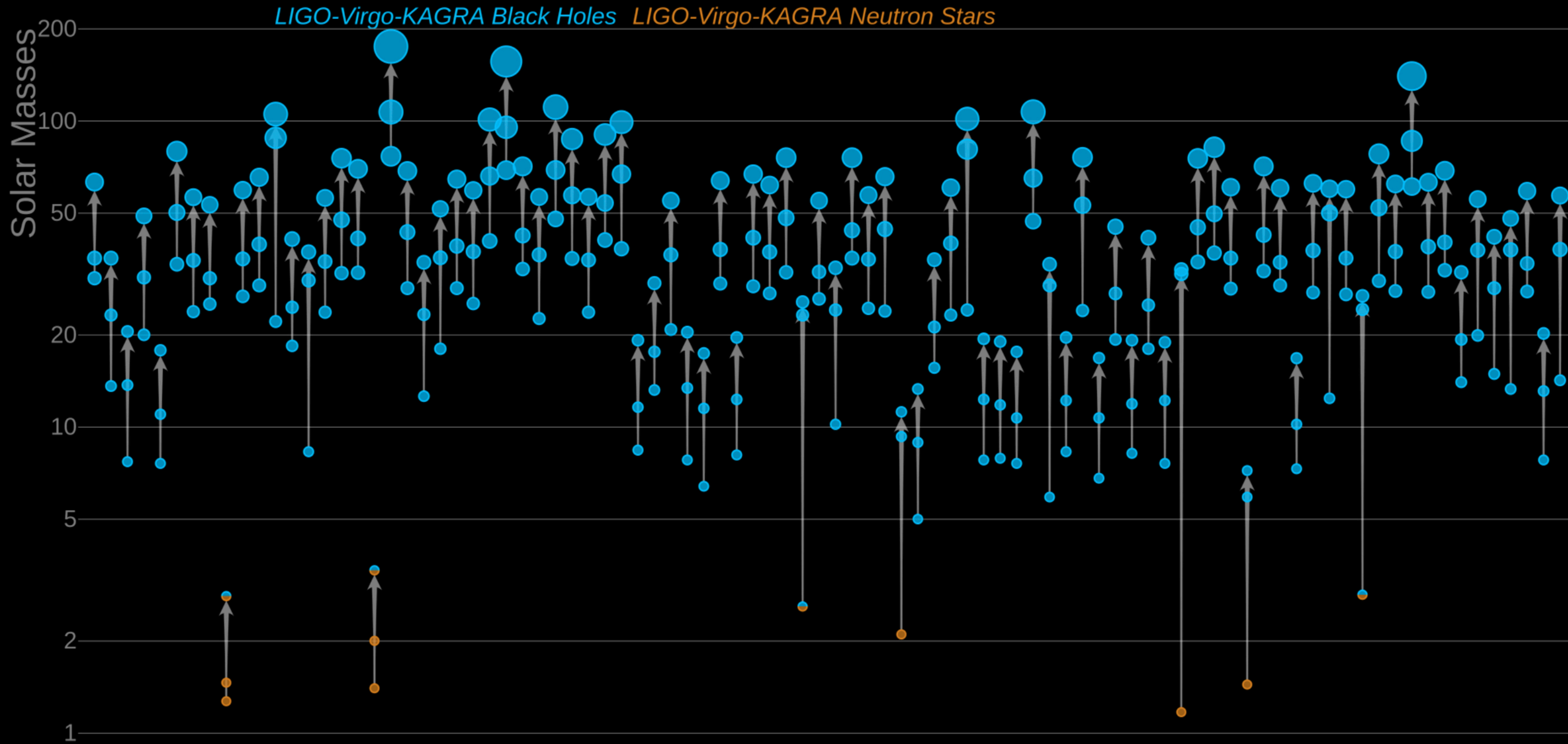


# Waveforms Systematics

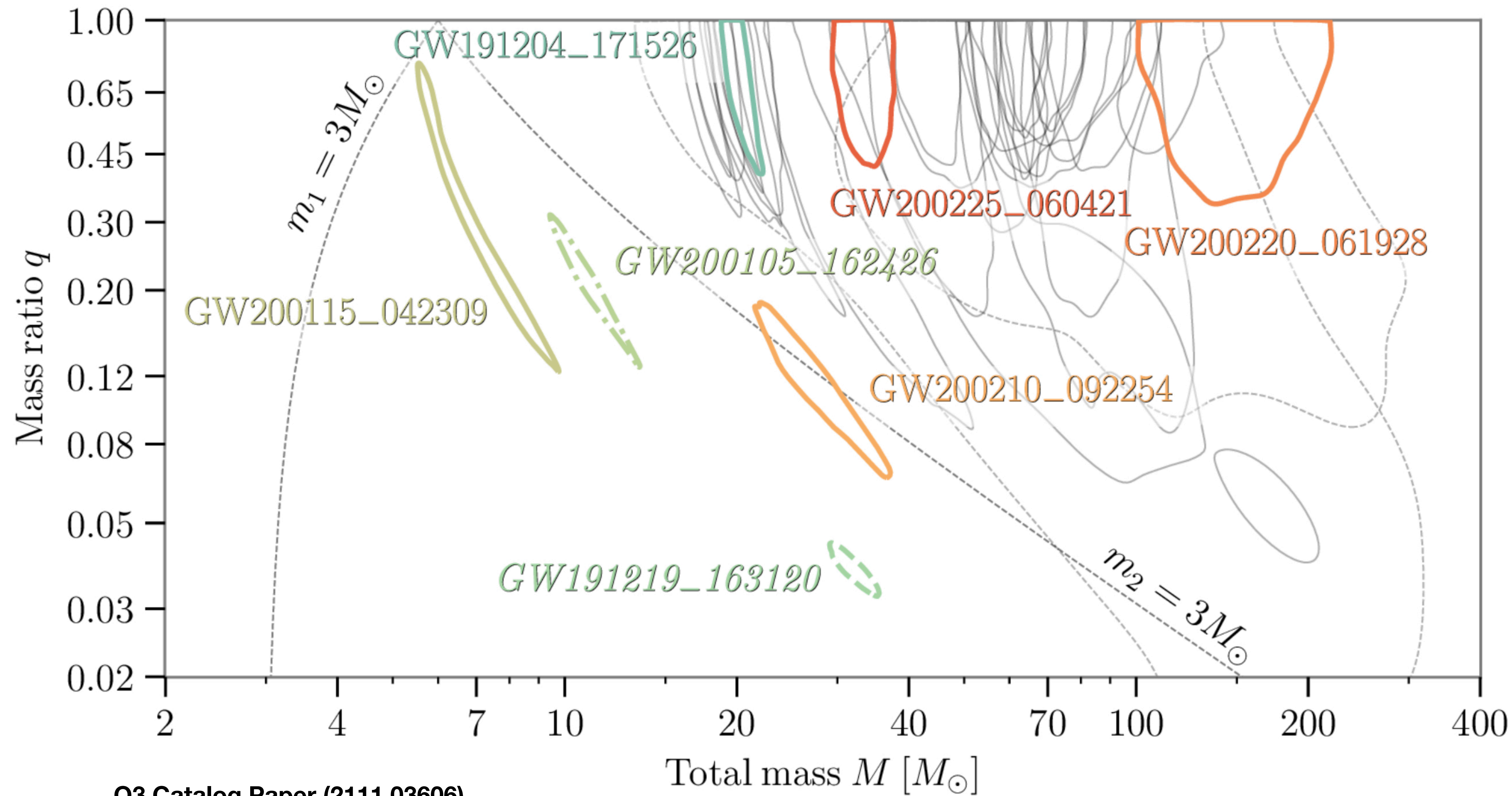
Deirdre Shoemaker  
Weinberg Institute  
Center for Gravitational Physics  
UT Austin

**Waveform Working Group Chairs: DS, Maarten van de Meent, Niels Warburton, Helvi Witek**  
**Waveform Working Package Chairs: Leor Barack, Anna Heffernan, Harald Pfeifer**

# Masses in the Stellar Graveyard









# LESSONS LEARNED



Stellar-mass BBHs waveforms were successful ( $\text{SNR} < 35$ )

- Waveform systematics cause false positives in testing GR: for example, <https://arxiv.org/abs/2112.06861>
- Different models lead to different parameters (missing precession or higher modes, lack of uniform NR coverage, ...): 2111.03606, Huang et al, Ossokine et al, Nita et al
- Extreme cases (high  $q$ , high spin, high inclination): Biscoveanu et al, Varma et al, Colleoni et al
- Degeneracies between GR parameters, and between GR and non-GR : Vallisneri et al, Ghosh et al, Johnson-McDaniel et al
- Few cycle waveform fit everything!



# This talk

- LISA waveform landscape
- Waveform challenges and priorities
- Dive into NR as an example

**Ensure the quality of GW science is not limited by our capability to solve Einstein's equations**

# THE WAVEFORM THEORETICAL LANDSCAPE

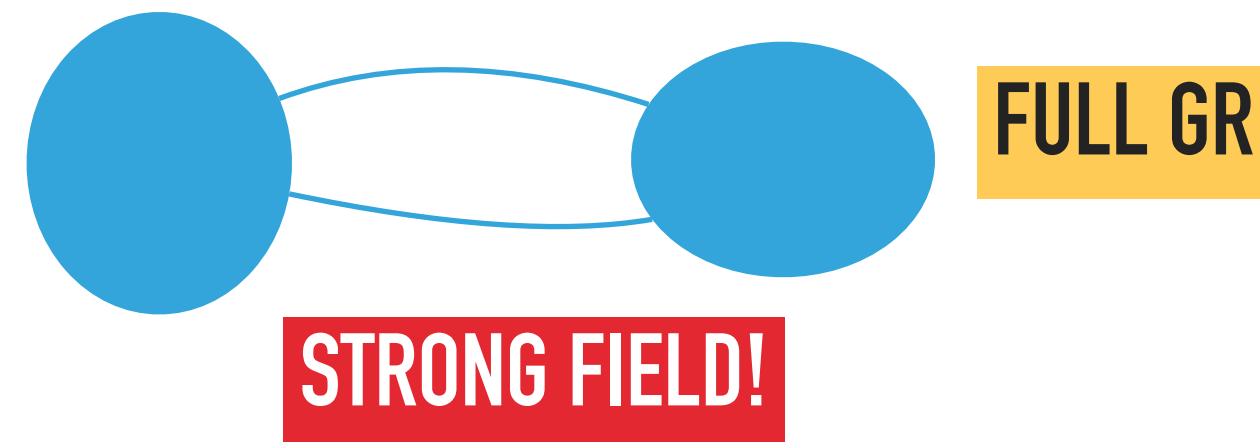
Post-Newtonian



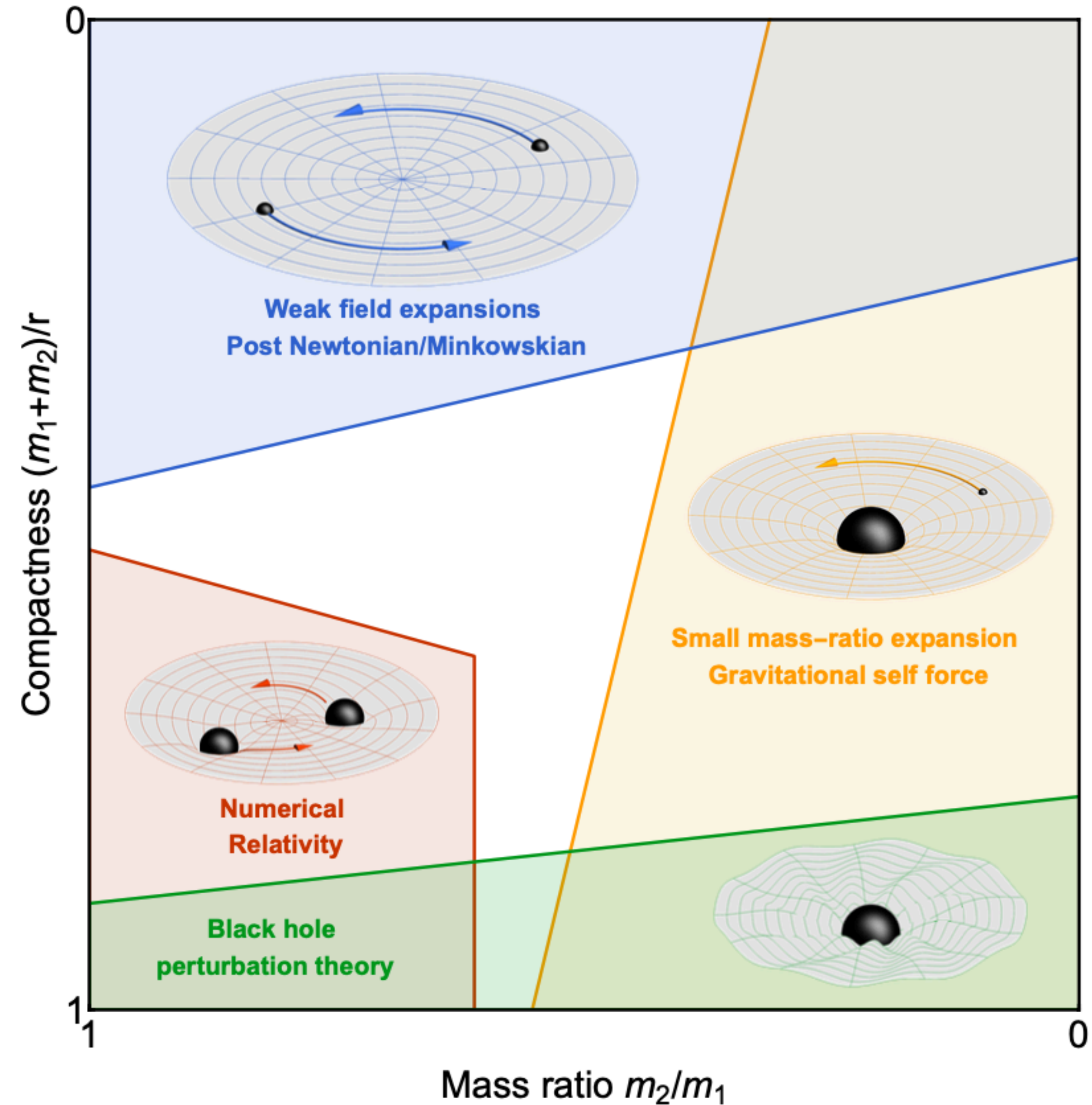
Perturbation



Numerical relativity



Models: EOBNR and Phenom style





# Demands of GW future

## 1. Different/broader frequency ranges

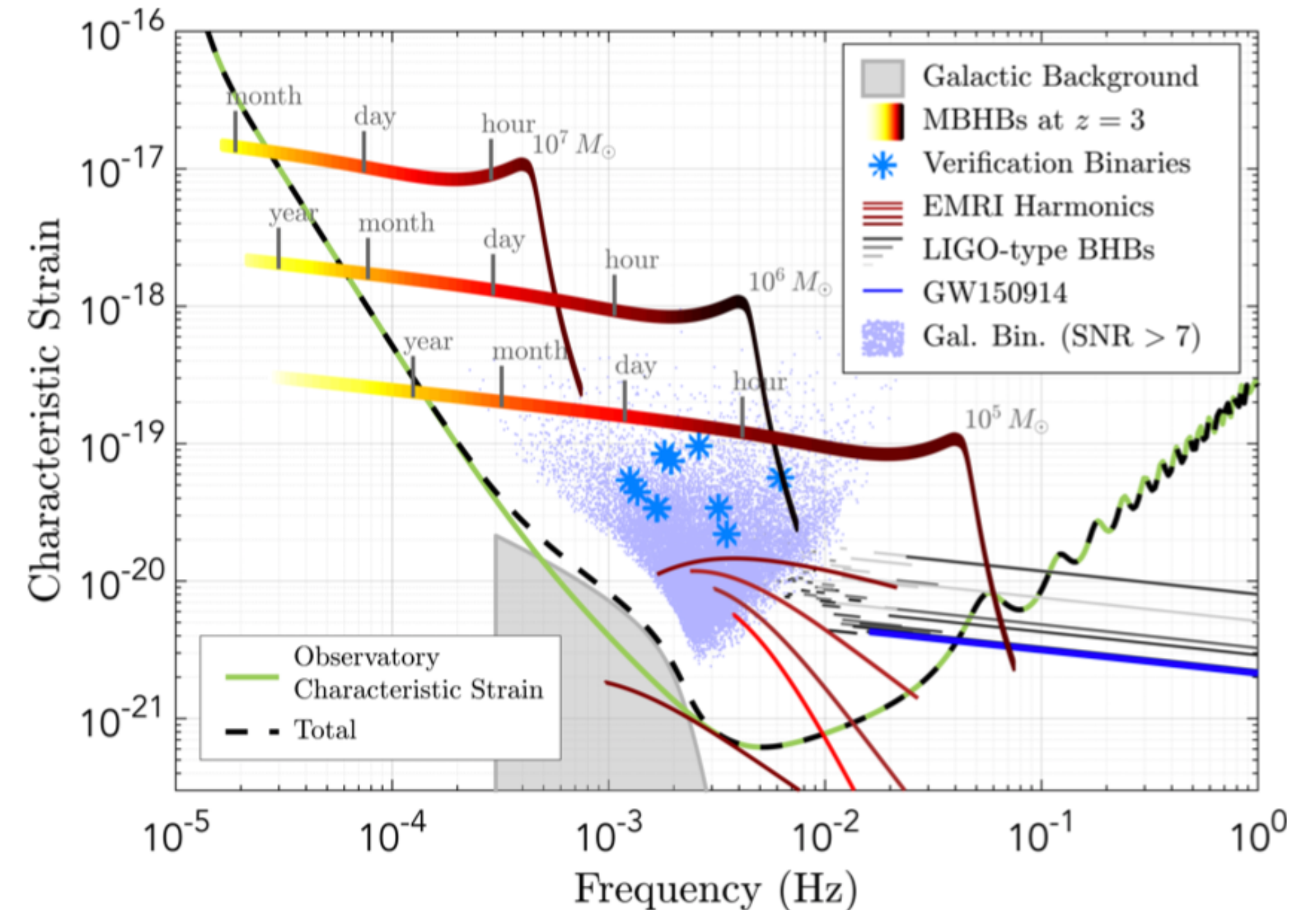
- sources (supermassive black hole mergers, intermediate mass ratio, extreme mass ratio inspirals, galactic white dwarf binaries, ...)
- increases length of signal in band
- observe early universe

## 2. High signal-to-noise ratios

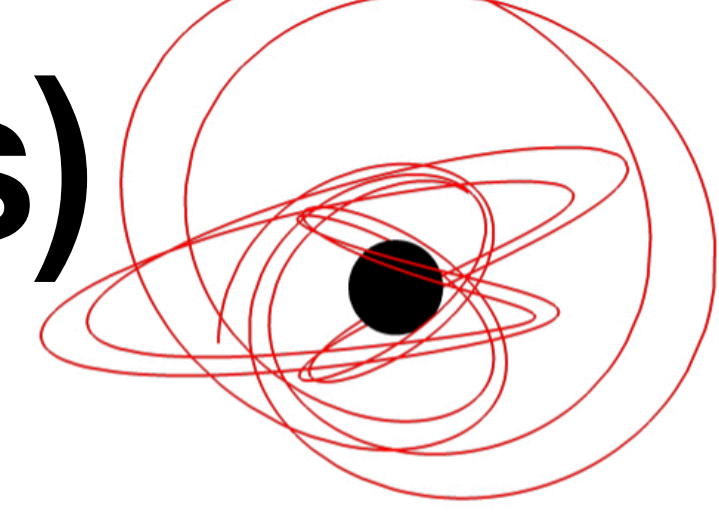
- game changer for precision tests of GR and accuracy of parameter estimation
- increases the accuracy necessary from waveform models
- increases the length of time for computing models and data analysis

## 3. Multiple signals - challenge for data analysis (global fit)

## 4. Speed



# Extreme Mass Ratio inspirals (EMRIs)



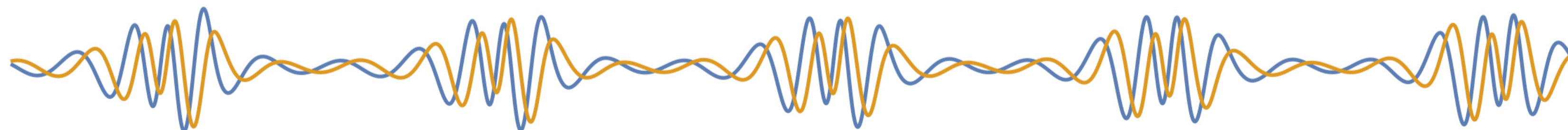
## Key Features:

- Binary with an extremely small mass ratio  $\epsilon = m_2/m_1 \sim 10^{-4} - 10^{-7}$
- Hundreds of thousands of orbits in strong field
- Visible for months to years in LISA band
- **No spin alignment** expected
- **Eccentricity up to  $\sim 0.7$**
- Rich waveform phenomenology

**Modelled using self-force approach:**  $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \epsilon h_{\alpha\beta}^{(1)} + \epsilon^2 h_{\alpha\beta}^{(2)} + \mathcal{O}(\epsilon^3)$

**Waveform phase:**  $\phi = \epsilon^{-1} \phi_{0PA} + \epsilon^0 \phi_{1PA} + \mathcal{O}(\epsilon)$

- 0PA models likely accurate enough some astrophysics. **Efficient** models that cover the full **parameter space** coverage possible if prioritised and resources available
- 1PA crucial for fundamental physics. Significant work over a 10+ years needed to cover **parameter space**.
- Work in environmental effects and beyond-GR corrections in infancy

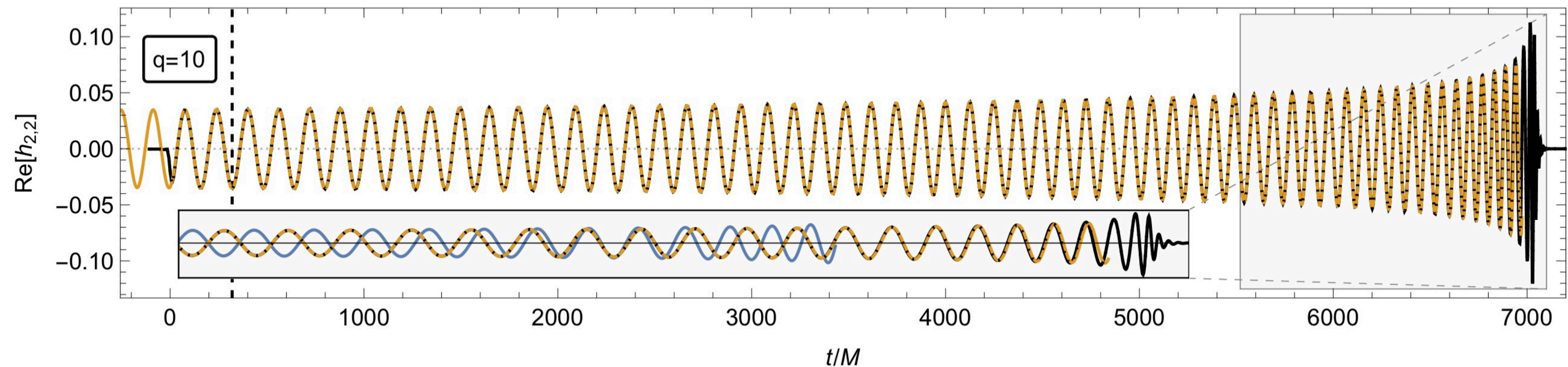


Slide courtesy of N. Warburton



# Intermediate Mass Ratio inspirals (IMRis)

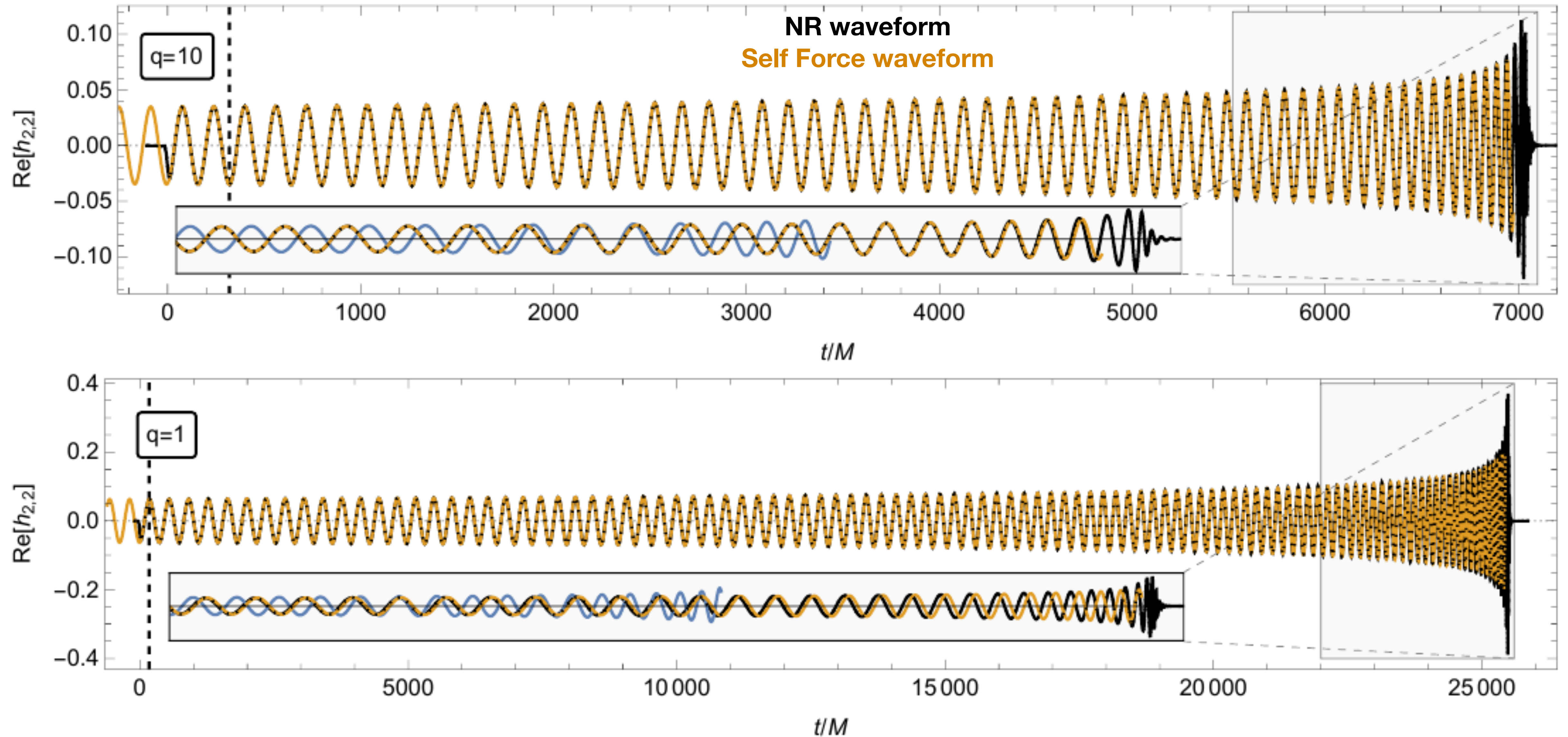
- Binaries involving an intermediate mass black hole (IMBH)
- Mass ratio from  $q \sim 10 - 10^4$
- Both **heavy IMRIs** (MBH + IMBH) and **light IMRIs** (IMBH + SOBH) detectable with LISA
- Event rate highly uncertain: potential discovery space.
- Until very recently modelling approach was unclear but recent self-force post-adiabatic (1PA) waveforms suggest this approach can model them well.



- Waveform generation is **efficient** (takes milliseconds)
- **Parameter space**: only quasi-circular, non-spinning binaries modelled so far



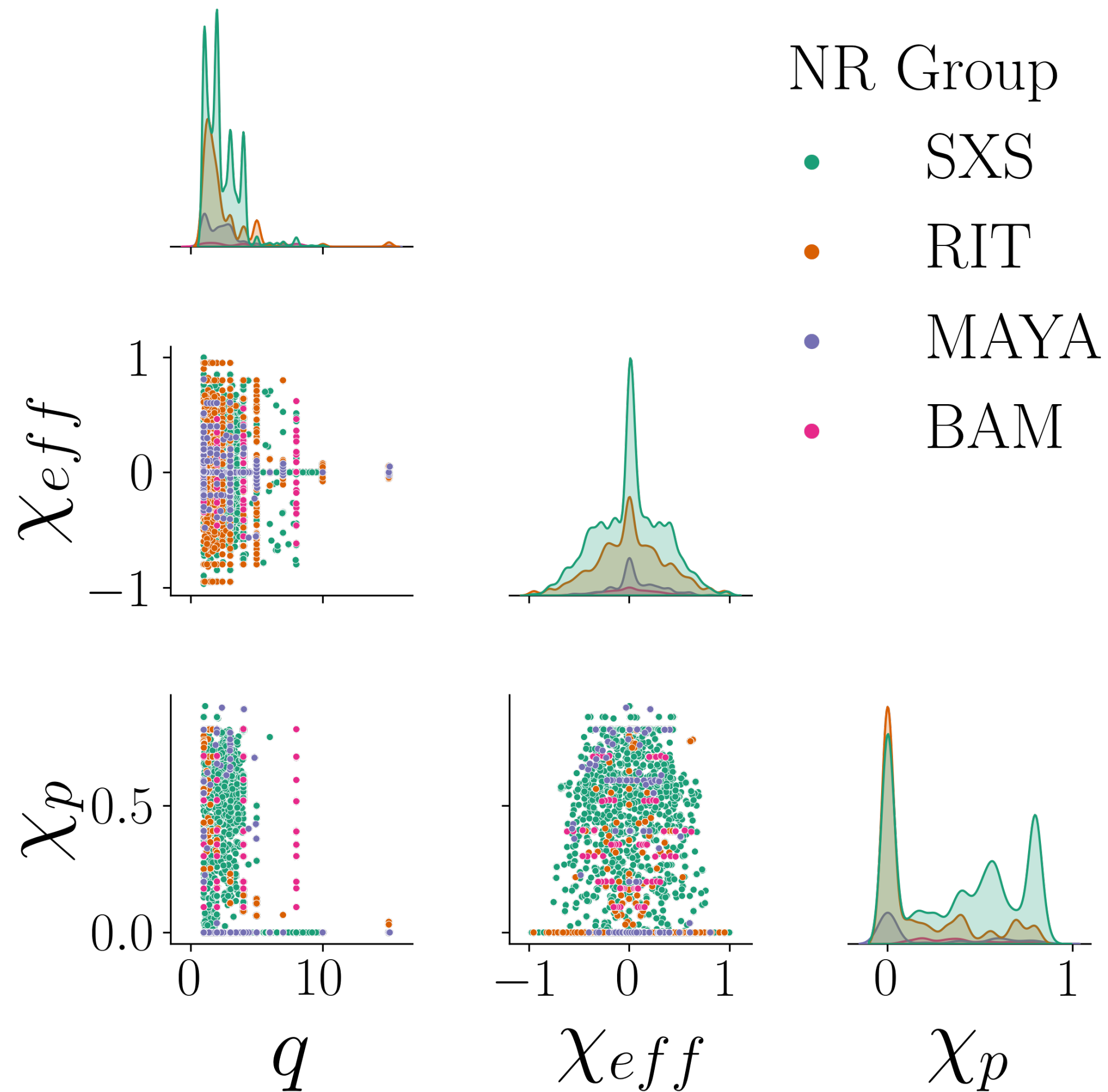
# Working toward highly-unequal waveforms



courtesy of L. Durkan, B. Wardell, N. Warburton, A. Pound J. Miller, and A. Le Tiec



# Massive Black Hole Binaries (MBHBs)



- Models can build upon work for ground-based detectors using Numerical Relativity (NR), effective-one-body (EOB), Phenomological models (Phenom), reduced-order model surrogates, etc, but...
- Accuracy** requirements are likely to be much higher to avoid biasing weaker signal in when performing the global fit
- Parameter space** coverage needs to be larger: high mass ratios, eccentricity, high spins

Public NR Waveform Catalogs: Maya ([cgp.ph.utexas.edu/waveform](http://cgp.ph.utexas.edu/waveform)), SXS ([black-hole.org](http://black-hole.org)), and RIT ([ccrg.rit.edu](http://ccrg.rit.edu))



# Challenges for BBH Numerical Relativity in future Gravitational Waves

## “Solved”

- Non-spinning, mass ratios  $q < 15$  (Jani et al, Healy et al, Mroue et al, Boyle et al, Gonzalez et al) and some up to 128 (Lousto et al)
- Moderate random spins for  $q < 8$  (Jani et al, Healy et al, Mroue et al) aligned spins up to 0.85,  $q < 18$  (Husa et al, Khan et al),  $q=1$ , aligned up to 0.99 (Zlochower et al, Scheel et al)
- Eccentric for  $q < 10$  (Hinder et al, Huerta et al, Gayathri et al)
- Order tens of orbits
- Harmonics unto  $\sim l=6$

## Challenging

- Faster, more efficient codes to achieve 100s of cycles and high mass ratios
- Accuracy for high SNR and lots of harmonics
- Means to set accuracy requirements
- Beyond GR to the point where we have waveforms
- Better gauge and extraction

Once above challenges met, they results need to feed into EOB, Phenom, and surrogate models for practical **efficient** templates for use in data analysis



# Table of NR Codes

Code	Open Source	Public catalog	Formulation	Hydro	Beyond GR
AMSS-NCKU [491, 546–548]	Y	–	BSSN/Z4c	–	Y
BAM [515, 549–551]	–	[463, 552, 553]	BSSN/Z4c	Y	–
BAMPS [554–556]	–	–	GHG	Y	–
COFFEE[557, 558]	Y	–	GCFE	–	Y
Dendro-GR [559, 560]	Y	–	BSSN/CCZ4	–	Y
Einstein Toolkit [561, 562]	Y	–	BSSN/Z4c	Y	Y
*Canuda [304, 305, 563]	Y	–	BSSN	–	Y
*IllinoisGRMHD [564]	Y	–	BSSN	Y	–
*LazEv [454, 565]	–	[566–569]	BSSN/CCZ4	–	–
*Lean [570, 571]	Partially	–	BSSN	–	Y
*MAYA [572]	–	[572]	BSSN	–	Y
*NRPy+ [573]	Y	–	BSSN	Y	–
*SphericalNR [574, 575]	–	–	spherical BSSN	Y	–
*THC [576–578]	Y	[552]	BSSN/Z4c	Y	–
ExaHyPE [579]	Y	–	CCZ4	Y	–
FIL[580]	–	–	BSSN/Z4c/CCZ4	Y	–
GR-Athena++ [581]	Y	–	Z4c	Y	–
GRChombo [582–584]	Y	–	BSSN/CCZ4	–	Y
HAD [585–587]	–	–	CCZ4	Y	Y
Illinois GRMHD [588, 589]	–	–	BSSN	Y	–
MANGA/NRPy+ [590]	Partially	–	BSSN	Y	–
MHDuet [591, 592]	–	–	CCZ4	Y	Y
SACRA-MPI [593]	–	–	BSSN/Z4c	Y	–
SpEC [462, 594]	–	[460, 462, 595]	GHG	Y	Y
SpECTRE [596, 597]	Y	–	GHG	Y	–
SPHINCS-BSSN [598]	–	–	BSSN	SPH	–

This Table will be published in  
the LISA Waveform Working  
Group WhitePaper and  
SNOWMASS (Foucart et al)

# Phenom Status

Waveform Family	Domain	Waveform Model	Spins	Mode Content		Eccentricity	Calibration Region
1st generation	FD	IMRPhenomA	✗		(2,±2)	no	$0.16 \leq \eta \leq 0.25$
2nd generation		IMRPhenomB	✓				NR calibration: $q \leq 4,  \chi_{1/2}  \leq 0.75$ $ \chi_{1/2}  \leq 0.85$ (for $q = 1$ )
		IMRPhenomC	✓				
		IMRPhenomP	✓✓				
3rd generation		IMRPhenomD	✓		(2,±2),(2,±1),(3,±3), (4,±3),(4,±4)	NR calibration: $q \leq 18,  \chi_{1/2}  \leq 0.85$ $-0.95 \leq \chi_{1/2} \leq 0.98$ (for $q = 1$ )	
		IMRPhenomPv2	✓✓	CP			
		IMRPhenomPv3	✓✓				
		IMRPhenomHM	✓				
		IMRPhenomPv3HM	✓✓	CP			
4th generation			IMRPhenomXAS	✓	(2,±2)	in development	NR calibration: $q \leq 18,  \chi_{1/2}  \leq 0.99$ Teukolsky calibration: $q \leq 1000$
	IMRPhenomXP		✓✓	CP			
	IMRPhenomXHM		✓	(2,±2),(2,±1),(3,±2), (3,±3),(4,±4)			
	IMRPhenomXPHM		✓✓		CP		
	TD	IMRPhenomT	✓	(2,±2)	in development	NR calibration: $q \leq 18,  \chi_{1/2}  \leq 0.99$ Teukolsky calibration: $q \leq 1000$	
		IMRPhenomTP	✓✓				CP
		IMRPhenomTHM	✓	(2,±2),(2,±1),(3,±3), (4,±4),(5,±5)			
		IMRPhenomTPHM	✓✓				CP
✗ no spins      ✓ spins aligned with orbital angular momentum      ✓✓ precessing spins      CP mode content in co-precessing frame							



# EOBNR Status

family	waveform model	spins	ecc.	NR-calib. region	CP-frame modes
1 <sup>st</sup>	*EOBNRv1 [1315]			$q \leq 4$	(2, 2)
2 <sup>nd</sup>	EOBNRv2 [1344]			$q \leq 3$	(2, 2)
	*EOBNRv2HM [1344]			$q \leq 6$	(2, 2), (2, 1), (3, 3), (4, 4), (5, 5)
	SEOBNRv0 [1347]	✓		$q = 1, \chi_{1,2} = \pm 0.4$	(2, 2)
3 <sup>rd</sup>	*SEOBNRv1 [1317]	✓		$q \leq 6, \chi_{1,2} = \pm 0.4$	(2, 2)
	*SEOBNRv2 [1319]	✓			
	*SEOBNRv2_ROM [1331, 1332]	✓			
	SEOBNRv2P [1160]	✓✓		$q \leq 8, \chi_{1,2} \leq 0.98$	(2, 2), (2, 1)
	*SEOBNRv3P [1319, 1320]	✓✓			
4 <sup>th</sup>	*SEOBNRv4 [684]	✓		$q \leq 8, \chi_{1,2} \leq 0.98$	(2, 2)
	*SEOBNRv4HM [1324]	✓			(2, 2), (2, 1), (3, 3), (4, 4), (5, 5)
	*SEOBNRv4HM_ROM [684]	✓			
	*SEOBNRv4PHM [1325]	✓✓			
	*SEOBNRv4PHM_ROM [1336, 1339]	✓✓			
	*SEOBNRv4EHM [687]	✓	✓		
5 <sup>th</sup>	*SEOBNRv5 [1357]	✓		$q \leq 15, \chi_{1,2} \leq 0.98$	(2, 2)
	*SEOBNRv5_ROM [1357]	✓			(2, 2), (2, 1), (3, 2), (3, 3), (4, 3), (4, 4), (5, 5)
	*SEOBNRv5HM [1357]	✓			
	*SEOBNRv5PHM [1358]	✓✓			

✓ aligned spin, ✓✓ arbitrary spin orientations, \*available in LALsuite or more recently in pySEOBNR [1329]).

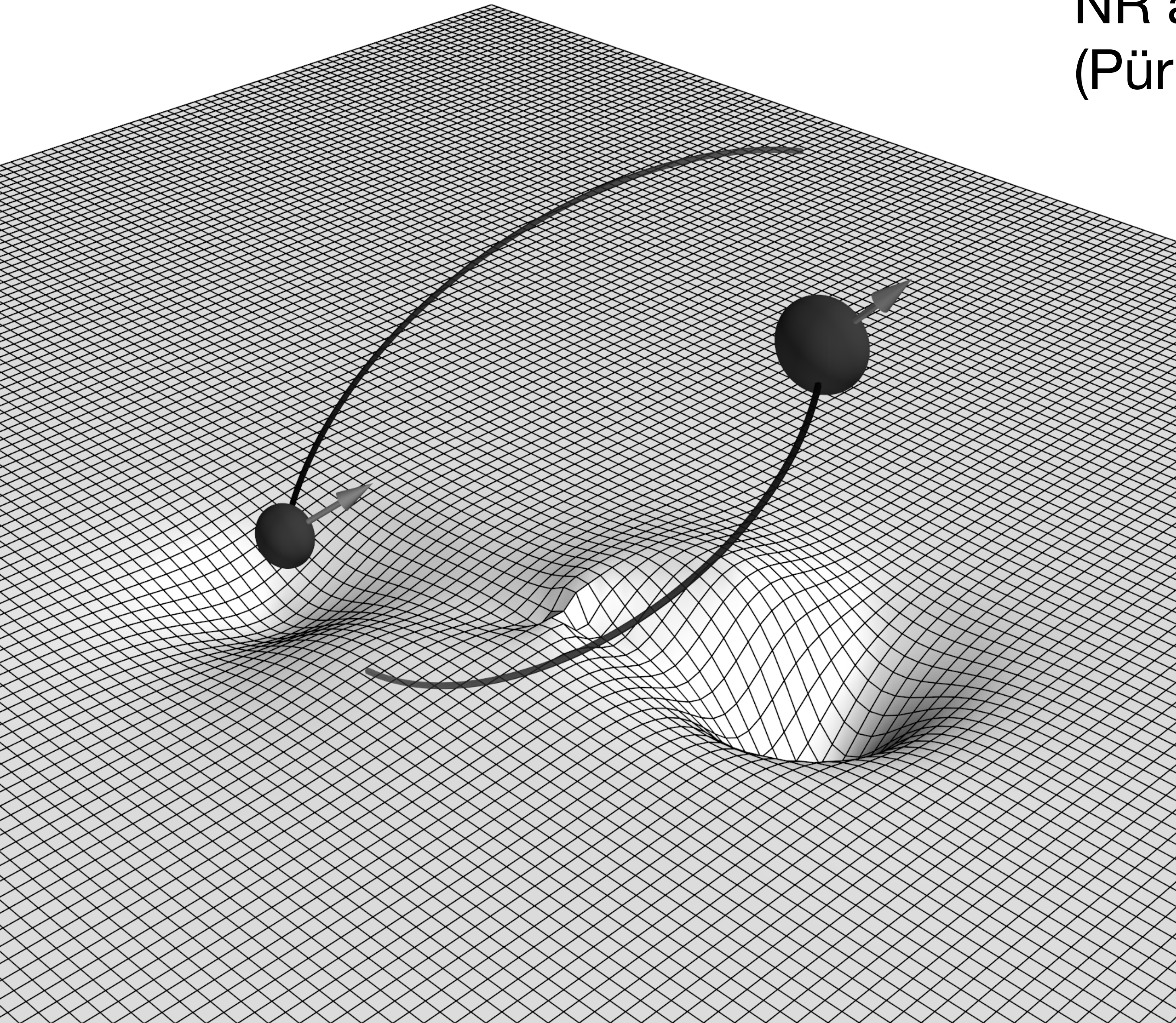
family	waveform model	spins	ecc.	NR-calib. region	CP-frame modes
1 <sup>st</sup>	EOBResum [1318]			$q \leq 6$	(2, 2)
2 <sup>nd</sup>	EOBResumS [1351]	✓		$q = 1, \chi_1 = \chi_2$	(2, 2)
3 <sup>rd</sup>	TEOBResumS [1326]	✓		$q \leq 8, \chi_{1,2} \leq \pm 0.85$	(2, 2)
	TEOBiResumS_SM [1327]	✓			
	TEOBiResumSE_SM [631]	✓			

✓ aligned spin, ✓✓ arbitrary spin orientations, \*available as public code



# Waveform Priorities: Dive into NR Accuracy

NR and Model communities addressing  
(Pürrer and Haster '19, Ferguson et al '21)





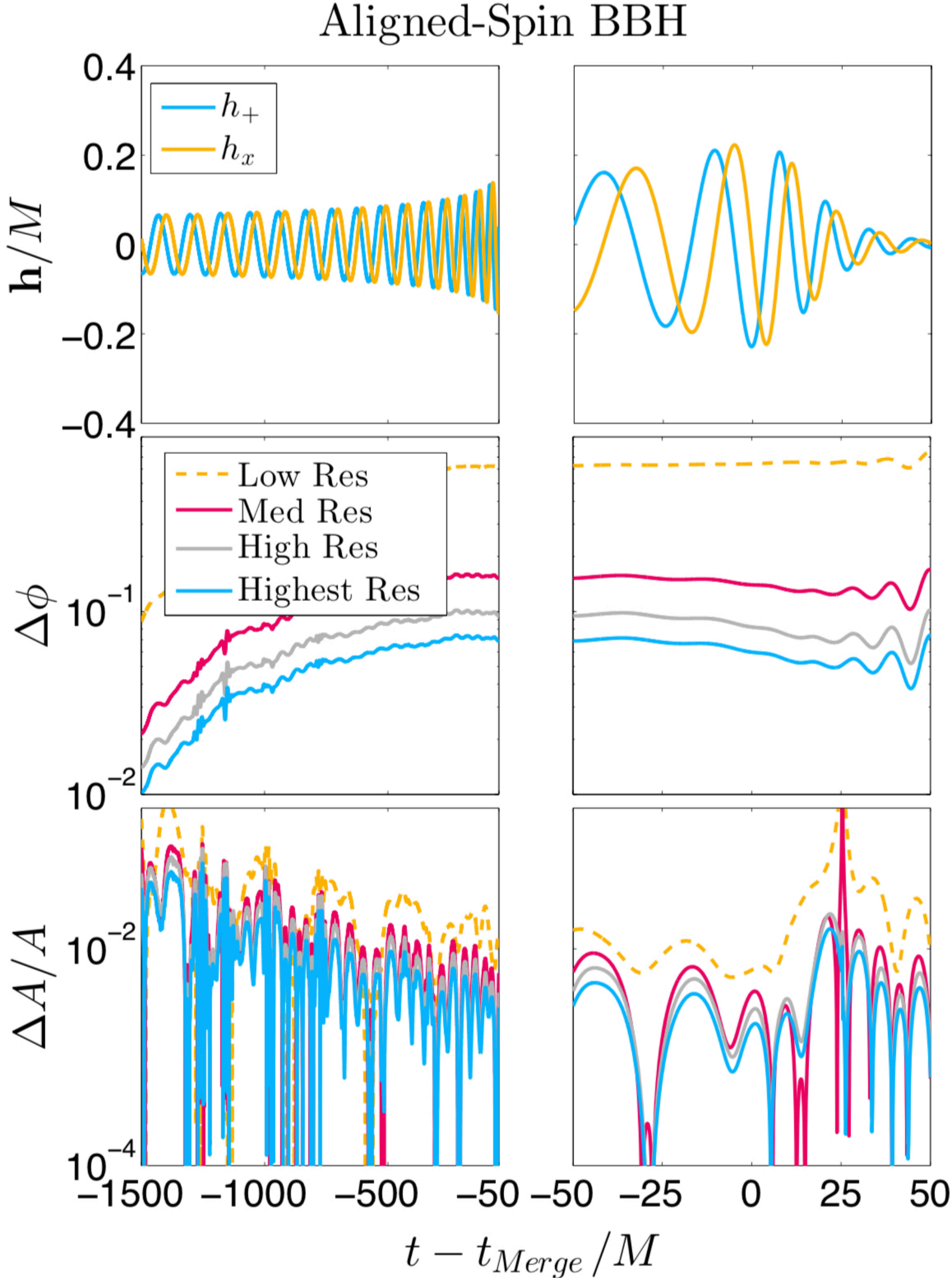
# NR Waveforms - The Sausage

- Getting the waveform from NR
  - Initial data choices
  - Coordinate choices, including frame
  - The numerics
- Going from NR to Strain for Data Analysis
  - Getting  $\Psi_4 - h(t)$
  - Extrapolating to infinity
- Preparing for DA
  - Windowing, and the evils of tapering
  - Models



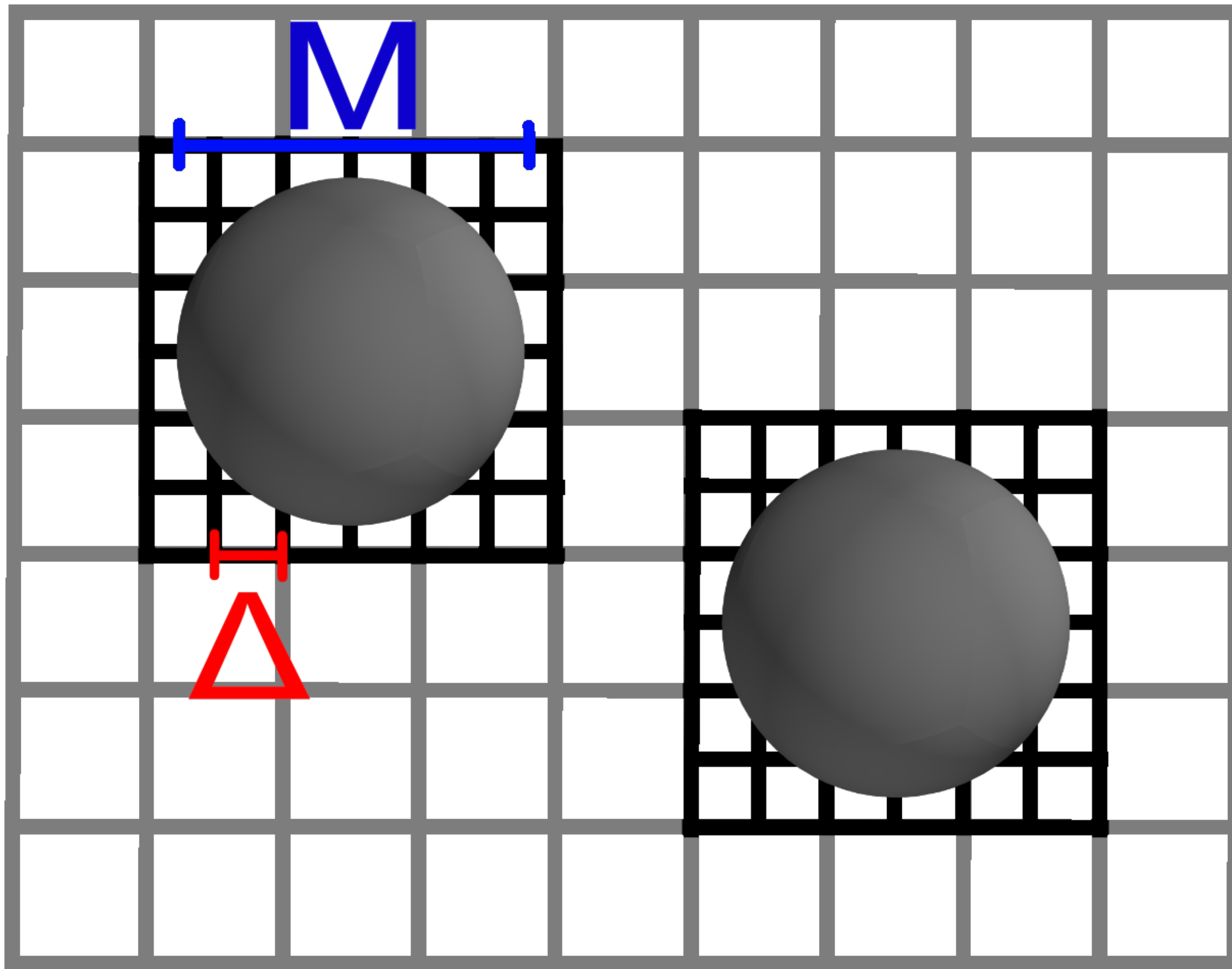


# Convergence Test





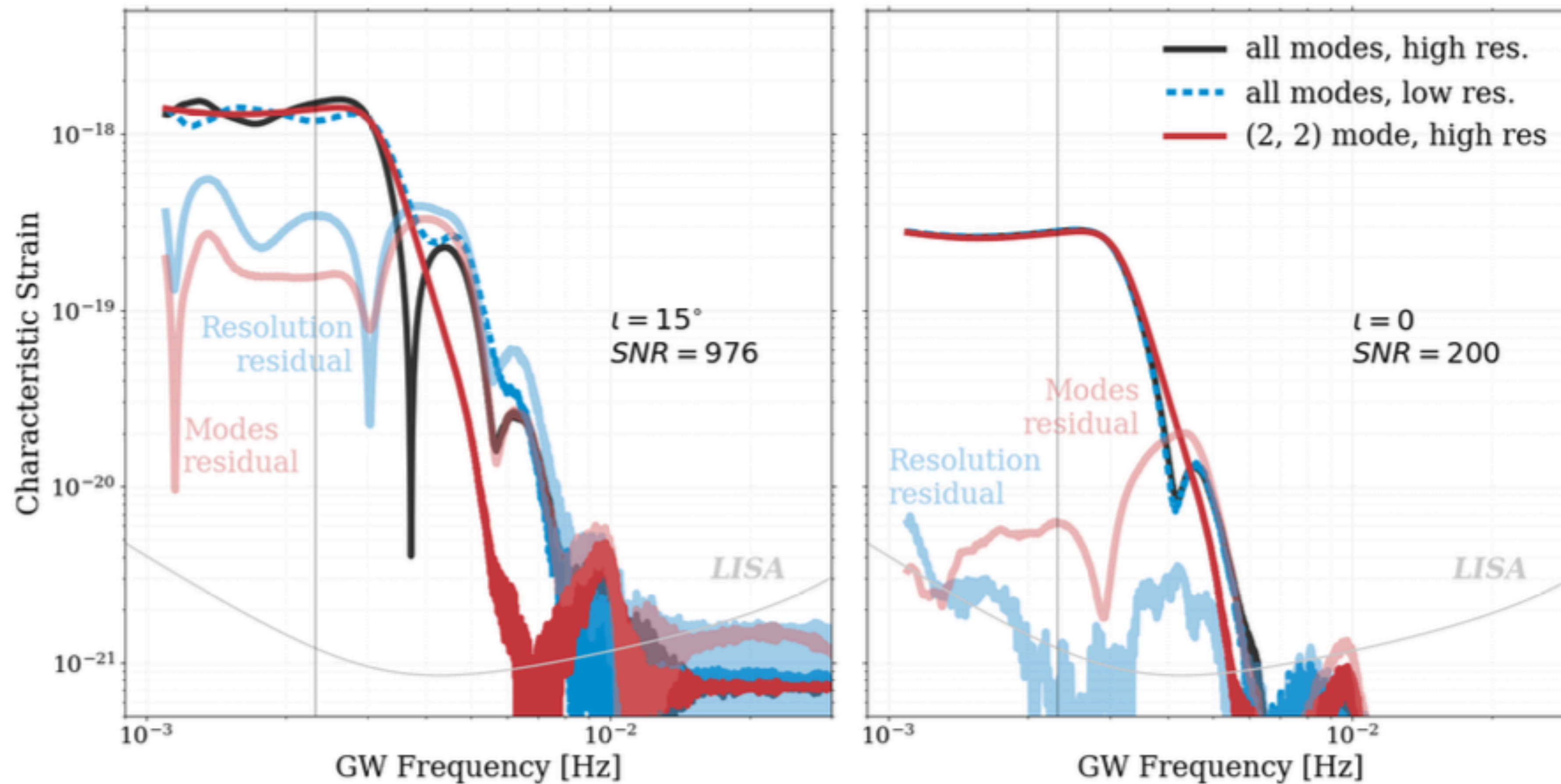
# Example: Accuracy



- NR simulations are performed on finite resolution grids with boundaries typically around  $400M$ .
- Typical simulations have at least  $\sim 48$  points across the BH horizon or a resolution of around  $\Delta = M/240$ .
- NR codes have been tested to converge to the correct solution with increasing resolution
- Waveforms generated from even the highest resolution simulations will have some intrinsic error
- How will this error manifest during the analysis of high SNR signals?

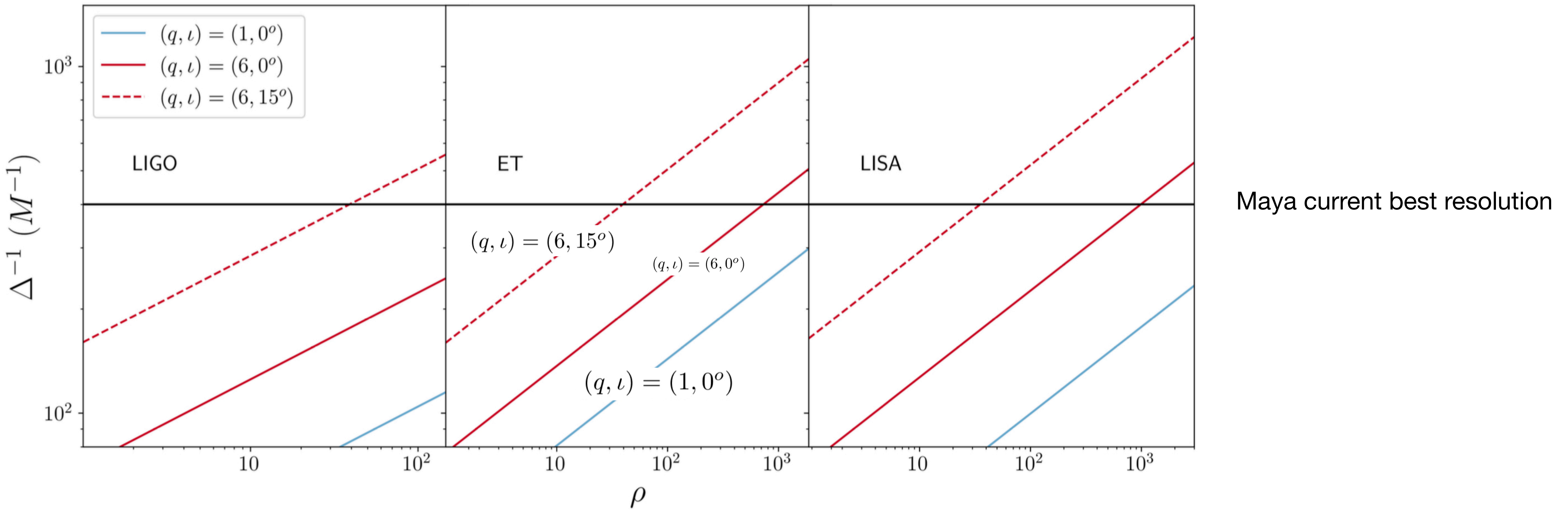
# Residuals due to finite resolution

- Strains produced by high (solid, black) and low (dashed, gray) resolution numerical relativity simulations
- Residual resulting from subtracting the low resolution waveform from the high resolution waveform



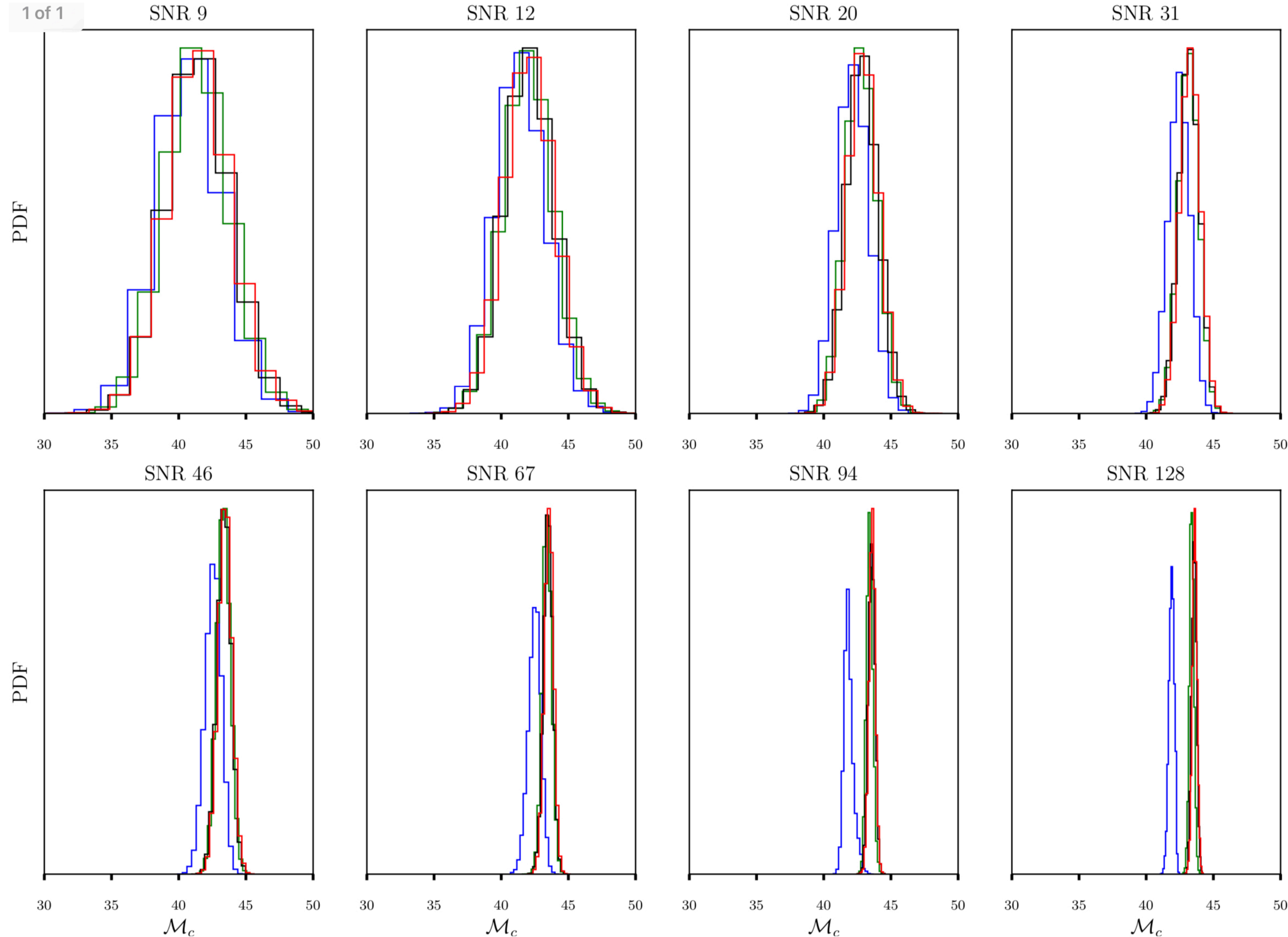


# Criteria for assessing accuracy



- Minimum resolution needed for waveforms to be indistinguishable from infinite resolution waveforms within LIGO as a function of signal-to-noise ratio for various detector frame total masses
- Vertical line is maximum SNR yet observed by LIGO ( $\rho = 33$ )

# Impact on Parameter Estimation



Hanford only, a  $q=1$   
spinning system using  
RIFT PE code

A. Jan et al in prep



# Computational Cost

- A simulation with a resolution of  $M/700$  would require 16 nodes on higher performance computer for 30 days to achieve 7 orbits before merger
- With our current allocation, that would allow us to perform 10 simulations per year
- To span LISA's band, we will likely want longer simulations
- Inclination will require even higher resolutions

# What are the requirements on waveform for fundamental physics with LISA?

- 1 sigma error in parameters might be “enough” for astrophysics
- But what is it for tests of GR? Residual below the noise curve?
- Answer probably depends on specific science case, but it would be great to do concrete examples

Waveforms + Fundamental Physics + LDC



# How do we develop a waveform pipeline?

- Berti showed us FoM using older waveform models
- Public catalogues and codes
  - NR public catalogues -> Models (some) -> LDC/DDPC?
- (Mock) Data Challenge involving the latest and greatest waveforms!

Waveforms + Fundamental Physics + LDC

# Summary

- Waveform templates needed to detect and characterise sources of gravitational waves
- Templates need to be (i) **accurate**, (ii) **efficient** to compute, and (iii) cover the **parameter space**
- These are drifting targets that must be routinely reviewed
- A handful of existing models achieve some of these goals but very significant development is needed to unlock full science potential of LISA
- In some cases the path to improving the models is clear but significant resources are needed. In other cases substantial development of theoretical and computational approaches are needed.

Most development is community driven and funded. To ensure models are ready for LISA coordination, management and funding are needed.