

Model independent tests of gravity

WG perspective

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“LISA in Copenhagen” workshop

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LUTH

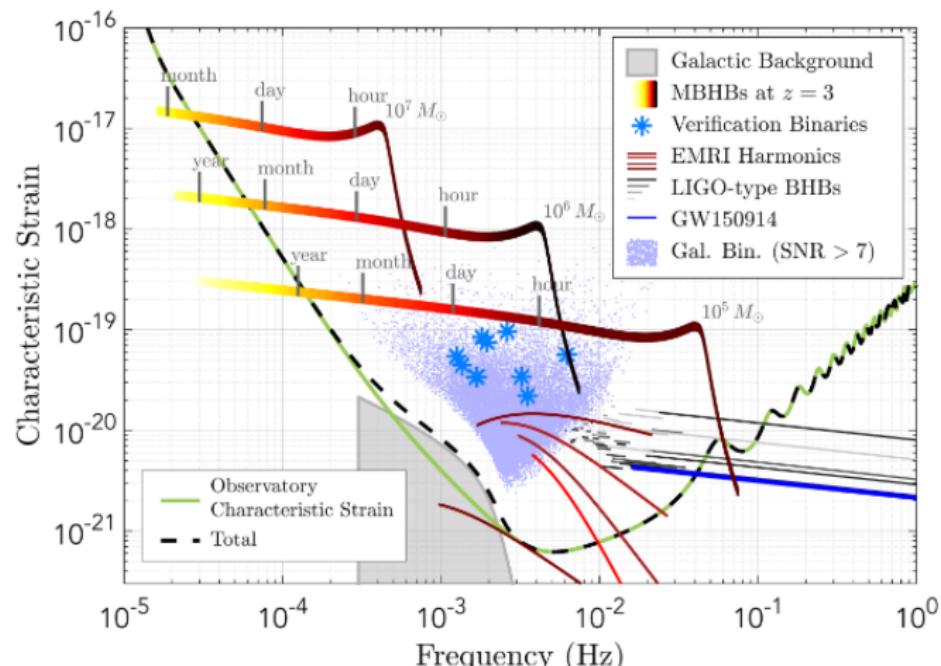
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Testing gravity with LISA

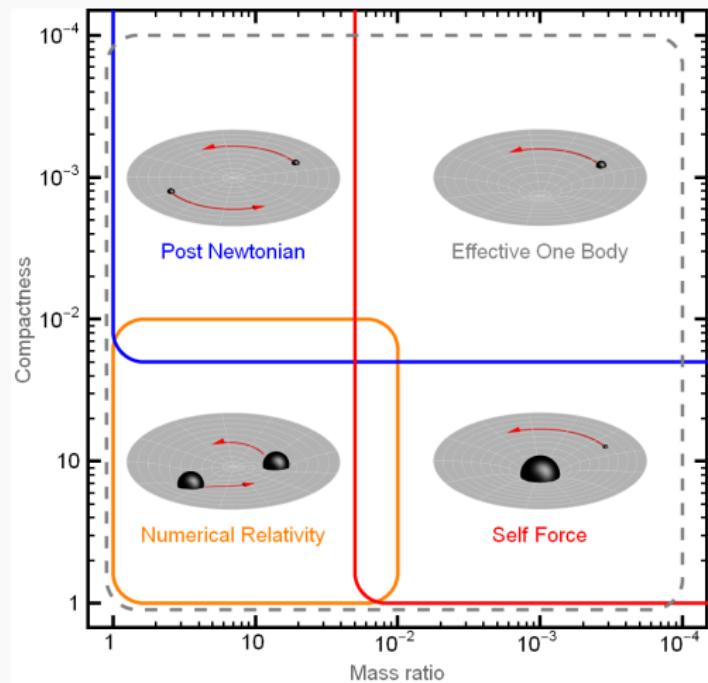


LISA science case

- ▷ EMRIs, long inspiral phase for MBHBs

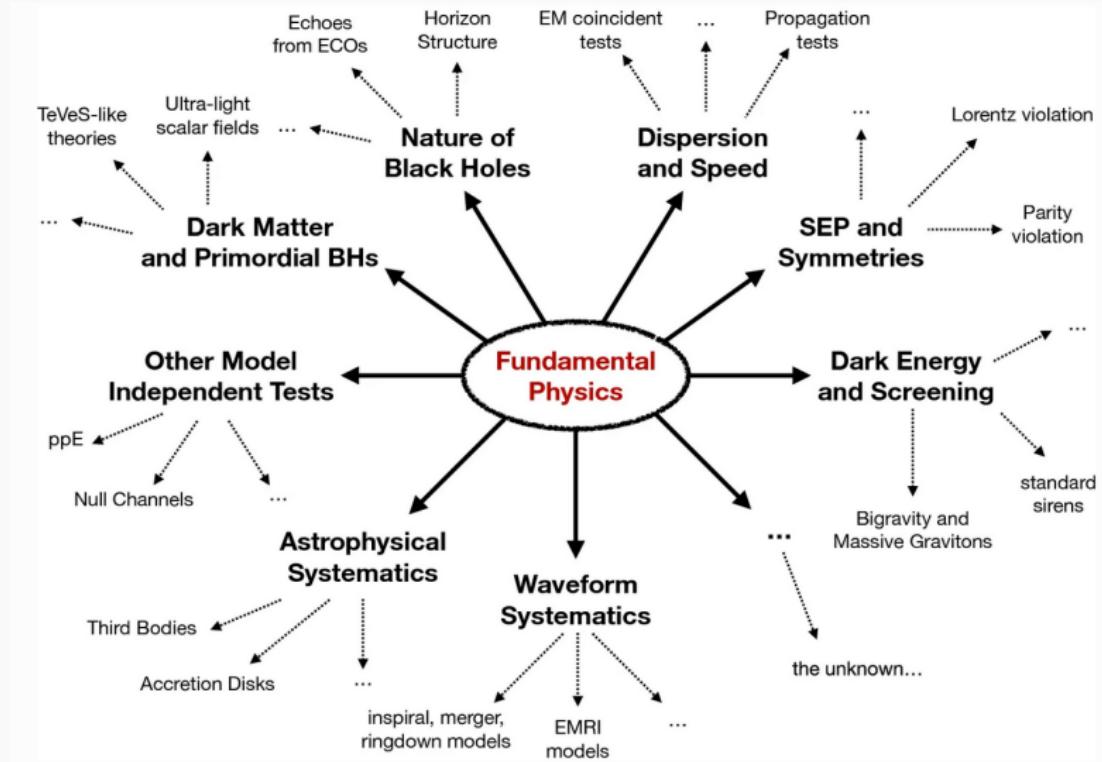
Gravitational wave modelling

$$h_+ - i h_\times = \sum_{l=2}^{+\infty} \sum_{m=-l}^l -2 Y_{lm} h_{lm}(t)$$



Credits: M. van de Meent

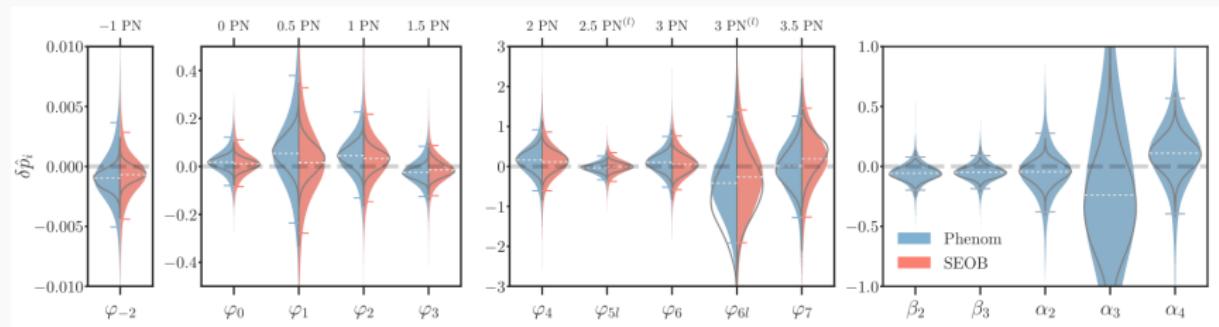
Going beyond GR



Currently in LIGO

Parametrized deviations from GR

$$h(f) = h_{\text{GR}}(f) e^{i \delta \varphi_n(f)}$$



LIGO-Virgo, 2021

Parametrized tests of gravity

A historical perspective: the PPN formalism

- ▷ pioneered by Will and Nordtvedt in the 70's
- ▷ parametrized post Newtonian: parametrizes deviation from GR in the 1PN metric

$$\begin{aligned} g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi \Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi) \Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi) \Phi_2 \\ & + 2(1 + \zeta_3) \Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi) \Phi_4 - (\zeta_1 - 2\xi) \mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3) w^2 U - \alpha_2 w^i w^j U_{ij} \\ & + (2\alpha_3 - \alpha_1) w^i V_i + \mathcal{O}(\epsilon^3), \end{aligned}$$

$$\begin{aligned} g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi) V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi) W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2) w^i U \\ & - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}), \end{aligned}$$

$$g_{ij} = (1 + 2\gamma U) \delta_{ij} + \mathcal{O}(\epsilon^2).$$

A historical perspective: PPN

- ▷ 10 meaningful parameters: $\gamma, \beta, \xi, \alpha_{1,2,3}, \zeta_{1,2,3,4}$

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much “nonlinearity” in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation	0	0	0
ζ_1	of total momentum?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

A historical perspective: PPN

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	2×10^{-4}	VLBI
$\beta - 1$	perihelion shift	8×10^{-5}	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	spin precession	4×10^{-9}	millisecond pulsars
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		4×10^{-5}	PSR J1738+0333
α_2	spin precession	2×10^{-9}	millisecond pulsars
α_3	pulsar acceleration	4×10^{-20}	pulsar \dot{P} statistics
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	\ddot{P}_p for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent [see Eq. (73)]

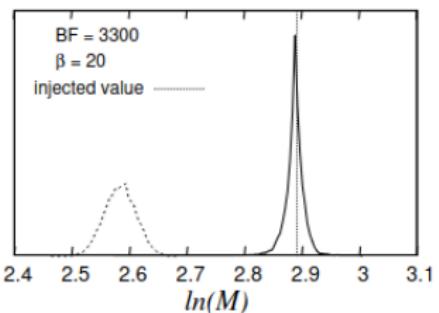
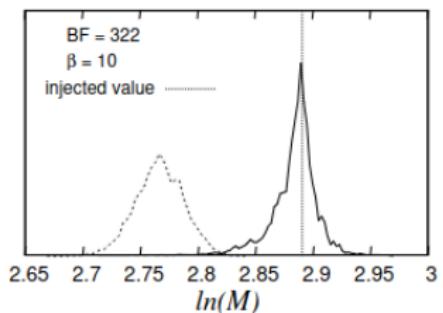
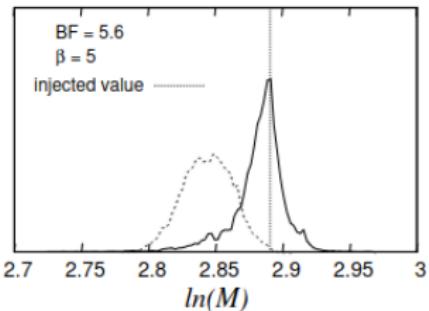
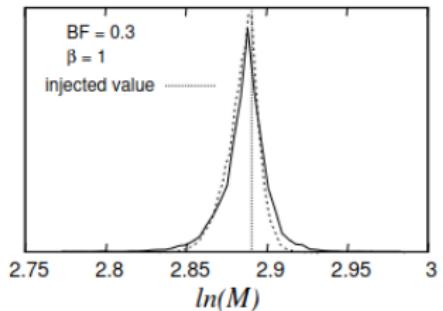
The parametrized post-Einsteinian (ppE) approach

modifies the most accurate GR waveform model in Fourier domain [Yunes & Pretorius, 2009]

$$h(f) = A_{\text{GR}}(f) [1 + \alpha_{\text{ppE}} v(f)^a] e^{i \Psi_{\text{GR}}(f) + i \beta_{\text{ppE}} v(f)^b}$$

- ▷ recover GR: $\Psi_{\text{GR}}(f) \propto v^{-5}$ and $\alpha_{\text{ppE}} = \beta_{\text{ppE}} = 0$
- ▷ applicable to many sources (MBHBs, EMRIs, *etc.*)
- ▷ $(\alpha_{\text{ppE}}, \beta_{\text{ppE}})$ controls the **magnitude of the deviation**
- ▷ (a, b) controls the **type of effects** to be constrained
 - $b = -7$: dipole radiation
 - $b = -13$: variation of G

The parametrized post-Einsteinian (ppE) approach



Mapping to specific theories

Focusing on the leading order contribution in the amplitude

$$a = 2n \implies n\text{PN order}$$

Theories	PPE Amplitude Parameters	
	Magnitude (α)	Exponent (a)
Scalar-Tensor [76, 90, 101]	$-\frac{5}{192}\eta^{2/5}(\alpha_1 - \alpha_2)^2$	-2
EdGB	$-\frac{5}{192}\zeta_{\text{EdGB}} \frac{(m_1^2 s_2^{\text{EdGB}} - m_2^2 s_1^{\text{EdGB}})^2}{m^4 \eta^{18/5}}$	-2
DCS	$\frac{57713}{344064}\eta^{-14/5}\zeta_{\text{dCS}} \left[-2\delta_m \chi_a \chi_s + \left(1 - \frac{14976\eta}{57713}\right) \chi_a^2 + \left(1 - \frac{215876\eta}{57713}\right) \chi_s^2 \right]$	+4
Einstein-Æther [99]	$-\frac{5}{96}\eta^{2/5} \frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1-s_1^{\text{EA}})(1-s_2^{\text{EA}})]^{4/3}} \left[\frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3 w_1^3} \right]$	-2
Khronometric [99]	$-\frac{5}{96}\eta^{2/5} \frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}} \sqrt{\bar{\alpha}_{\text{kh}}} \left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})} \right]^{3/2}$	-2
Noncommutative	$-\frac{3}{8}\eta^{-4/5}(2\eta - 1)\Lambda^2$	+4
Varying- G [92]	$\frac{5}{512}\eta_0^{3/5} \dot{G}_{C,0} [-7m_0 + (s_{1,0} + s_{2,0} - \delta_{\dot{G}})m_0 + 13(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]$	-8

Mapping to specific theories

Focusing on the leading order contribution in the phase

$$b = -5 + 2n \implies n\text{PN order}$$

Theories	PPE Phase Parameters		Binary Type	
	Magnitude (β)	Exp. (b)		
Scalar-Tensor [95, 96]	$-\frac{5}{7168}\eta^{2/5}(\alpha_1 - \alpha_2)^2$	-7	Any	
EdGB [97]	$-\frac{5}{7168}\zeta_{\text{EdGB}} \frac{(m_1^2 s_2^{\text{EdGB}} - m_2^2 s_1^{\text{EdGB}})^2}{m^4 \eta^{8/5}}$	-7	Any	
DCS [82, 98]	$\frac{481525}{3670016}\eta^{-14/5}\zeta_{\text{DCS}} [-2\delta_m \chi_a \chi_s + (1 - \frac{4992\eta}{19261})\chi_a^2 + (1 - \frac{72052\eta}{19261})\chi_s^2]$	-1	BH/BH	
Einstein-Æther [99]	$-\frac{5}{3584}\eta^{2/5} \frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1 - s_1^{\text{EA}})(1 - s_2^{\text{EA}})]^{4/3}} \left[\frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3w_1^3} \right]$	-7	Any	
Khronometric [99]	$-\frac{5}{3584}\eta^{2/5} \frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1 - s_1^{\text{kh}})(1 - s_2^{\text{kh}})]^{4/3}} \sqrt{\bar{\alpha}_{\text{kh}}} \left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})} \right]^{3/2}$	-7	Any	
Noncommutative [100]	$-\frac{75}{256}\eta^{-4/5}(2\eta - 1)\Lambda^2$	-1	BH/BH	
Varying- G [92]	$-\frac{25}{851968}\eta_0^{3/5}\dot{G}_{C,0} [11m_0 + 3(s_{1,0} + s_{2,0} - \delta_{\dot{G}})m_0 - 41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]$	-13	Any	

Extending ppE for precessing binaries

$$h(f) = \sum_{K=(l,m,n)} h_{(K)}^{\text{GR}}(f) \left[1 + \alpha_{(K)}^{\text{ppE}} v(f)^{a_{(K)}} \right] e^{i \beta_{(K)}^{\text{ppE}} v(f)^{b_{(K)}}}$$

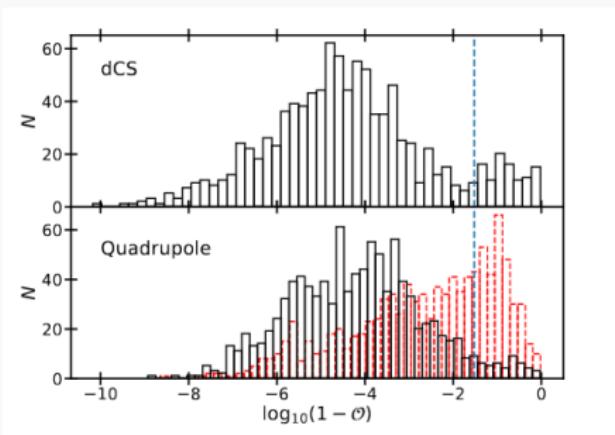


FIG. 5. Mismatch between the precessing ppE waveforms and non-GR waveforms containing corrections in all harmonics for dCS gravity (top) and non-axisymmetric quadrupoles (bottom), with N the number of systems. The vertical dashed line corresponds to a mismatch of 0.03, or overlap of 0.97, which is a commonly used as a requirement for waveform accuracy. For dCS gravity, 89.6% of systems have mismatches below this requirement, while for non-axisymmetric quadrupoles 66.7% or 95.2% are below this if one include four harmonics (red histogram) or five harmonics (black histogram) in the ppE waveform, respectively.

Loutrel & Yunes, 2022

- ▷ still needs further development

Ringdown tests - parametrization

Parametrized master equations (non-rotating BHs)

$$f \frac{d}{dr} \left(f \frac{d\Phi}{dr} \right) + [\omega^2 - f\mathbf{V}] \Phi = 0, \quad f = 1 - \frac{r_H}{r}$$

with potential: $V_{ij} = V_{ij}^{\text{GR}} + \frac{1}{r_H^2} \sum_{k \geq 0} \alpha_{ij}^{(k)} \left(\frac{r_H}{r} \right)^k$

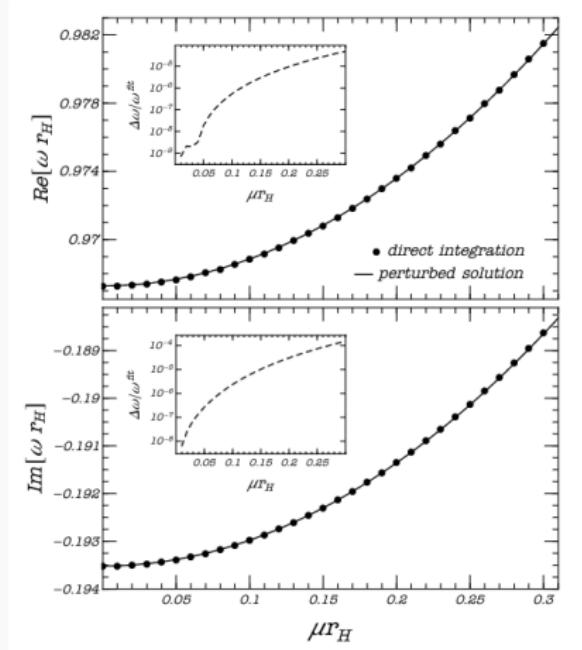
- $\alpha_{ij}^{(k)}$ small parameters
- ▷ corrected QNM frequencies:

$$\omega \approx \omega_0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \alpha'_{pq}^{(k)} d_{(k)}^{ij} d_{(s)}^{pq} + \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq}$$

- ▷ coefficients $d_{(k)}^{ij}$ and $e_{(ks)}^{ijpq}$ computed numerically

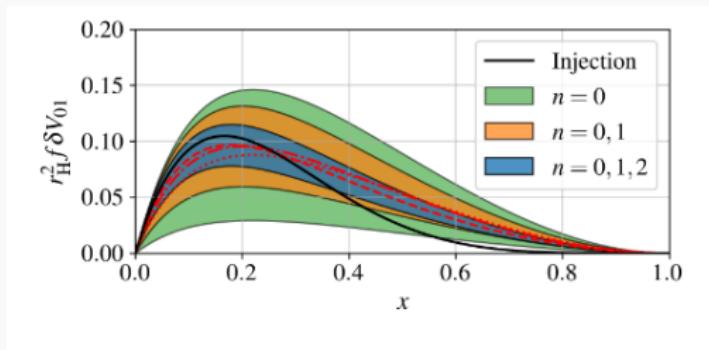
Ringdown tests - parametrization

k	Axial-Scalar		Polar-Scalar	
	$r_H e_{(kk)}^{1221}$	$r_H e_{(kk)}^{1221}$	$r_H e_{(kk)}^{1221}$	$r_H e_{(kk)}^{1221}$
2	-0.0388 - 0.00196 i		-0.0386 - 0.00135 i	
3	-0.0146 + 0.000930 i		-0.0155 + 0.00162 i	
4	-0.00567 - 0.000484 i		-0.00644 + 0.00000923 i	
5	-0.00228 - 0.00116 i		-0.00288 - 0.000923 i	
10	0.000457 - 0.000387 i		0.000318 - 0.000545 i	



Ringdown tests - inverse problem

- ▷ reconstructing the potential



Völkel et al., 2022

- ▷ mapping back to constraints on theories
- ▷ other approach: parametrized metric [Völkel & Franchini, 2022]

Ringdown tests - rotating case

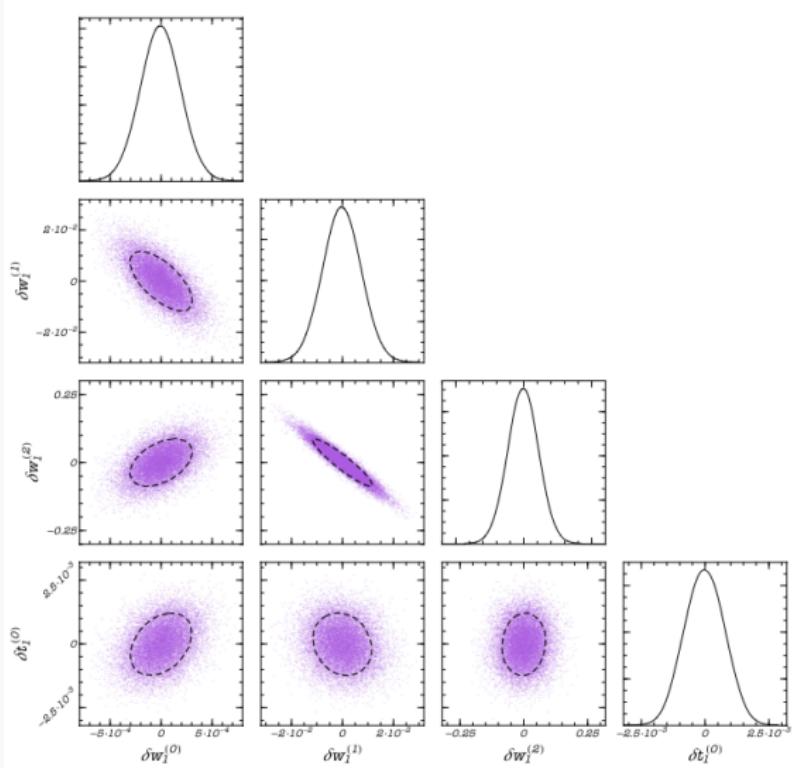
Parametrized spectroscopy (ParSpec) [Maselli et al., 2019]

$$\omega_i^{(lm)} = \frac{1}{M_i} \sum_{n=0}^D \chi_i^n \omega_{lm}^{(n)} (1 + \gamma_i \delta\omega_{lm}^{(n)})$$

$$\tau_i^{(lm)} = M_i \sum_{n=0}^D \chi_i^n t_{lm}^{(n)} (1 + \gamma_i \delta t_{lm}^{(n)})$$

- γ_i are coupling parameters, for i ringdown detections
- expansion in spin parameters to order D

ParSpec



Parametrized plunge-merger-ringdown waveforms

Parametrized EOB waveform models

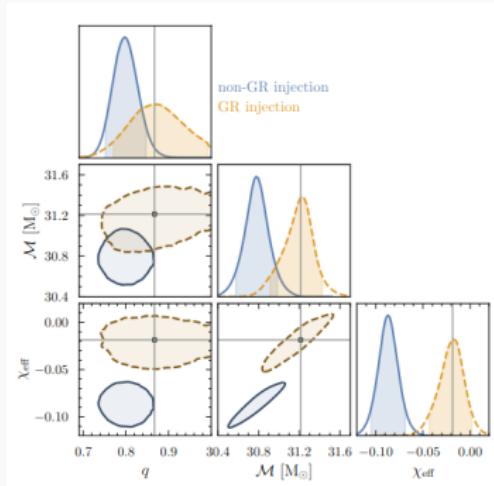
- ▷ mode amplitudes and frequencies
(NR informed)

$$|h_{lm}^{\text{NR}}| \rightarrow |h_{lm}^{\text{NR}}| (1 + \delta A_{lm})$$

$$\omega_{lm}^{\text{GR}} \rightarrow \omega_{lm}^{\text{GR}} (1 + \delta \omega_{lm})$$

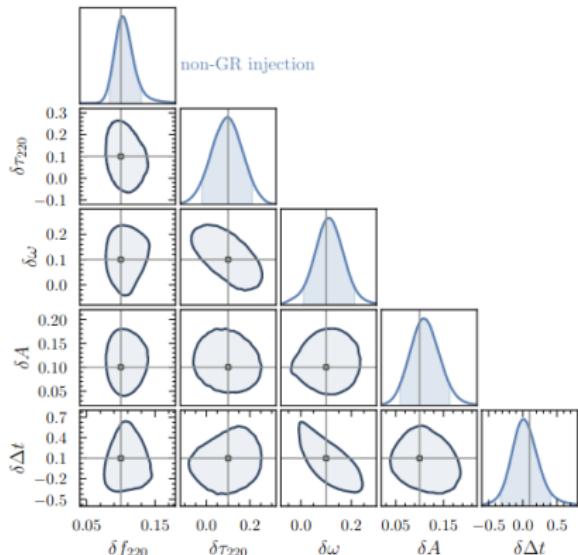
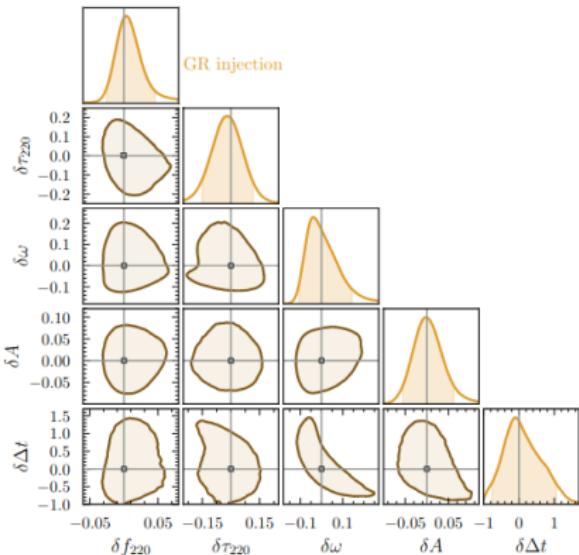
- ▷ time-lag parameters modification
(ringdown)

$$|\Delta t_{lm}^{\text{GR}}| \rightarrow |\Delta t_{lm}^{\text{GR}}| (1 + \delta \Delta t_{lm})$$



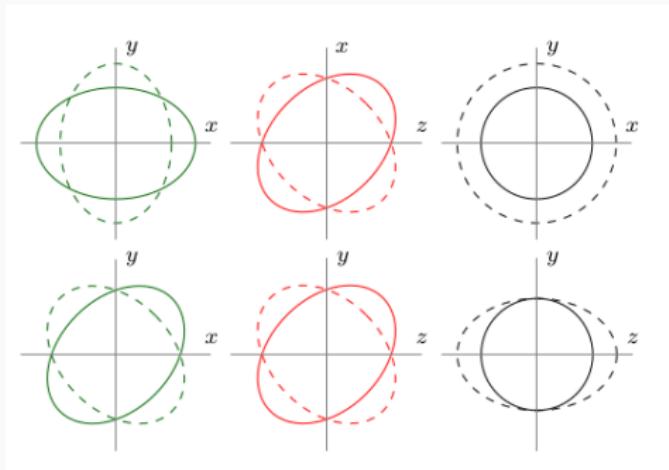
Maggio et al., 2022

Plunge-merger-ringdown tests



Other tests

Polarization tests



$$h \sim \begin{pmatrix} h_b + h_+ & h_x & h_x \\ h_x & h_b - h_+ & h_y \\ h_x & h_y & h_l \end{pmatrix}$$

Examples:

- ▷ Lorentz violating theories: 2 tensor + 1 scalar + 0 to 2 vectors
- ▷ Horndeski: 2 tensor + 1 scalar
- ▷ massive gravity: 2 tensor + 0 to 1 scalar + 2 vectors

The extended ppE formalism

- ▷ Single detector, higher order harmonics (including l=1)

$$\tilde{h}_{\text{ppE},1}^{\text{SD}}(f) = \mathcal{A} u_2^{-7/2} e^{-i\Psi_{\text{GR}}^{(2)}} \left[1 + c \beta u_2^{b+5} \right] + \gamma u_1^{-9/2} e^{-i\Psi_{\text{GR}}^{(1)}} e^{i\beta u_1^b}$$

- ▷ Multiple detectors, all harmonics

$$\begin{aligned} \tilde{h}_{\text{ppE},1}^{\text{MD}}(f) &= \tilde{h}_{\text{MD}}^{\text{GR}}(1 + c \beta u_2^{b+5}) e^{2i\beta u_2^b} + [\alpha_b F_b \sin^2 \iota + \alpha_L F_L \sin^2 \iota + \alpha_{sn} F_{sn} \sin \iota + \alpha_{se} F_{se} \sin 2\iota] \frac{\mathcal{M}^2}{D} u_2^{-7/2} e^{-i\Psi_{\text{GR}}^{(2)}} e^{2i\beta u_2^b} \\ &\quad + [\gamma_b F_b \sin \iota + \gamma_L F_L \sin \iota + \gamma_{sn} F_{sn} + \gamma_{se} F_{se} \cos \iota] \eta^{1/5} \frac{\mathcal{M}^2}{D} u_1^{-9/2} e^{-i\Psi_{\text{GR}}^{(1)}} e^{i\beta u_1^b}. \end{aligned} \quad (146)$$

GW propagation tests

Propagation of GWs other cosmological distance

$$h_{ij}'' + \left(3\frac{H'}{H} + \nu \right) h_{ij}' + \left[c_T^2 \left(\frac{k}{aH} \right)^2 + \frac{\mu^2}{H^2} \right] h_{ij} = \frac{\Gamma}{H^2} \gamma_{ij}$$

- ν : additional damping term
 - μ : mass of the graviton
 - c_T : speed of propagation
 - Γ : additional source term \Rightarrow oscillations in the amplitude
- ▷ DHOST, massive gravity, higher dim., Lorentz-viol., etc.
- ▷ recover GR with: $c_T = c$ and $\mu = \nu = \Gamma = 0$
- ▷ can be mapped to the ppE formalism

GW propagation tests: killing theories

Current constraints with GWs

$$\left| \frac{c_T}{c} - 1 \right| \lesssim \mathcal{O}(10^{-15}) , \quad m_g = \frac{\mu h}{c^2} \leq 4.7 \times 10^{-23} \text{ eV}/c^2$$

- ▷ implications for Horndeski theories

$$\mathcal{L}_{\text{st}} = R + X - V(\phi) + \mathcal{G}_2(\phi, X) + \mathcal{G}_3(\phi, X)\square\phi + \mathcal{G}_4(\phi)R$$

Modified dispersion relation

$$\frac{v_g}{c} \approx 1 + (\alpha - 1)A_\alpha E^{\alpha-2}$$

- α, A_α parametrizes quantum gravity effects, extra dim. etc.
- massive graviton for $\alpha = 0$ and $A_0 = m_g^2 c^4$

Perspectives

Perspectives

Path to improvement

- ▷ Parametrized ringdown models
- ▷ Adding eccentricity to ppE, improving for polarisations, spins, etc.

What was not mentionned

- ▷ Consistency tests of GR
 - ▷ residual tests, IMR tests, multipolar GW tests
- ▷ stochastic GW background
- ▷ ???