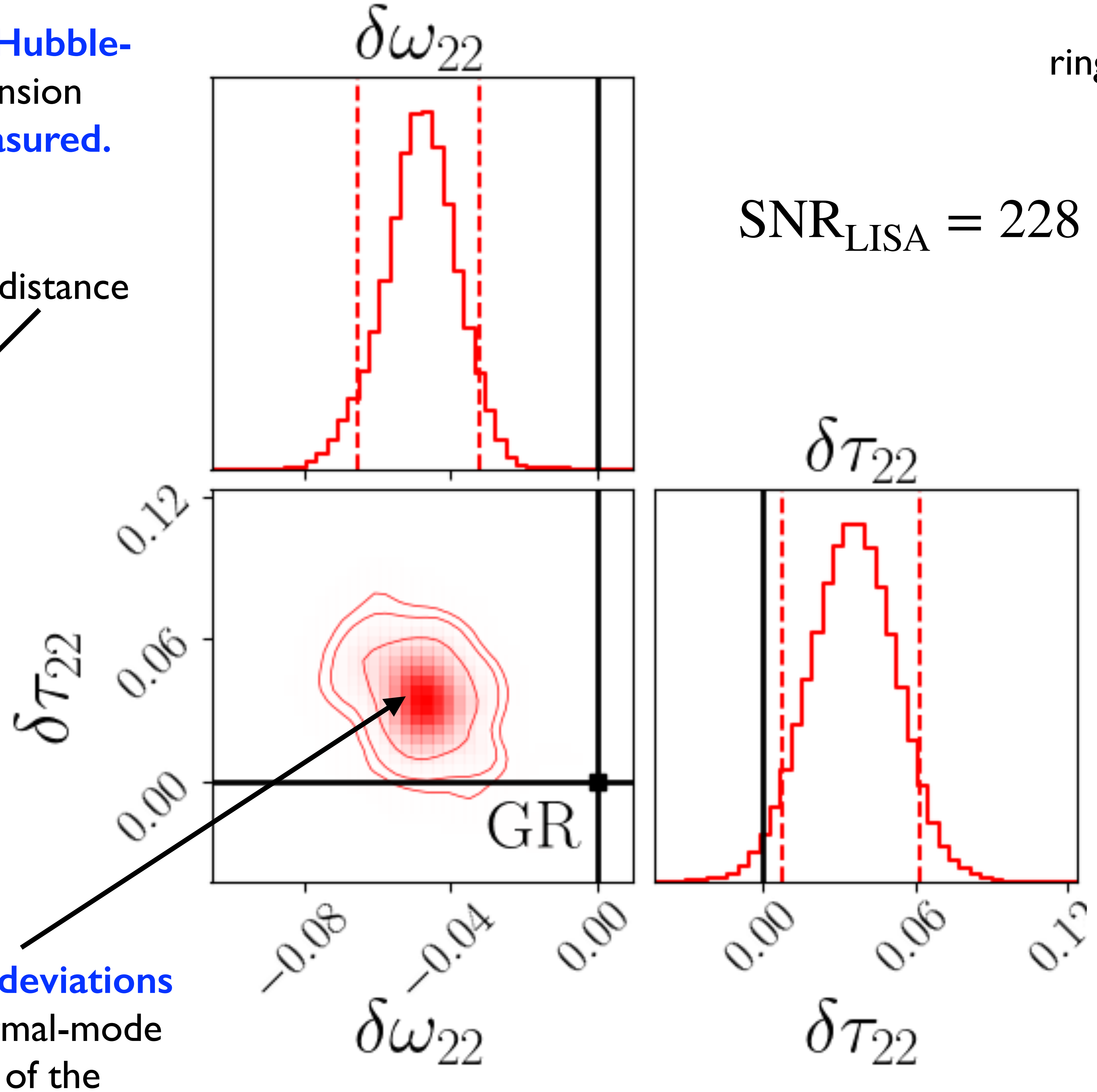
(see also Gupta+24)

(Dhani, Völkel, AB, Estellés, Gair, Pfeiffer Pompili & Toubiana arXiv:2404.05811)

- Massive BH binary **with moderate mass ratio and spins.**



$$\mathrm{SNR}_{\mathrm{LISA}} = 228$$



- Due to systematics, **false deviations from GR** in the quasi-normal-mode frequency and decay time of the ringdown **are measured.**

(Toubiana, Pompili, AB, Gair & Katz arXiv:2307.15086)

Impact of Systematics for LVK-like BBH Population with XG Detectors

quasi-circular, spin-precessing case

- $\delta\theta$: **bias between SEOBNRv5PHM** and **IMRPhenomXPHM** *(Flanagan & Hughes 98; Cutler & Vallisneri 07)*

- $\Delta\theta$: **statistical error** (1σ) using Fisher Information Matrix

XG= ET/CE detectors

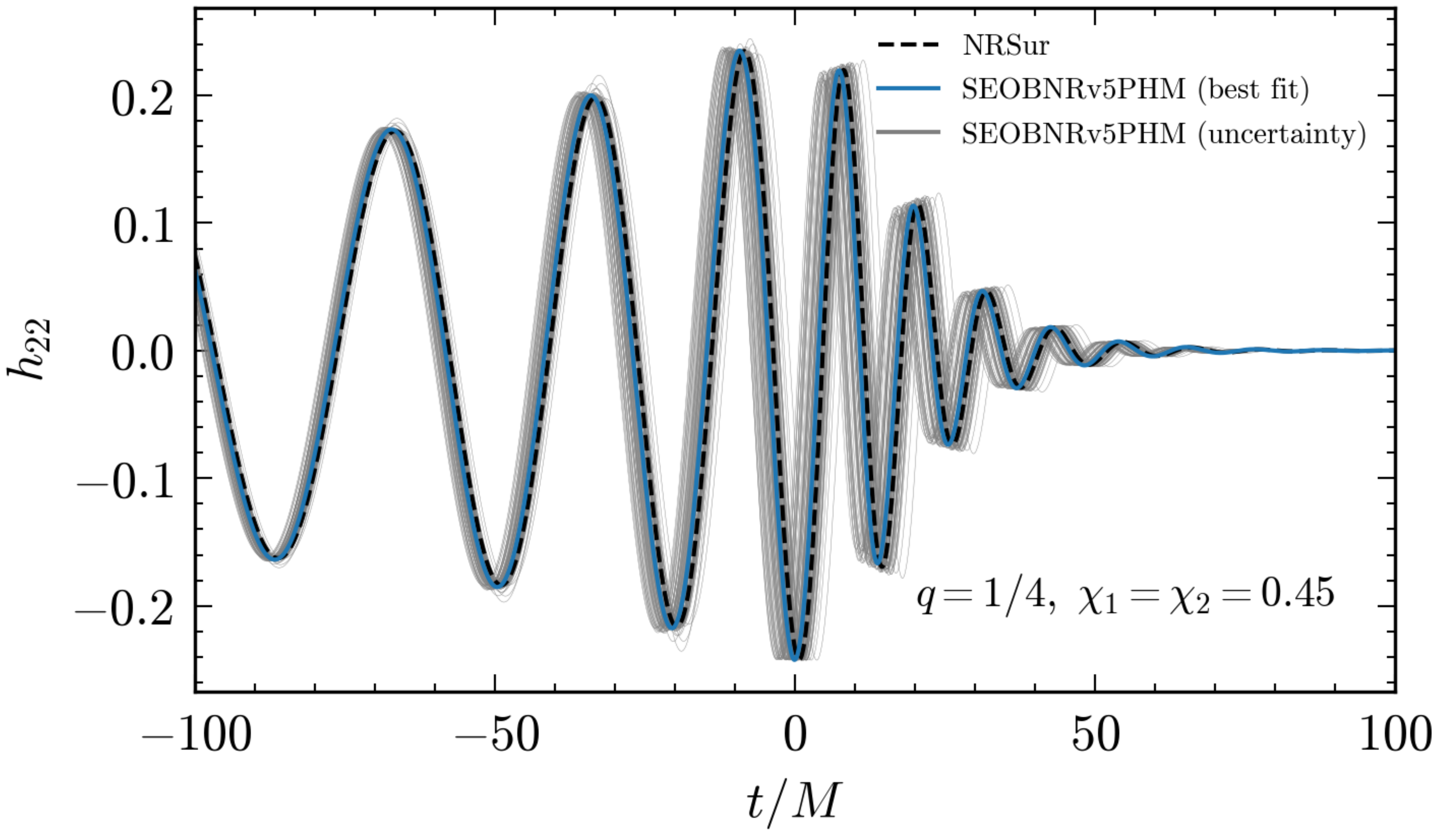
(Dhani, Völkel, AB, Estellés, Gair, Pfeiffer Pompili & Toubiana arXiv:2404.05811)

(see also Kapil+24)

- **SEOBNRv5PHM and IMRPhenomXPHM are not independent** since the latter is also calibrated to SEOBNR waveforms.

Addressing Systematics by Including Calibration Uncertainty

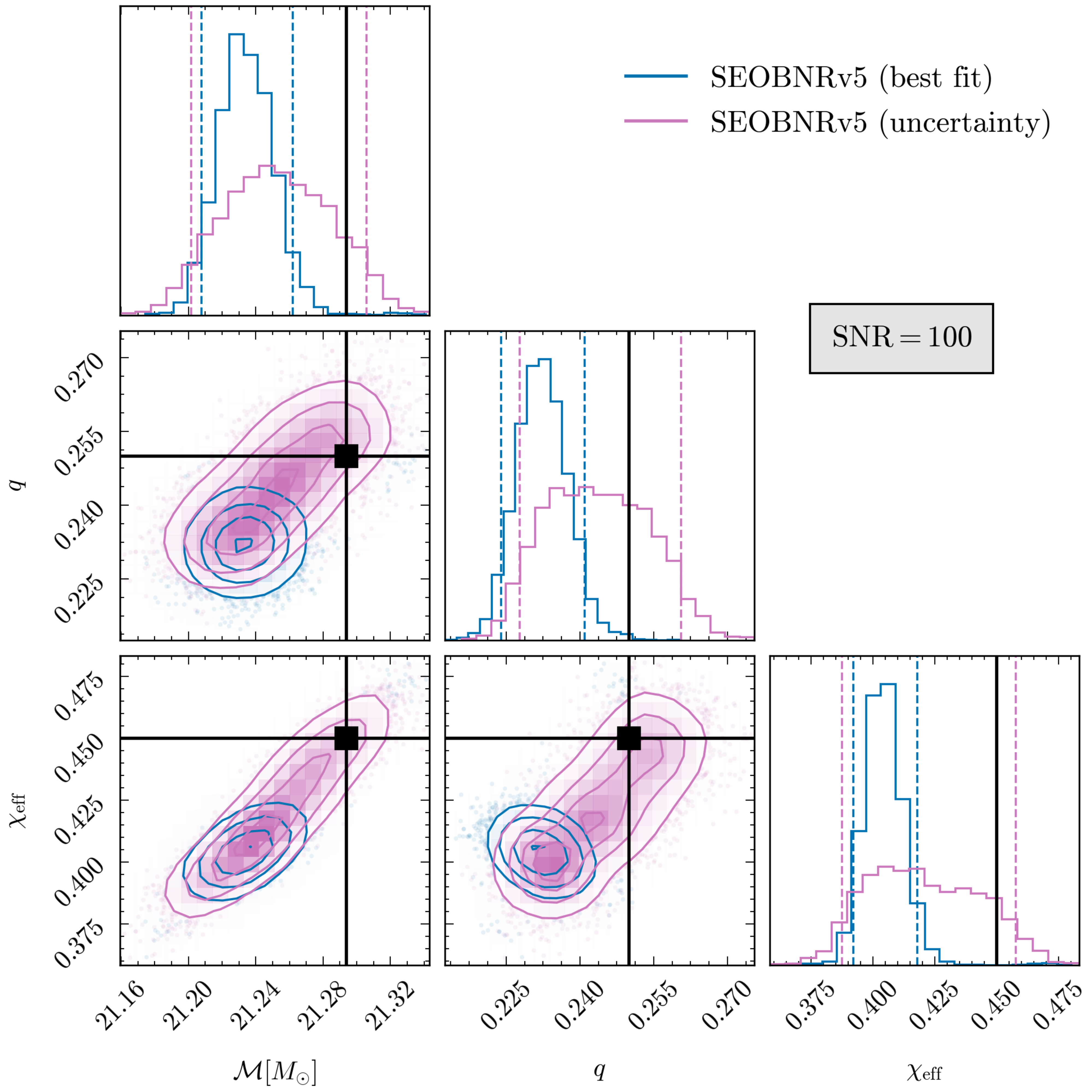
- **Uncertainty is included in the waveform by drawing 100 samples** from posterior distributions of calibration parameters.



(Pompili, AB & Pürrer, in prep 24)

(see also Andrade+ 23, Khan 24)

- **Posterior distributions shift toward true values and become broader** when calibration parameters are marginalized over.

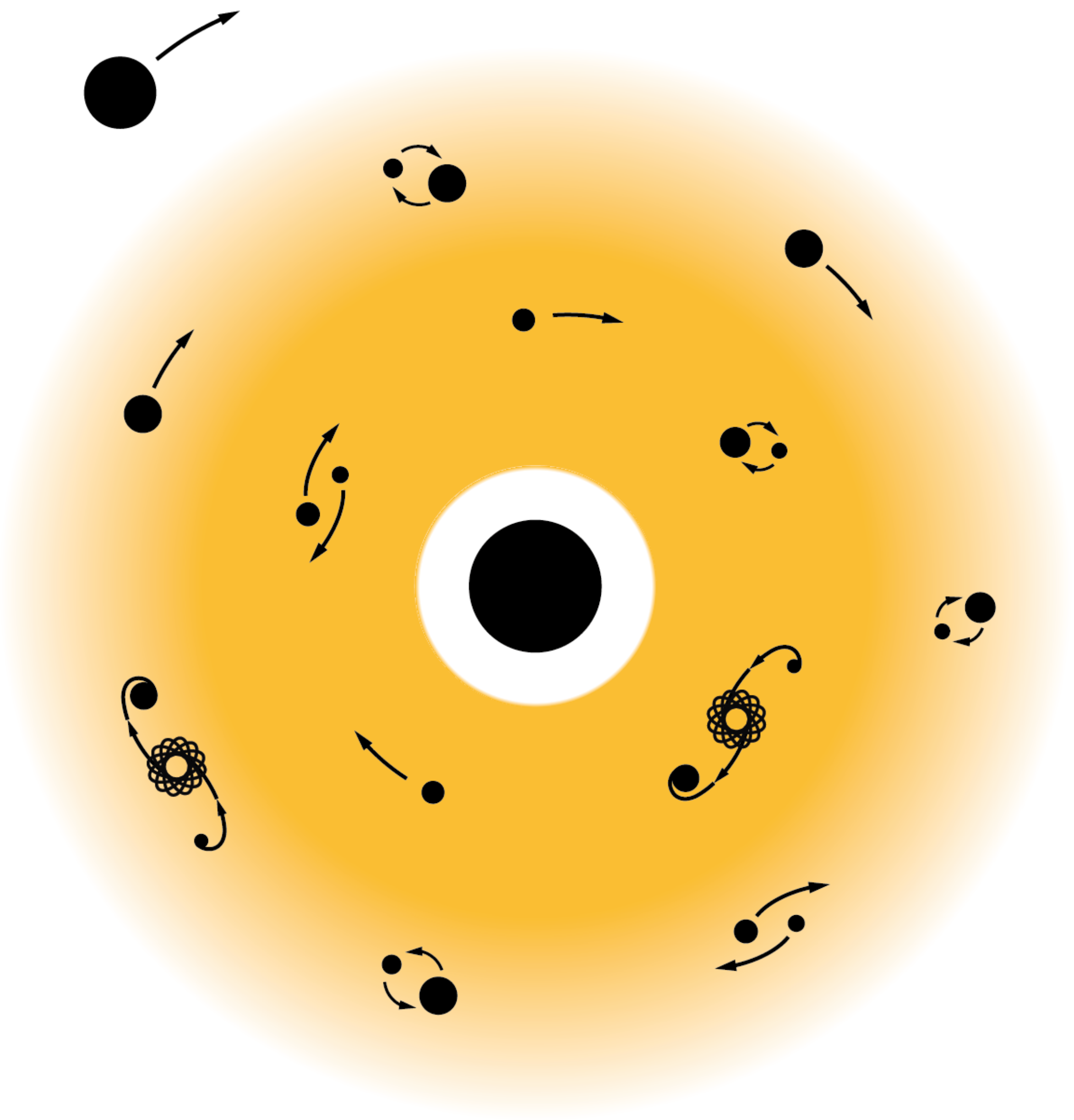


Theoretical Advances to Enable Precision GW Astronomy

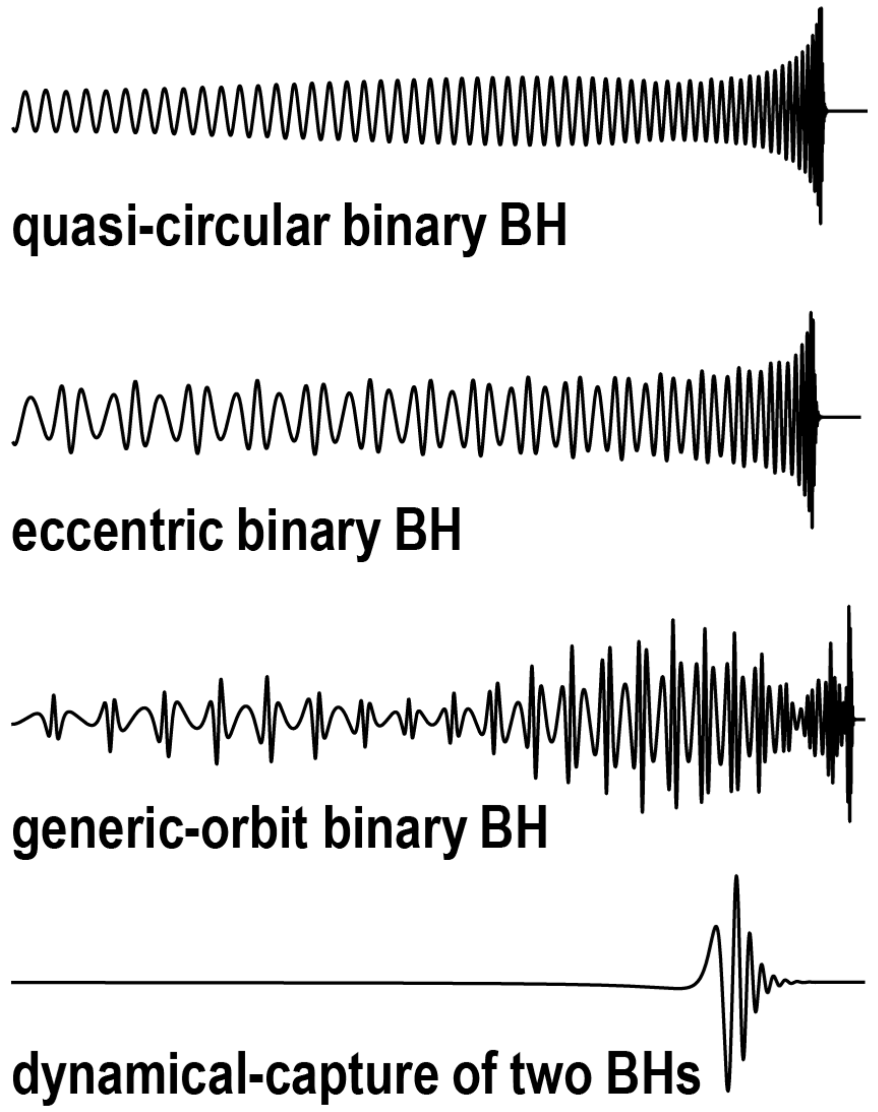
- **The accuracy of current waveform models** (for comparable mass binaries) would need **to be improved by 2 orders of magnitude**. **Numerical-relativity simulations** would also need to become **more accurate for BBHs,** and especially **BNS/NSBHs**.

(Pürrer & Halster 19, Samajdar & Dietrich 18, Gamba+21, Dhani+24)

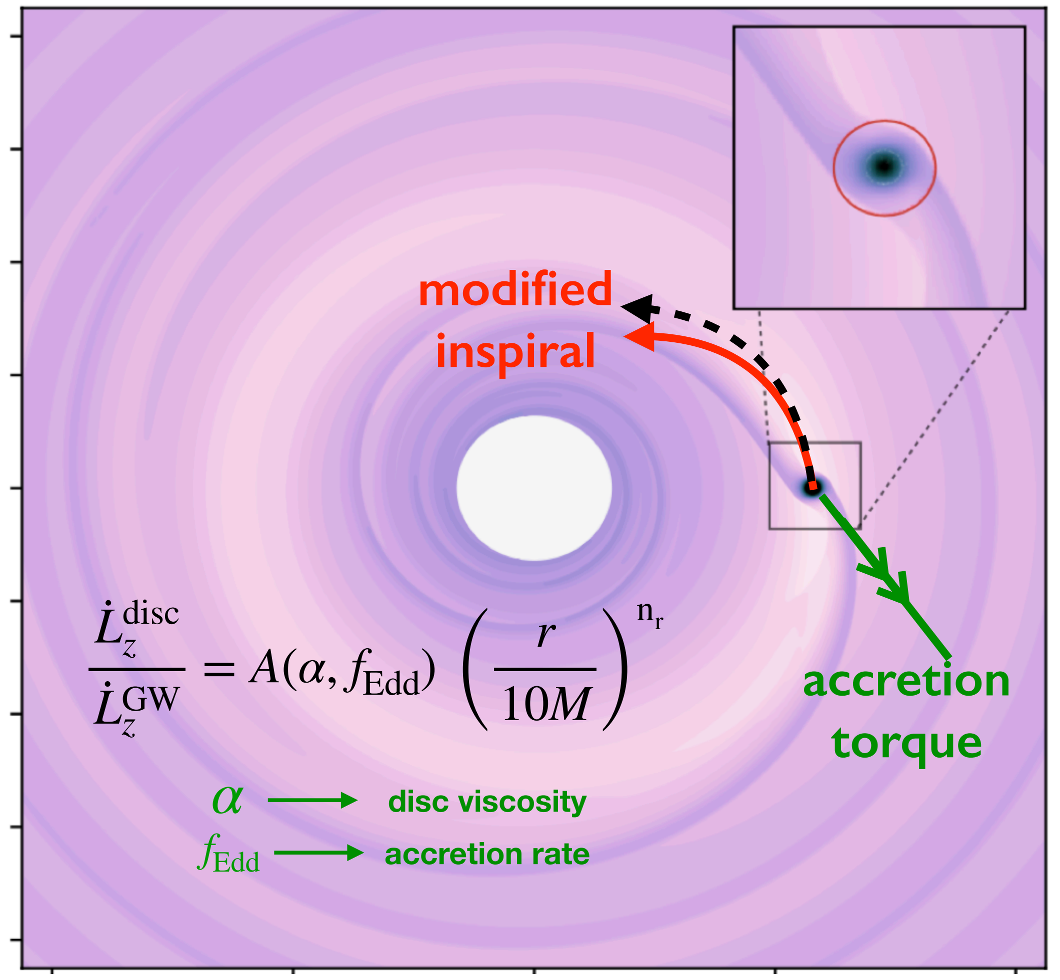
- **All physical effects** would **need to be included in waveform models** (memory effects, generic orbits, astrophysical environmental effects, new physics beyond-GR, gravitational lensing, etc.) **to avoid wrong scientific** conclusions.



(credit: Ana Carvalho)



- GWs can place constraints on **astrophysical environment,** e.g., accretion disks, triple systems, resonant tidal interactions, etc.



(credit: Derdzinski)

(Levin 03, Barausse+14, Speri+23, Zwick+23)

Theoretical Advances to Enable Precision GW Astronomy (contd.)

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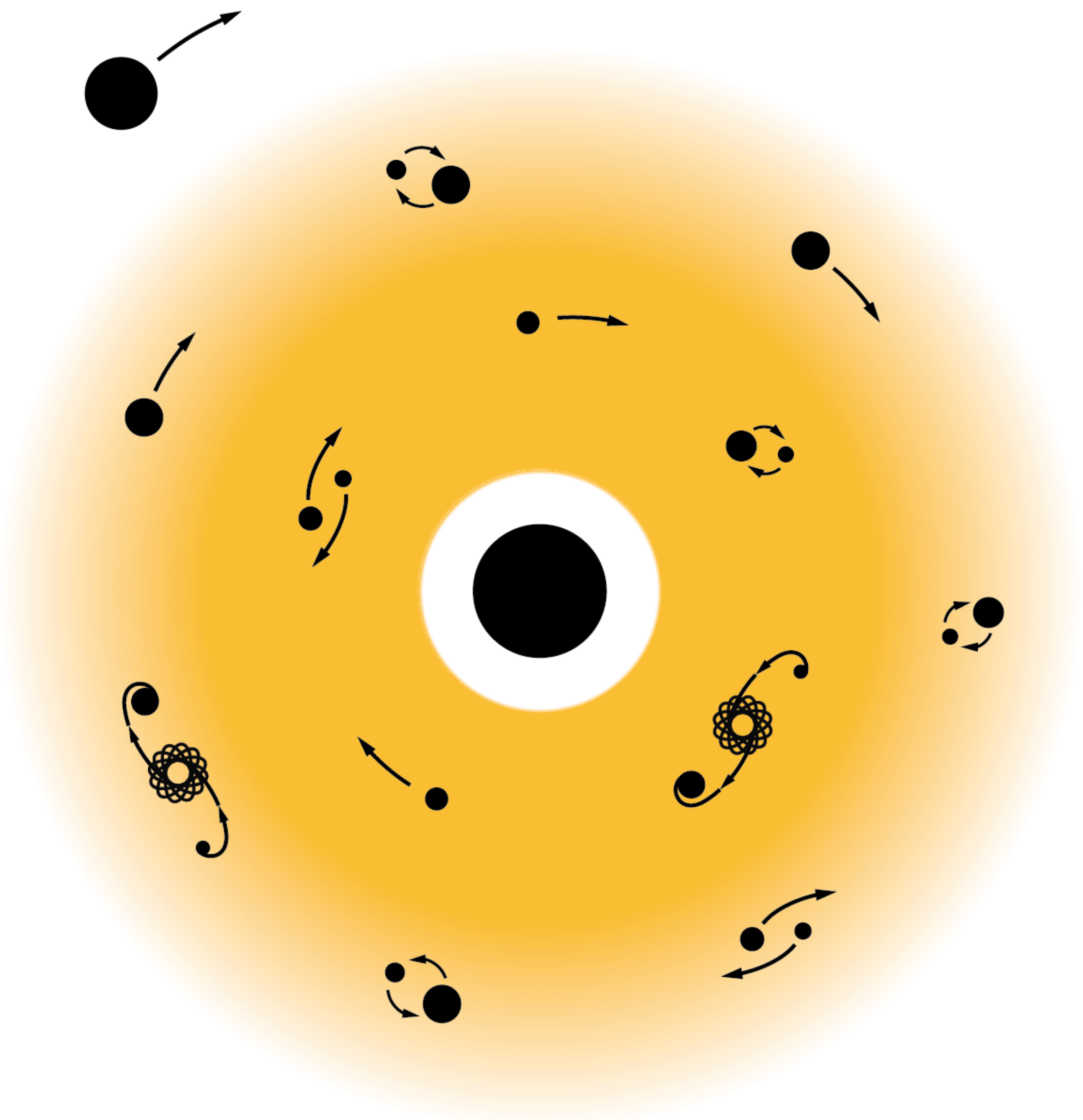
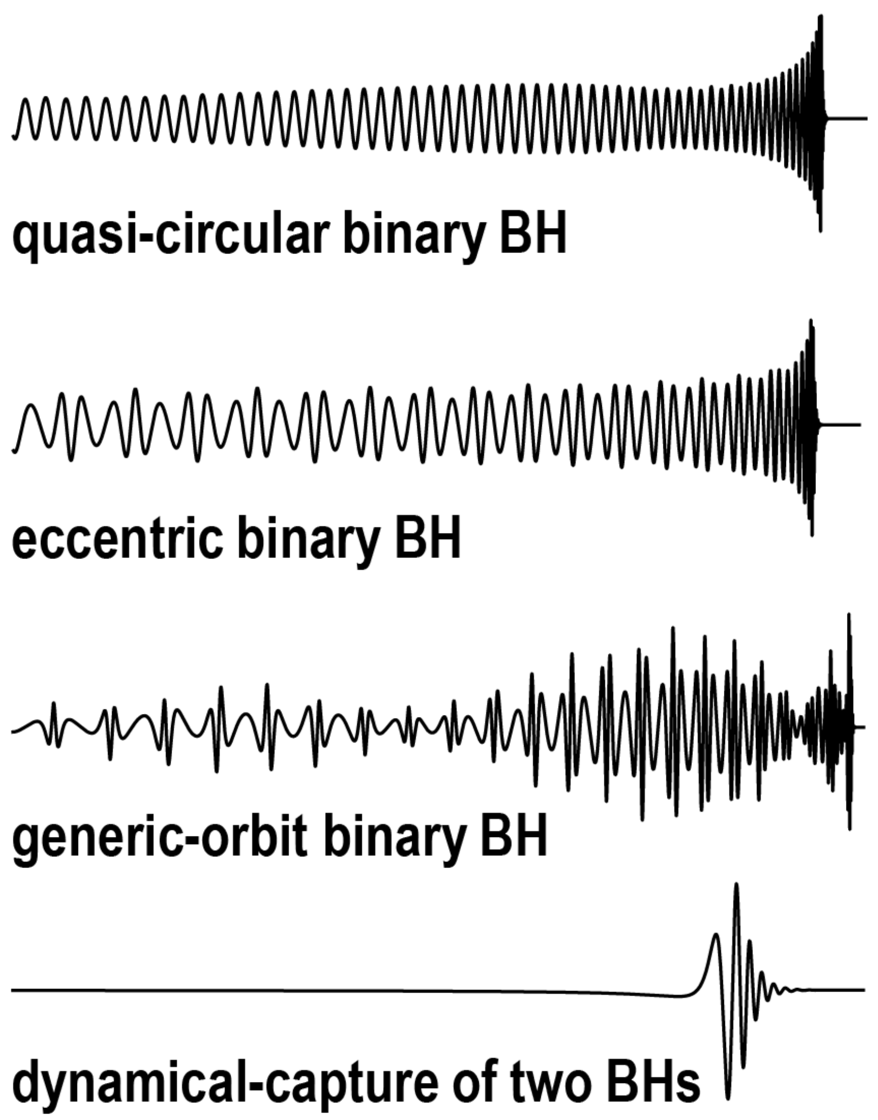
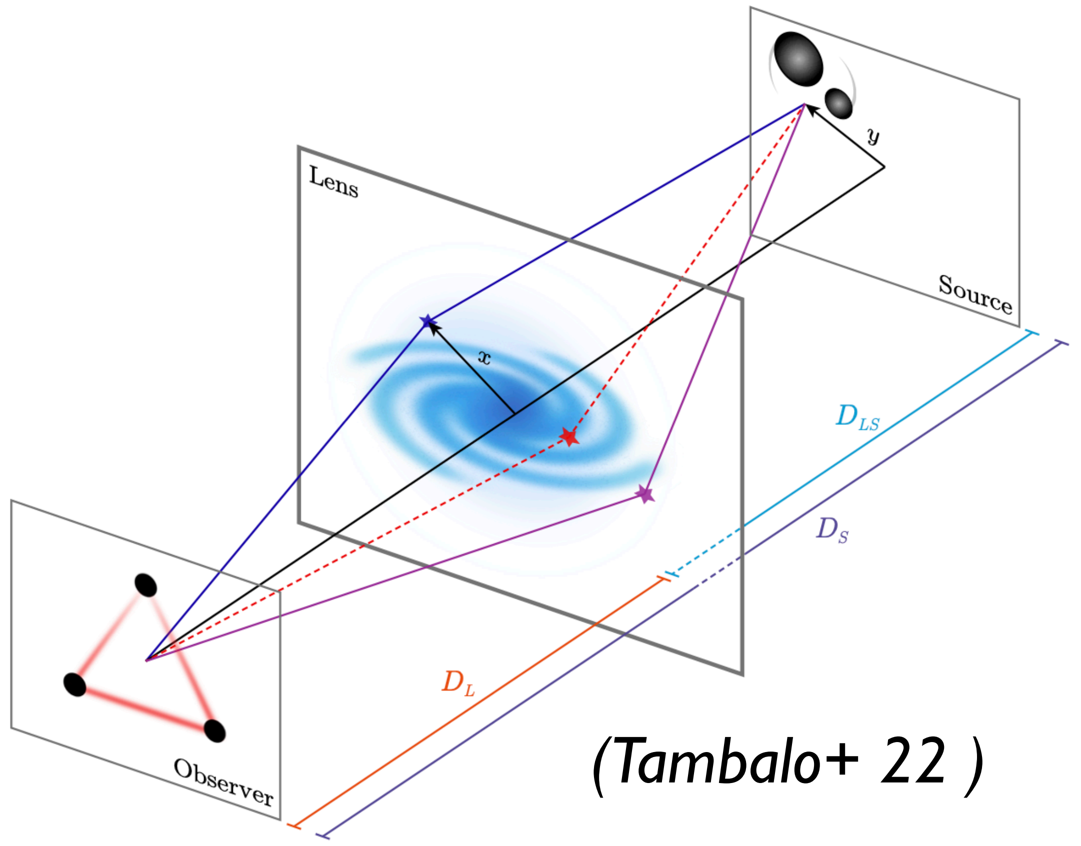
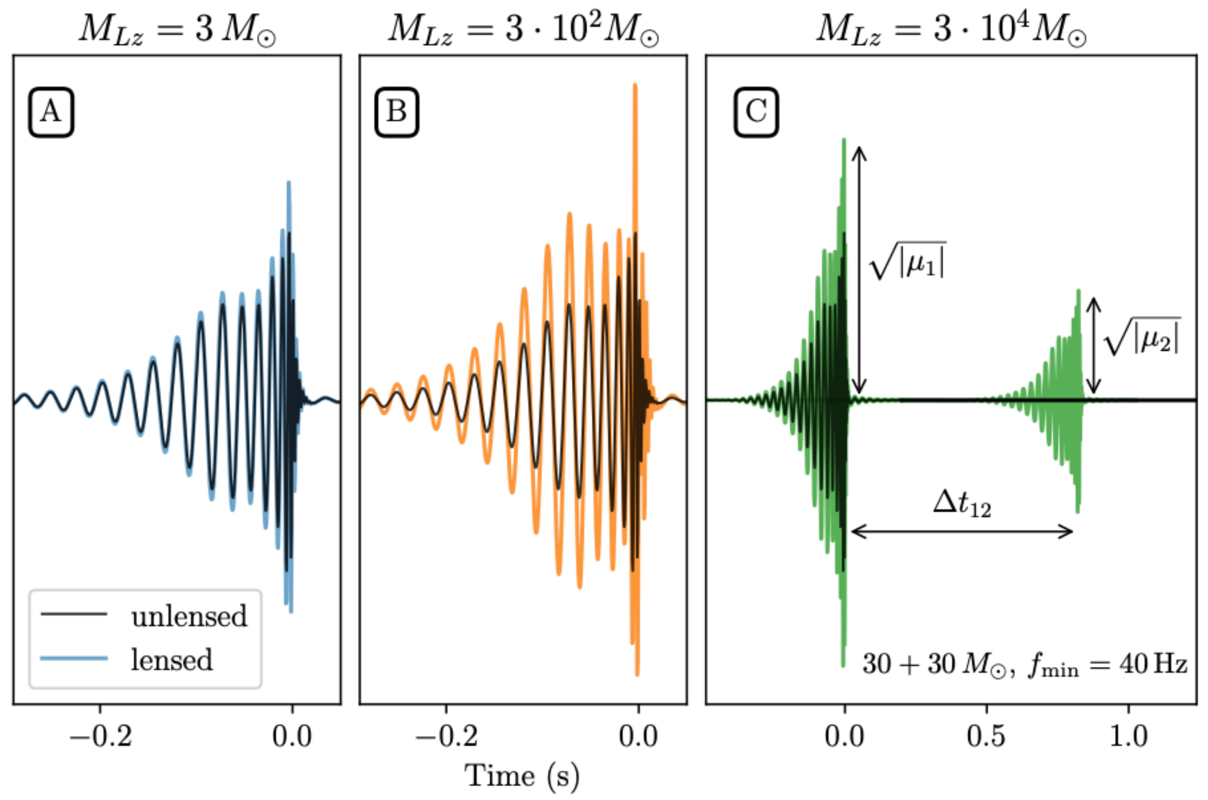


(credit: Ana Carvalho)





(Tambalo+ 22)

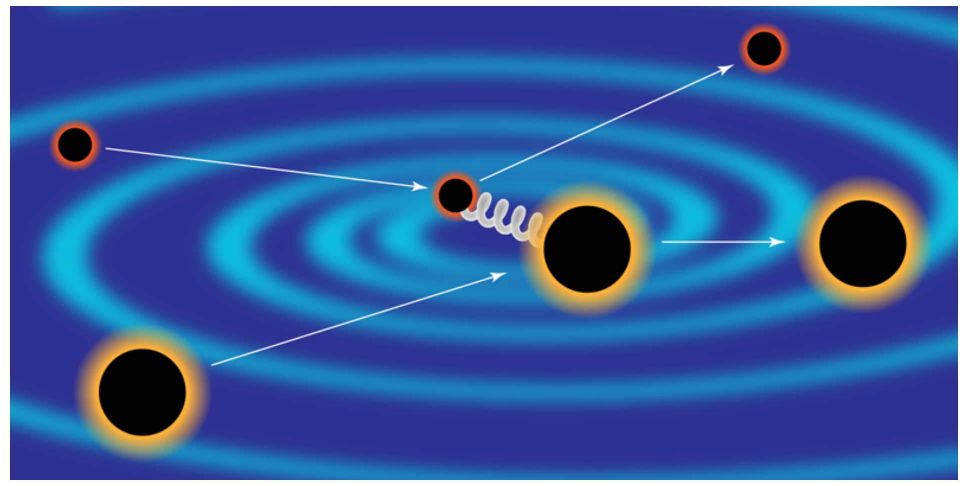


Theoretical Advances to Enable Precision GW Astronomy (contd.)

- **PN, PM, GSF** should be **pushed at higher order** and **combined in EOB** approach **more effectively and in novel ways** to largely **improve analytical solutions** of two-body problem. **Calibration** to NR should be made **more effective**.

- **Scattering-amplitude/effective-field-theory/quantum-field-theory** methods from high-energy physics **have brought new tools to solve two-body problem in classical gravity**.

  (Damour 17; Bjerrum-Bohr+18, Vines+18, Cheung+19; Bern+19, Kosower+19, Cristofoli+19, Damgaard+19, Blümlein+20, Bern+20, Kälin+20, Cheung & Solon 20, Parra-Martinez+20, Mogull+21, Brandhuber+21, Bern+21, Dlapa+21, Liu+21, Jakobsen+22, Bern+23, Jakobsen+23, Driesse+24, Dlapa+24, Bern+24, Bini+24)

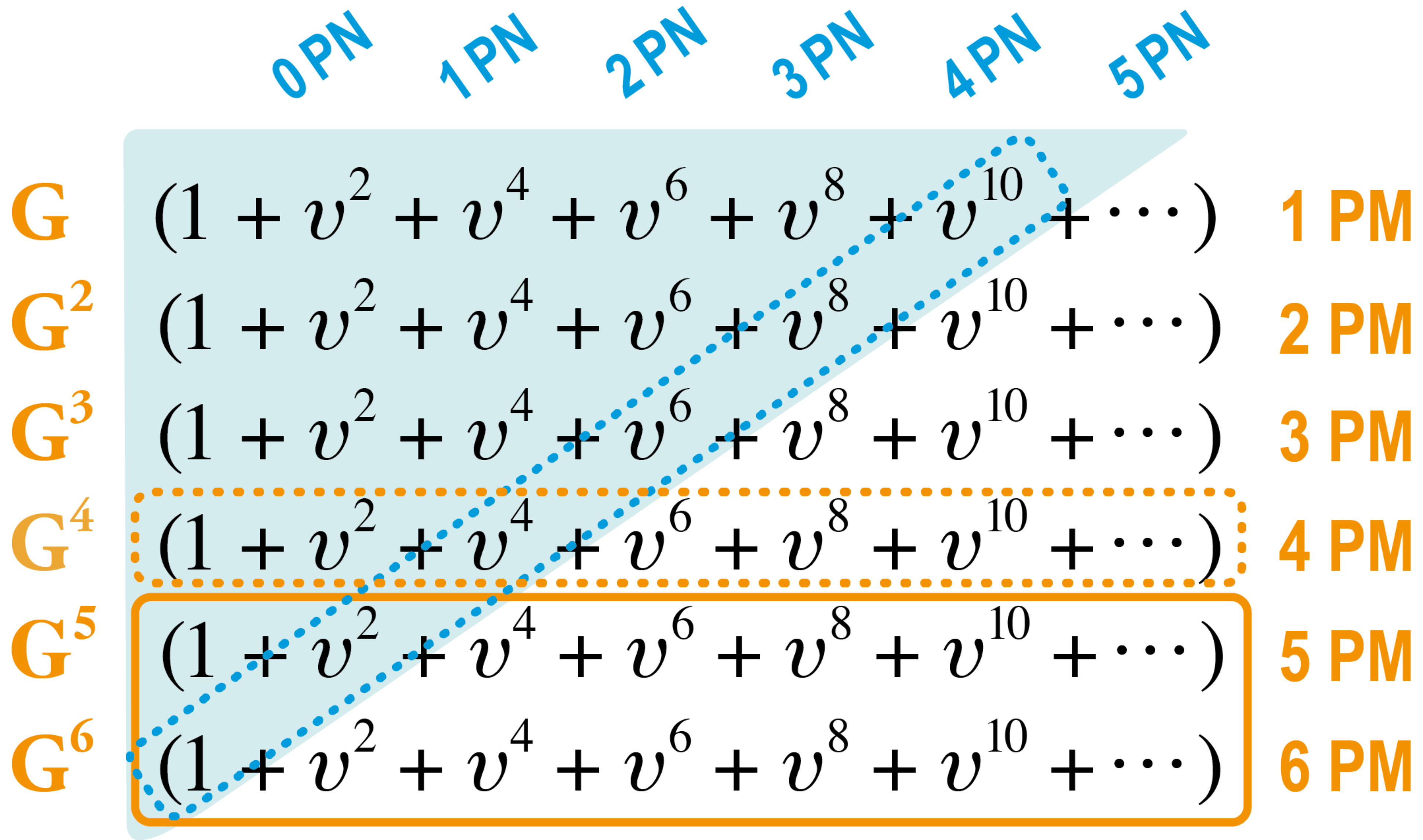


(APS/Stonebraker)

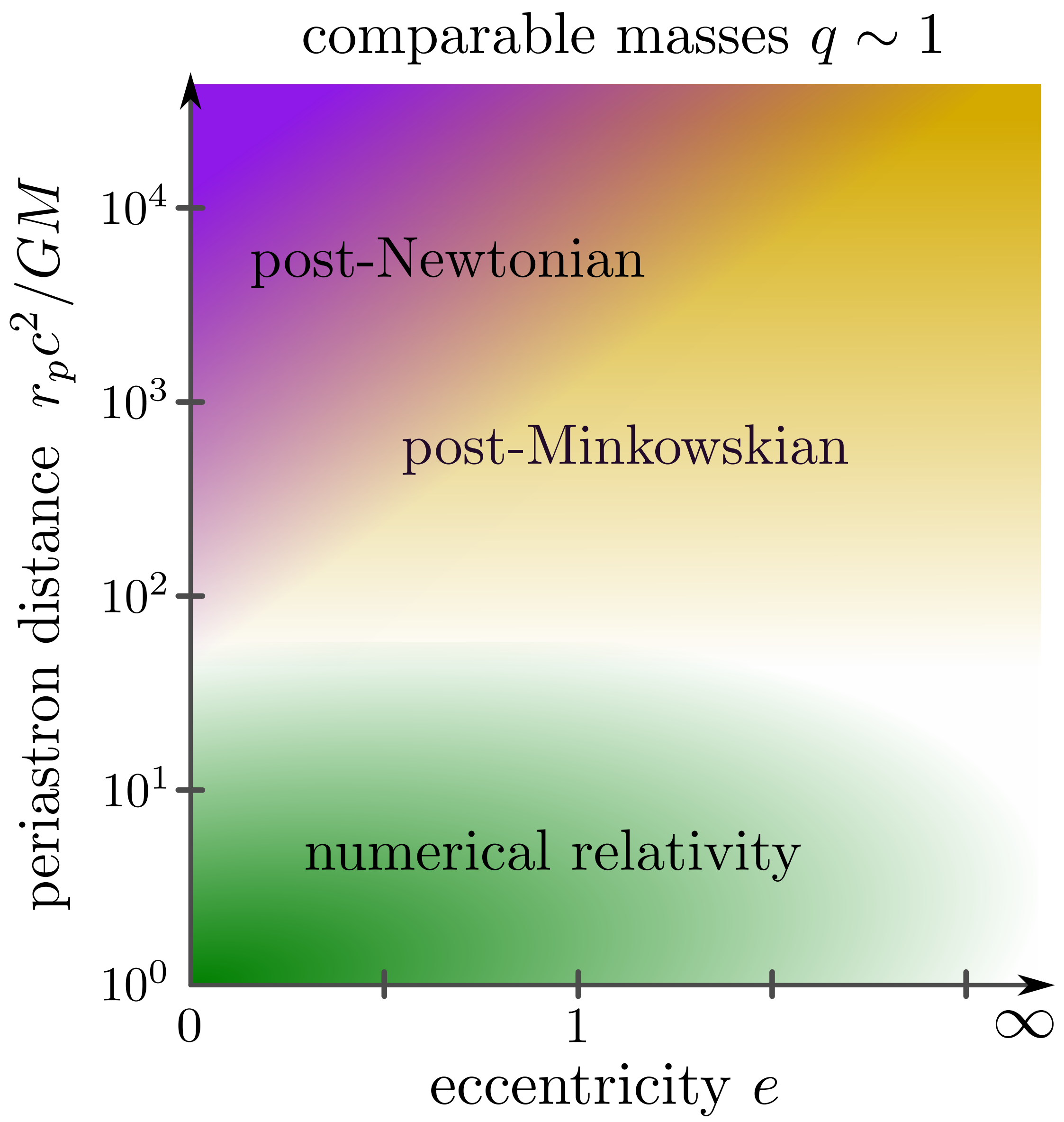
- **Traditional PN methods continue to make important progress.**

  (Blanchet+23, Trestini+23, Blanchet+24)

- **Frontier in analytical, perturbative calculations:** 6PM/5PN.



- The **PM approximation is more accurate than PN** for scattering encounters **at large velocities,** or equivalently **large eccentricities at fixed periastron distance**.



(Khalil+ arXiv: 2204.05047)

Theoretical Advances to Enable Precision GW Astronomy (contd.)

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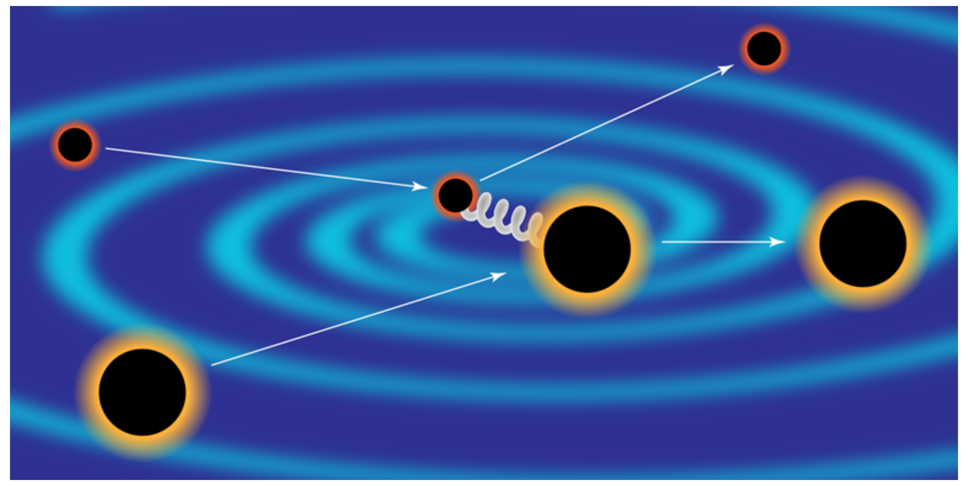
- **Scattering-amplitude/effective-field-theory/quantum-field-theory** methods from high-energy physics **have brought new tools to solve two-body problem in classical gravity**.

  (Damour 17; Bjerrum-Bohr+18, Vines+18, Cheung+19; Bern+19, Kosower+19, Cristofoli+19, Damgaard+19, Blümlein+20, Bern+20, Kälin+20, Cheung & Solon 20, Parra-Martinez+20, Mogull+21, Brandhuber+21, Bern+21, Dlapa+21, Liu+21, Jakobsen+22, Bern+23, Jakobsen+23, Driesse+24, Dlapa+24, Bern+24, Bini+24)

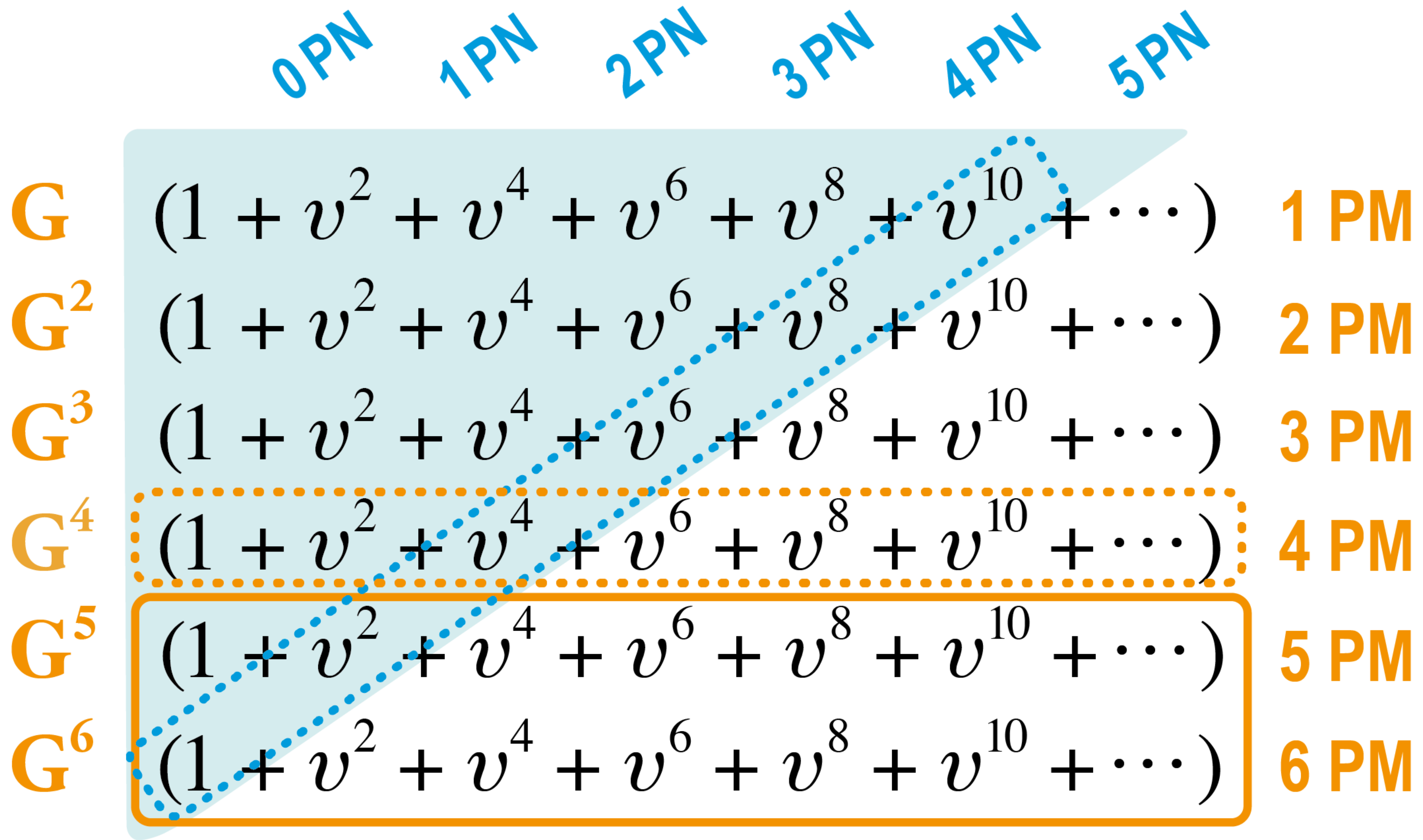


(APS/Stonebraker)

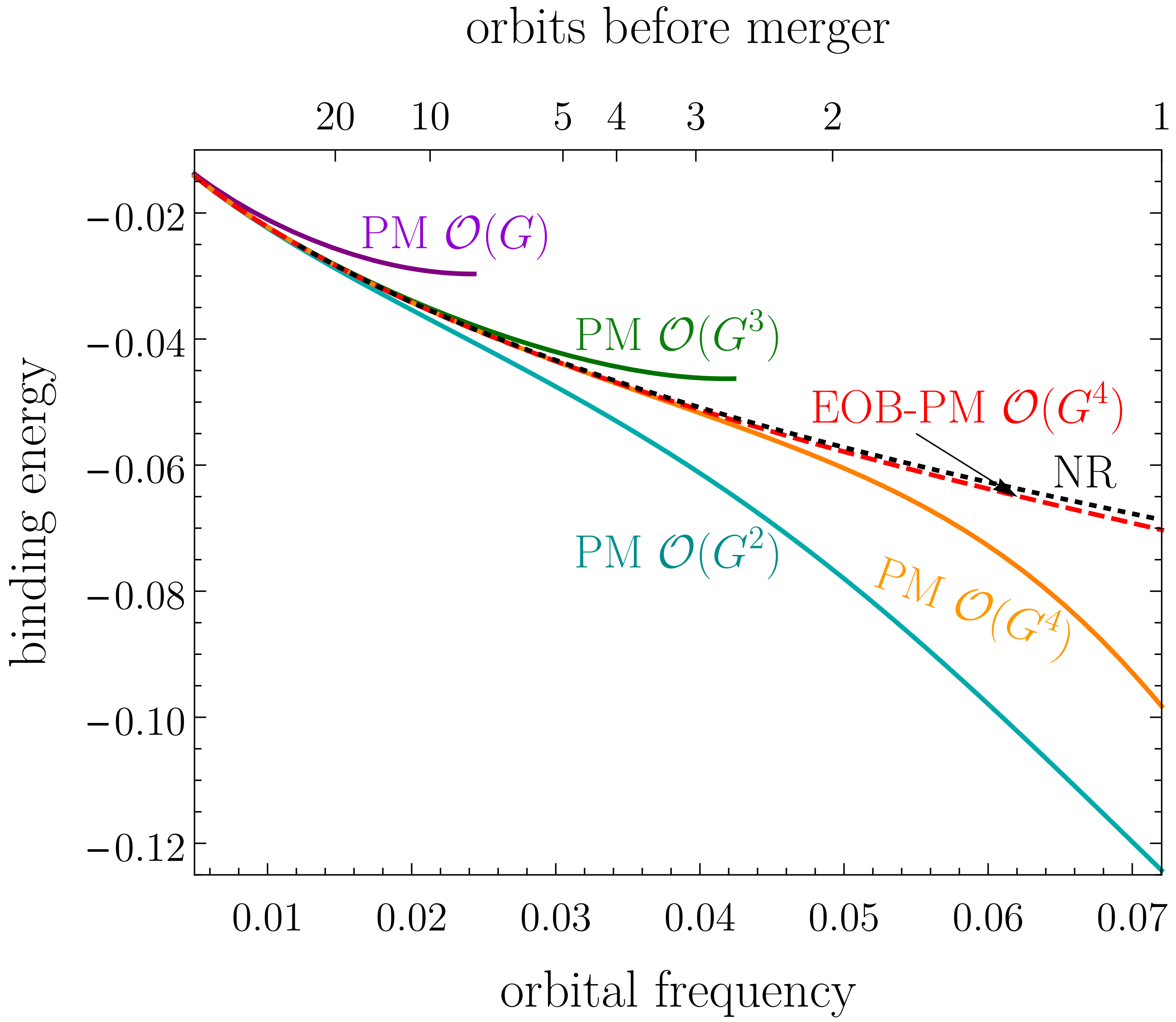
- **Traditional PN methods continue to make important progress.**

  (Blanchet+23, Trestini+23, Blanchet+24)

- **Frontier in analytical, perturbative calculations:** 6PM/5PN.



- **PM results resummed** in the EOB formalism.



(Khalil+ arXiv: 2204.05047)

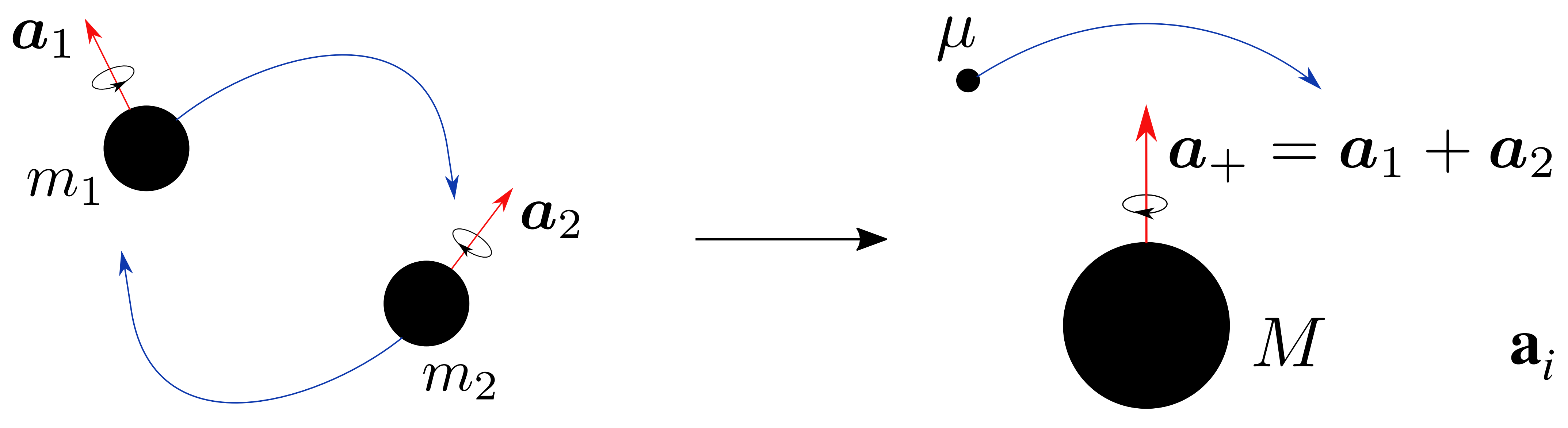
EOB Approach Meets the PM Theory for Bound Orbits

- **Two-body dynamics is mapped** onto the dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

$\mu = m_1 m_2/M \qquad M = m_1 + m_2 \qquad \nu = \mu/M \qquad 0 \leq \nu \leq 1/4$

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

$G = 1 = c$



(credit: Khalil)

$\mathbf{a}_i = m_i\,\chi_i \quad i = 1,2$

$0 \leq \chi_i \leq 1$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Rettegno, Marrtinetti, Nagar et al. 19; Khalil, Steinhoff, Vines & AB 20; Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)

- **PM results for conservative dynamics** (last 5 years)

(Guevara, Ochirov & Vines 19, Chen, Chung, Huang, & Kim 22, Bern, Kosmopoulos, Luna, Roiban & Teng 23, Aoude, Haddad & Helset 23, Bautista 23)

(Bern, Cheung, Roiban, Shen, Solon & Zeng 19, Kälin, Liu & Porto 20, Cheung & Solon 20, Di Vecchia, Heissenberg, Russo & Veneziano 20, Jakobsen & Mogull 22, 23, Febres Cordero, Kraus, Lin, Run & Zeng 23, Brandhuber+21)

(Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon et al. 22, Dlapa, Kälin, Liu & Porto 22, Jakobsen, Mogull, Plefka, Sauer & Xu 23, Jakobsen, Mogull, Plefka & Sauer 23, Dlapa, Kälin, Liu & Porto 24, Damour & Bini 24)

(Driesse, Jakobsen, Mogull, Plefka, Sauer & Usovitsch 24)

(AB, Jakobsen & Mogull arXiv: 2402.12342)

	S^0 (Spin-0)	S^1 (Spin-1/2)	S^2 (Spin-1)	S^3 (Spin-3/2)	S^4 (Spin-2)	S^5 (Spin-5/2)
1PM (tree level)	G	G^2	G^3	G^4	G^5	G^6
2PM (1 loop)	G^2	G^3	G^4	G^5	G^6	G^7
3PM (2 loops)	G^3	G^4	G^5	G^6	G^7	G^8
4PM (3 loops)	G^4	G^5	G^6	G^7	G^8	G^9
5PM (4 loops)	G^5	G^6	G^7	G^8	G^9	G^{10}

tails

PM Theory Meets the EOB Approach for Bound Orbits

- **The SEOB-PM Hamiltonian** is a **deformation of the Kerr Hamiltonian**, it is **informed by *available* PM results,** and it is **complemented by PN bound-orbit** corrections.

(Bini+17-18, Antonelli, AB+19, Khalil, AB+22, Khali, AB+23, AB, Jakobsen & Mogull 24)

(AB, Mogull, Patil & Pompili arXiv: 2405.19181)

$$H_{\rm eff} = \frac{M\, p_\phi\, (g_{a_+}\, a_+ + g_{a_-}\, \delta\, a_-)}{r^3 + a_+^2\,(r+2M)} + \sqrt{A\left(\mu^2 + \frac{p_\phi^2}{r^2} + (1+B_{\rm np}^{\rm Kerr})\, p_r^2 + B_{\rm npa}^{\rm Kerr}\, \frac{p_\phi^2\, a_+^2}{r^2}\right)}$$

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu}-1\right)}$$

$G = 1 = c$

$$A = \frac{(1 - 2u + \chi_+^2\, u^2 + \Delta A)}{[1 + \chi_+^2\, u^2\,(2u+1)]} \qquad g_{a_\pm} = \frac{\Delta g_{a_\pm}}{u^2} \qquad u = M/r$$

$$a_i = m_i\, \chi_i \quad M\chi_\pm = a_1 \pm a_2 \quad 0 \le \chi_i \le 1 \quad \delta = (m_1 - m_2)/M$$

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tails

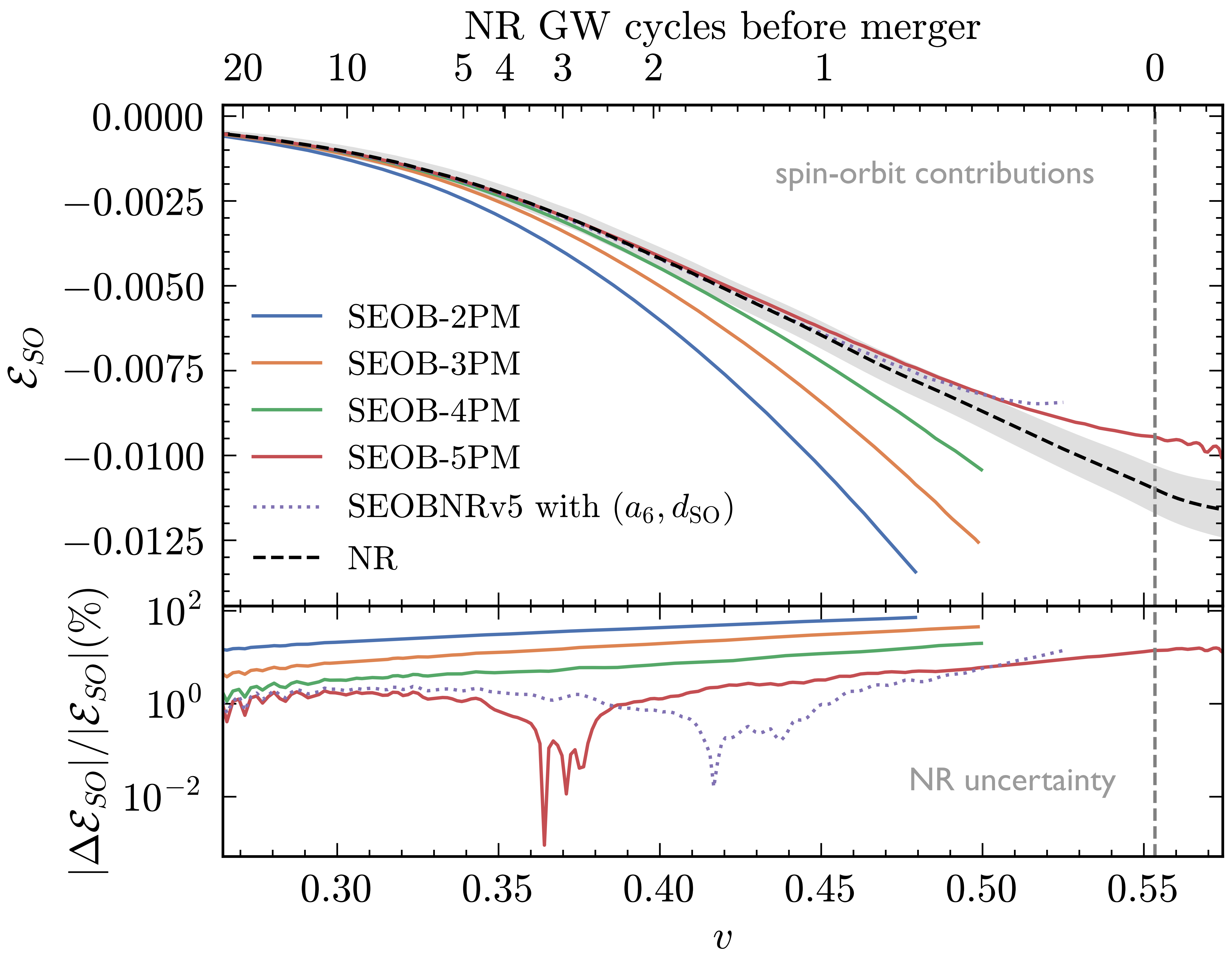
Comparing SEOB-PM Binding Energy with Numerical Relativity

$G = 1 = c$

(AB, Mogull, Patil & Pompili arXiv: 2405.19181)

binding energy

$q = \frac{m_1}{m_2} = 1$



- **Despite not being calibrated to NR, SEOB-PM shows excellent agreement with NR**, with a clear convergence. Its **accuracy is somewhat better than SEOBNRv5**, despite the latter being calibrated in the non-spinning (a_6) and spin-orbit coupling ($d_{\rm SO}$) sectors.

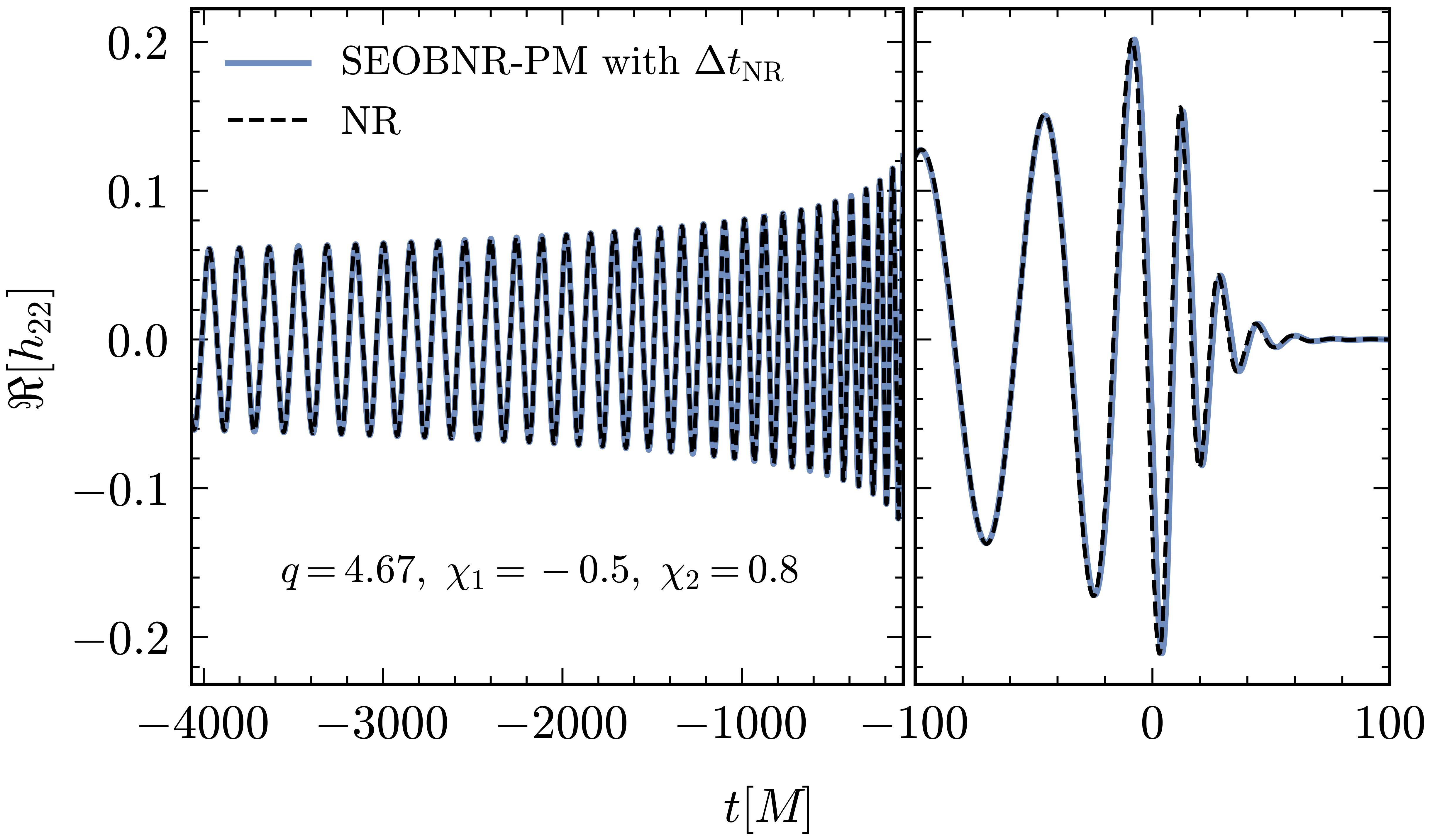
Comparing SEOBNR-PM Waveforms with Numerical Relativity

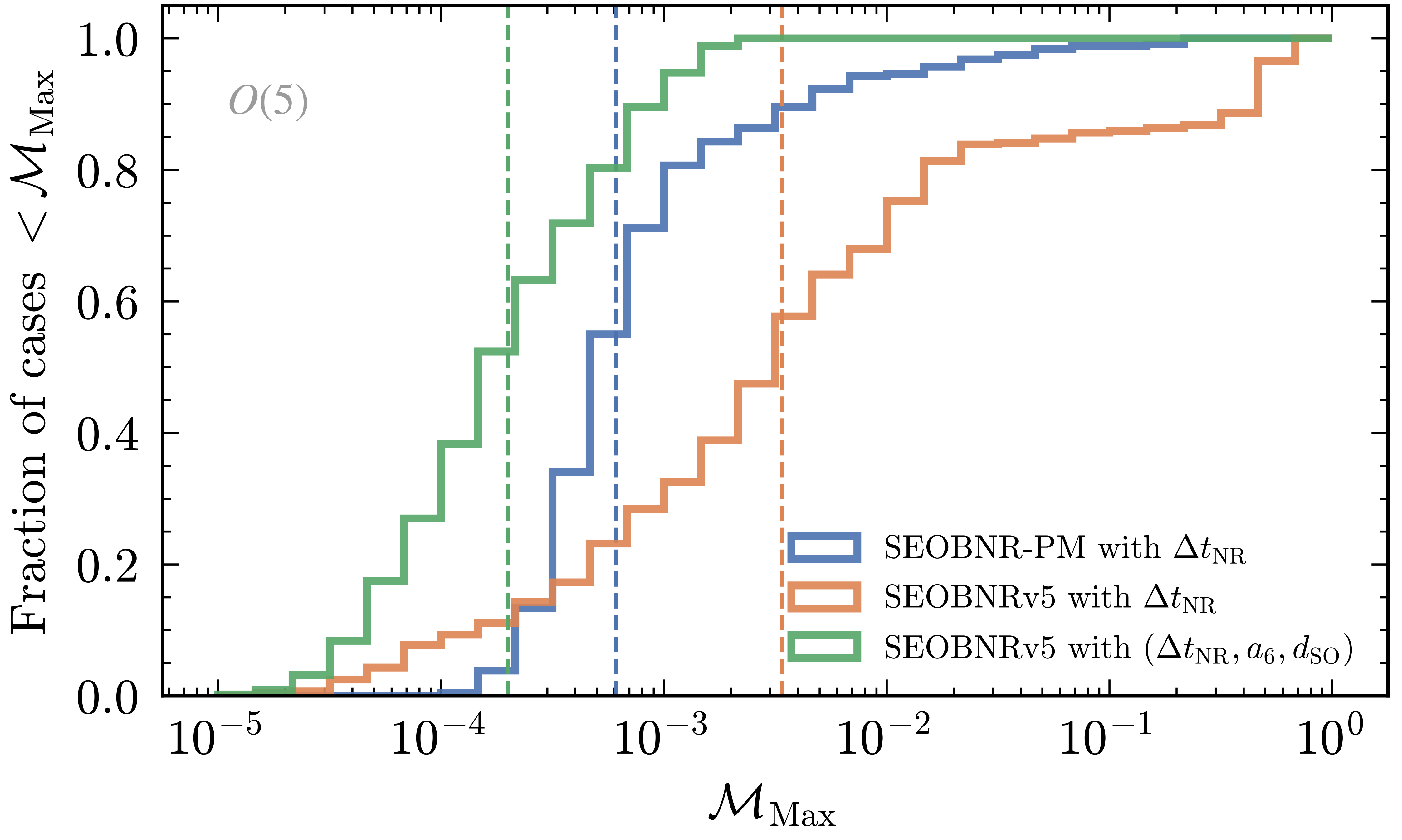
pySEOBNR

(AB, Mogull, Patil & Pompili arXiv: 2405.19181)

Mismatch $\mathscr{M} = 0$ implies models & NR match perfectly

- **Inspiral-Merger-Ringdown** waveform **with PM information**
- **Mismatch against 441 NR SXS waveforms**





- **SEOBNR-PM has remarkably good agreement with NR** when calibrating only the time to merger (Δt_{NR}). The accuracy with NR is **better than when using the SEOBNR model based** entirely **on PN**.

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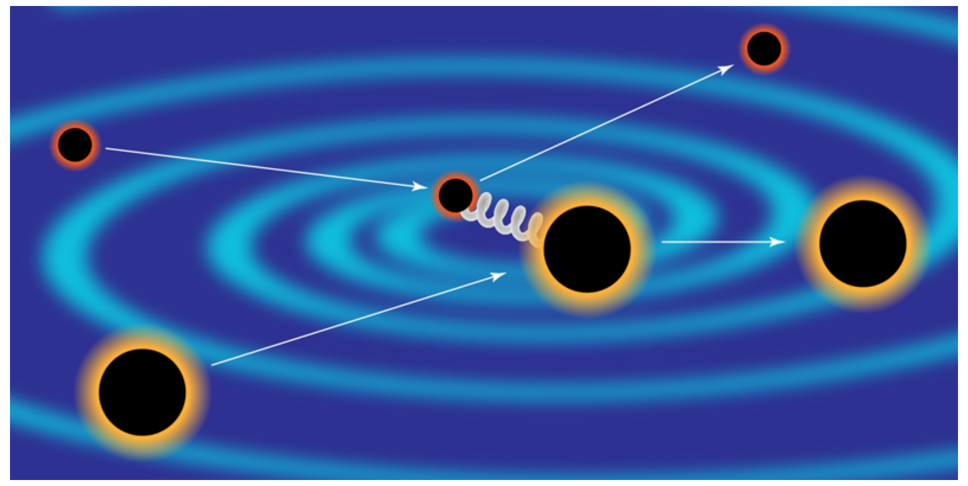

(APS/Stonebraker)

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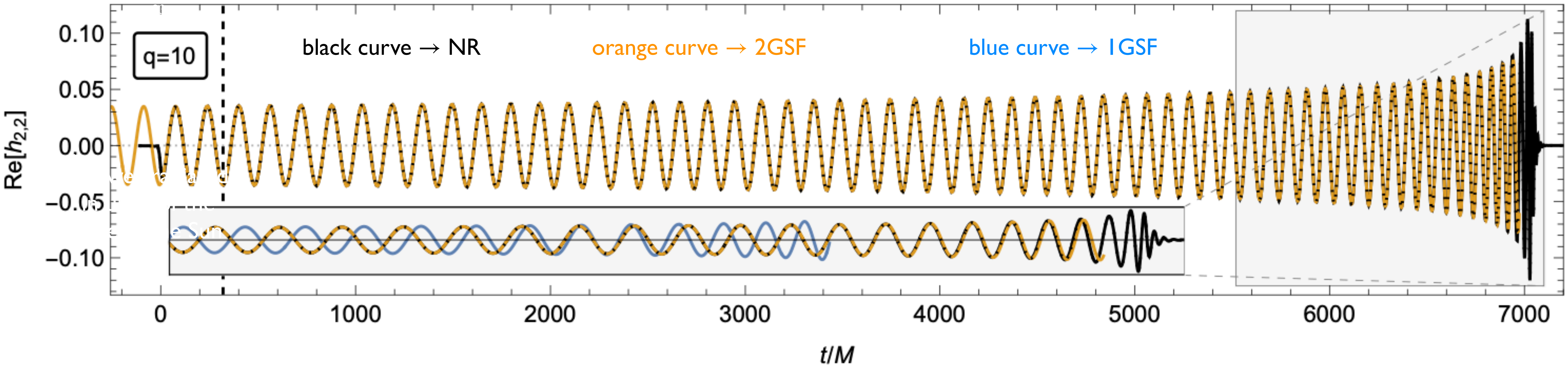
  (Blanchet+23, Trestini+23, Blanchet+24)

- **Frontier in analytical, perturbative calculations:** 6PM/5PN

  & 2GSF. *(Pound+20, Warburton+21, Wardell, Pound, Warburton, Miller, Durkan & Le Tiec 23)*

non spinning

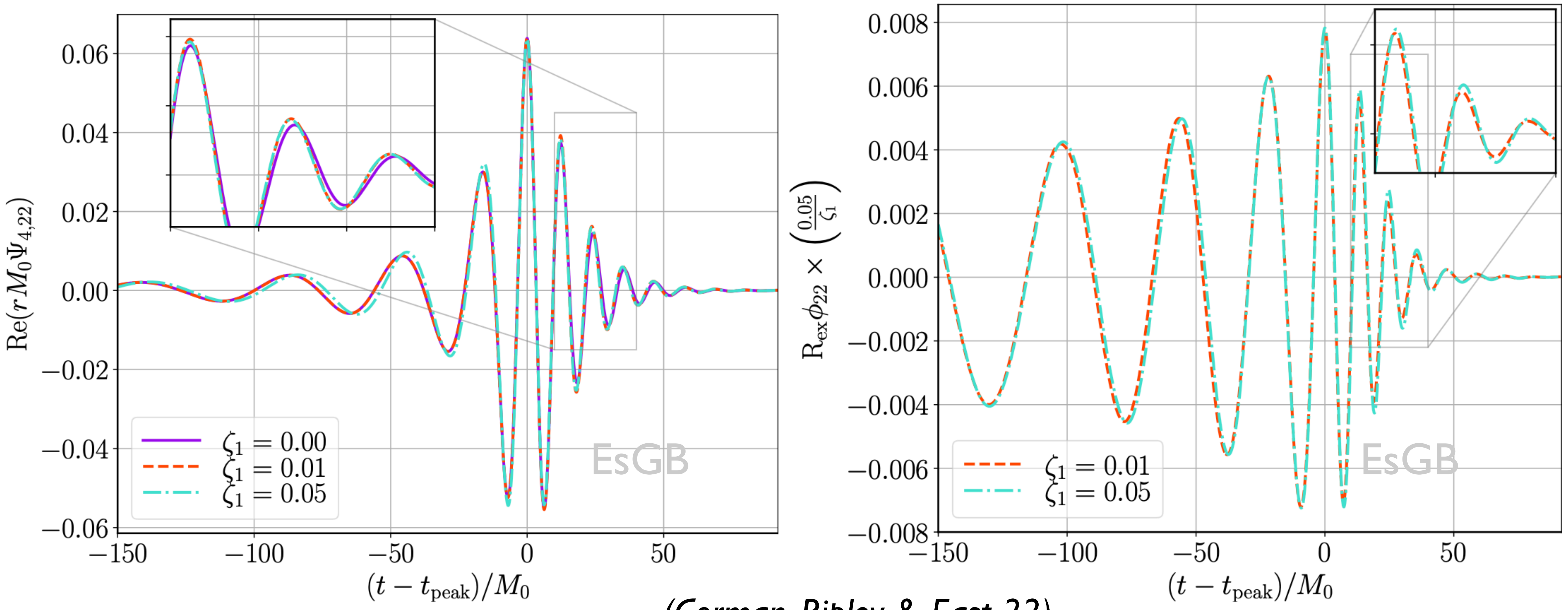


Toward Waveforms in beyond-GR Theories

- Rapidly growing results of **binary coalescences in beyond-GR theories using NR**.

(Witek+19, Okounkova+17-20, East+21, Figueras+22, Corman+22, Barausse+22, Lara+22, Cayuso+23, Corman+24, Figueras+24, Lara+24, Nee+24)

EsGB: Einstein scalar Gauss Bonnet



(Corman, Ripley & East 22)

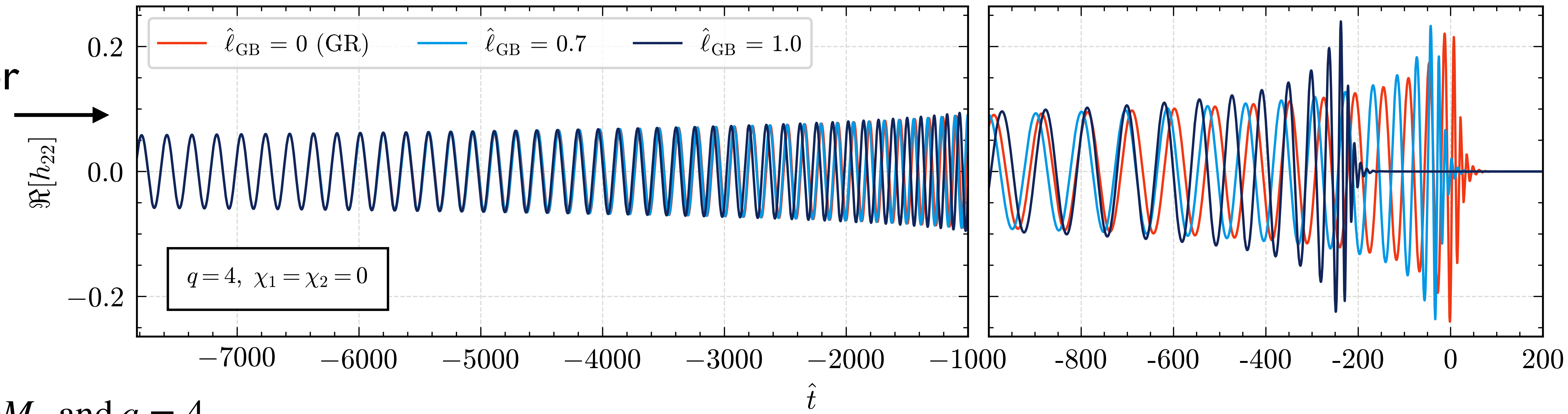
- Important progress in **analytical results** for the inspiral stage **in beyond-GR theories** in recent years.

(Lang 14-15, Sennett+16, Julié & Deruelle 17, Julié 17, Khalil+18, Julié 18, Shiralilou+20, Bernard+22, Jain+22, Julié+22)

- EOB inspiral-merger-ringdown waveforms for **scalarized BHs** in (dilatonic) **EsGB gravity.**

(Julié, Pompili, AB 24)

pySEOBNR



- Current bound: $l_{GB} \lesssim 1.65\,\mathrm{km}$ (GW230529)

(Gao+24, Sänger+24)

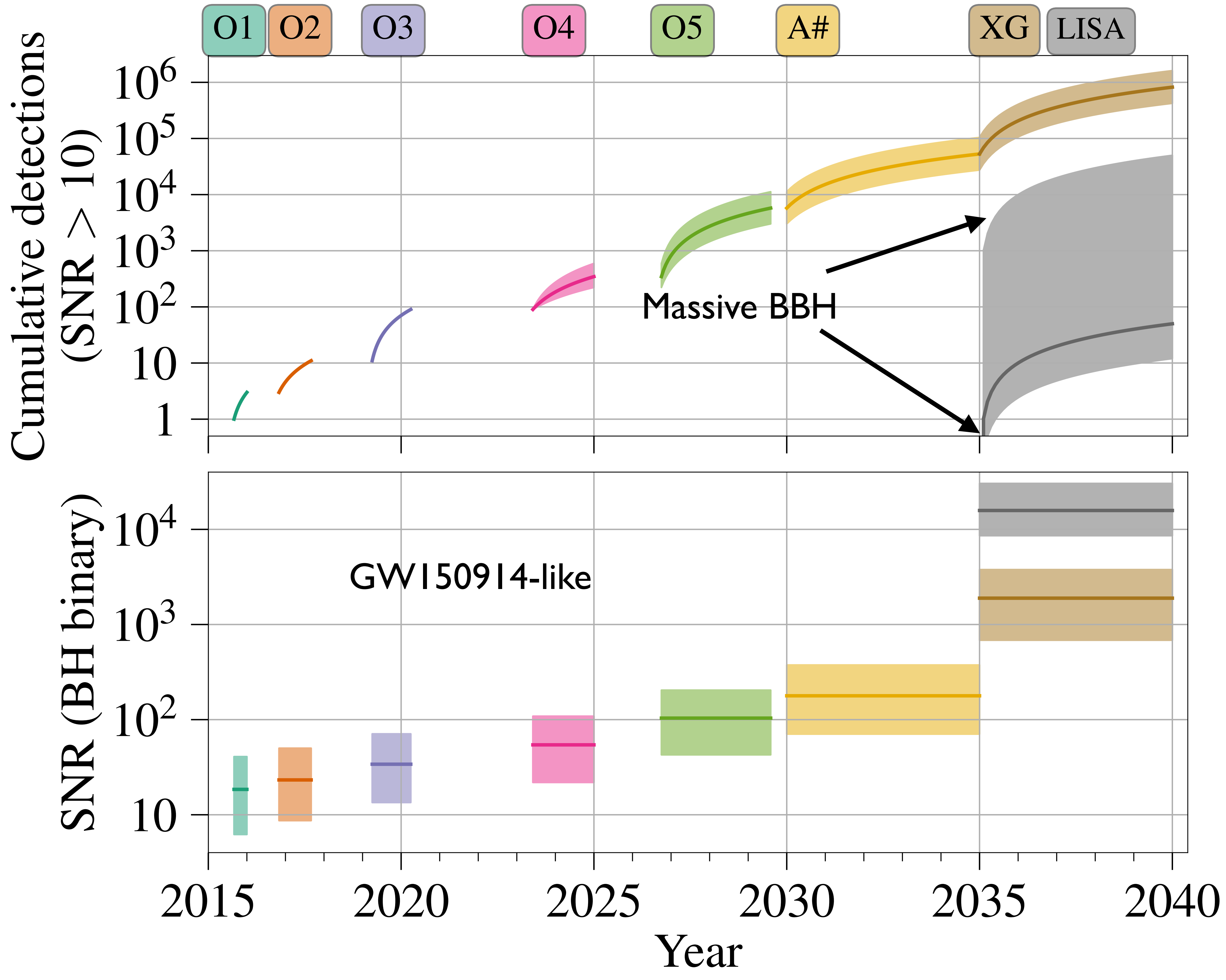
$\hat{l}_{GB} \lesssim 0.11$ for $M = 20 M_\odot$ and $q = 4$

Summary & Outlook

- **Solving the relativistic two-body problem** with the accuracy needed for today's GW observations has been **a long-standing challenge**. The last 20 years have seen **tremendous progress thanks also to the synergistic work** at the interface between **analytical and numerical relativity**.

- Traditionally, approximation methods like **PN, PM, GSF have progressed independently**. However, there are **now cross-fertilization and validation**. **Phenom** waveforms **excel in efficiency**. The **EOB approach can** successfully **push the boundaries of analytical accuracy,** while **NR surrogate models** have been **highly effective where applicable**.

- Achieving a **100-fold improvement in accuracy** for vacuum-GR and **incorporating all physical effects** (generic orbits, beyond-GR, matter/environment) is a **significant challenge**. Yet, it's **essential to accurately interpret** future GW observations **and avoid misinterpreting** scientific results.



(credit: Dhani)

The “Astrophysical and Cosmological Relativity” Division


Retreat at Ringberg Castle


Einstein’s Haus in Caputh


Workshop: Connecting the Dots

The material presented is based upon work supported by NSF's LIGO Laboratory, which is a major facility fully funded by the NSF, by the STFC, and the Max Planck Society, and by the Virgo Laboratory through the European Gravitational Observatory (EGO), INFN, CNRS, and the Netherlands Organization for Scientific Research, and of many other national research agencies of the members of the LIGO-Virgo-KAGRA Collaboration.

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LISA CONSORTIUM

ET EINSTEIN TELESCOPE

Thank you!