Gravitational waves from the primordial universe & scalar-field cosmology.

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DESY/U.Hamburg

New horizons for Psi school,
IST Lisbon, 01-07-2024

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Universität Hamburg

These lectures:

Gravitational-wave signatures of axion early cosmology .

Other lectures at this school as well as multiple talks at the workshop discuss GW signatures from axions & light bosons in the late universe: superradiance, boson clouds. See also DM axion density fluctuations directly sourcing noise in GW detectors (H. Kim)

These lectures:

Within standard Einsten gravity (no modified gravity!)

Non-standard physics comes from particle physics, not from the gravity side

These lectures

Lecture 1

— Generalities about primordial GW backgrounds
Review of the best-motivated sources: short versus long-lived sources

- ▫ First-order phase transitions
- ▫ Inflation
- ▫ Cosmic strings

— Axion-like-particles (ALPs): cosmological probes and dark matter

Lecture 2

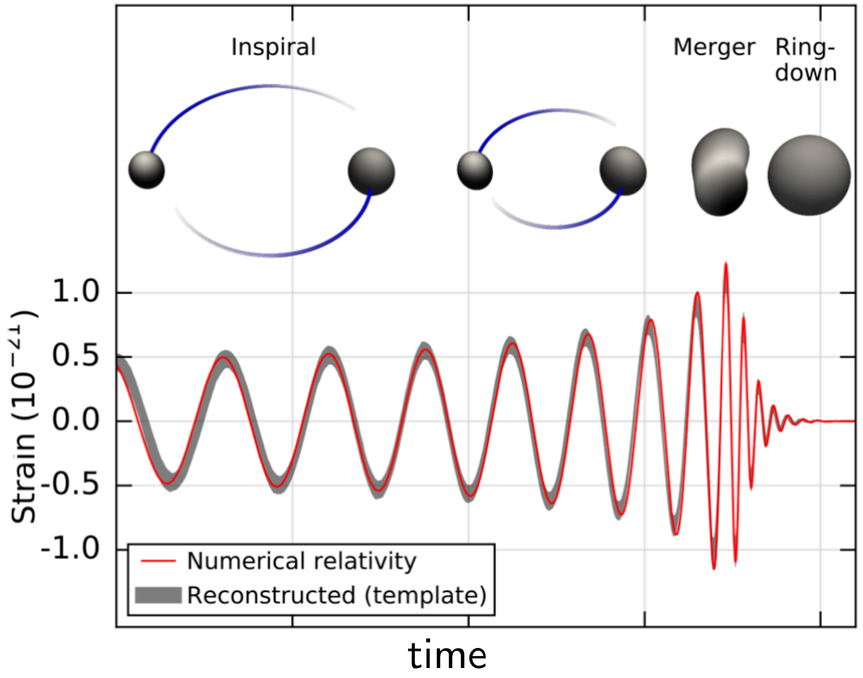
— GW backgrounds from axion early-universe dynamics:
3 distinct sources:

- ▫ GWs from the Peccei-Quinn phase transition
- ▫ GWs from axionic (global) cosmic strings
- ▫ GW signatures from kination induced by rotating axions
- ▫ GWs from axion fragmentation

Two types of gravitational-wave (GW) signals .

- **Astrophysical signals (in the late universe)**

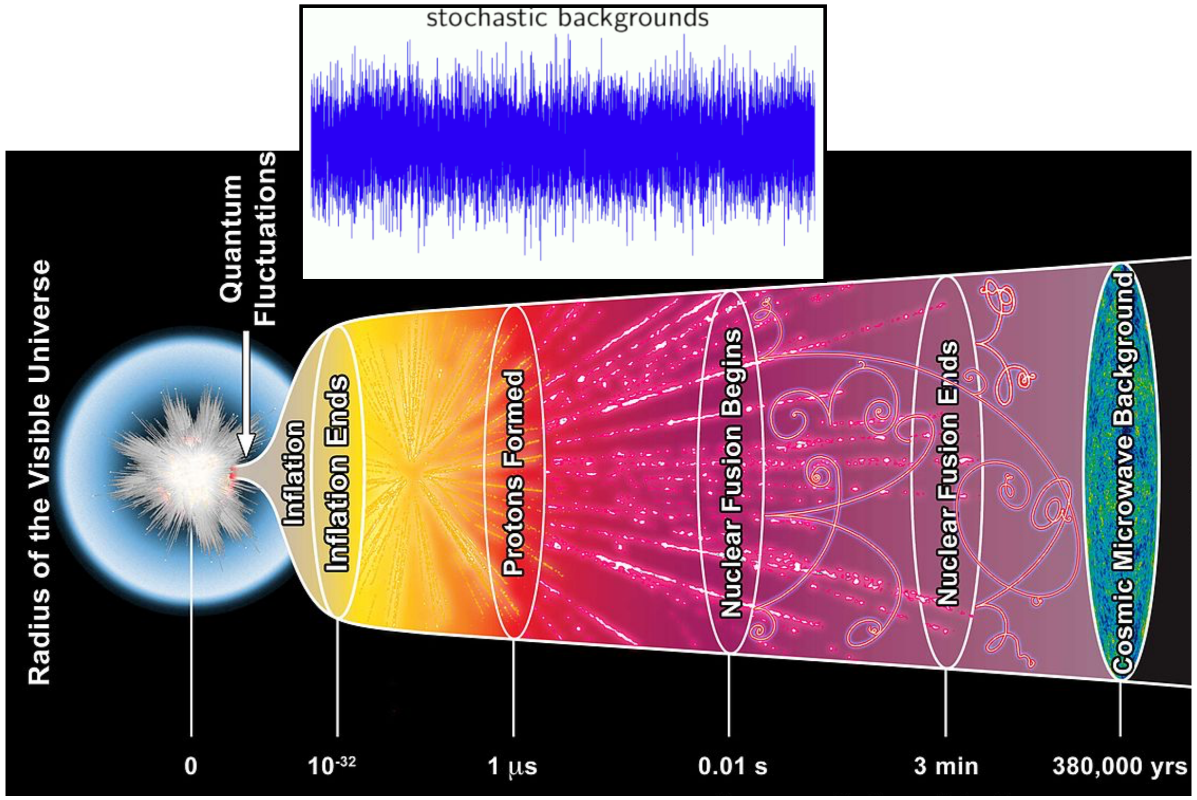
✔ **detected**



LIGO&Virgo, arXiv:1602.03841

- **Cosmological background filling the whole universe (a relic from the early universe)**

✘ **not yet detected, some hint (PTAs?)**

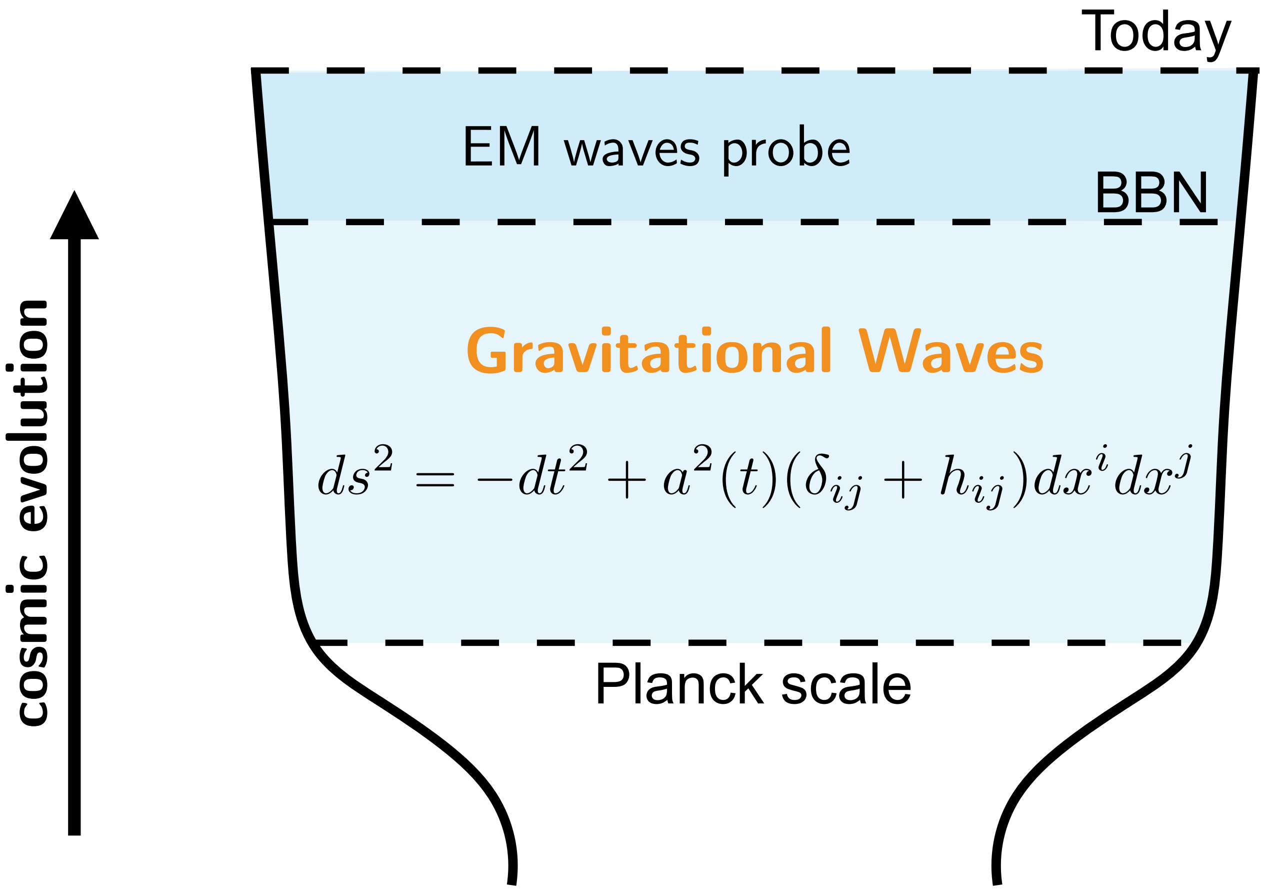


Note: Astrophysical signals can lead to a stochastic background if they cannot be resolved.

Primordial gravitational waves: Fossil radiation .

superposition of GW generated by an enormous number of causally independent sources, arriving at random times and from random directions.

Individual waves are not detectable, sources can not be resolved but instead we can only observe a Stochastic GW Background. For most of the cosmological sources, it is homogeneous, isotropic, gaussian and unpolarized and appears as a noise in the detector.

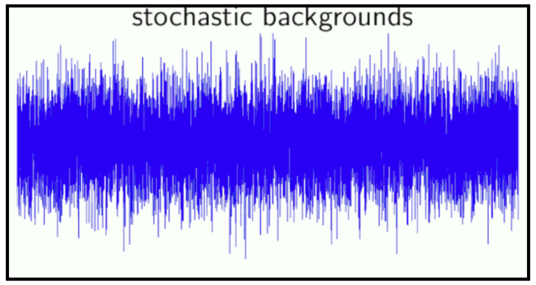


Random h_{ij}

$\Downarrow$

Stochastic GW background (SGWB)
characterized
by energy density

$$\rho_{\mathrm{GW}} = \frac{\langle \dot{h}_{ij}\dot{h}^{ij} \rangle}{32\pi G}$$



Ensemble average
$\Updownarrow$
time/space average

Probing high-energy physics with gravitational waves

Interaction rate of GW

$$\frac{\Gamma_{\rm GW}(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{\rm pl}} = \left(\frac{T}{M_{\rm pl}}\right)^3 \ll 1$$

Expansion rate

GW produced below the Planck scale are decoupled: They propagate freely in the universe until today.

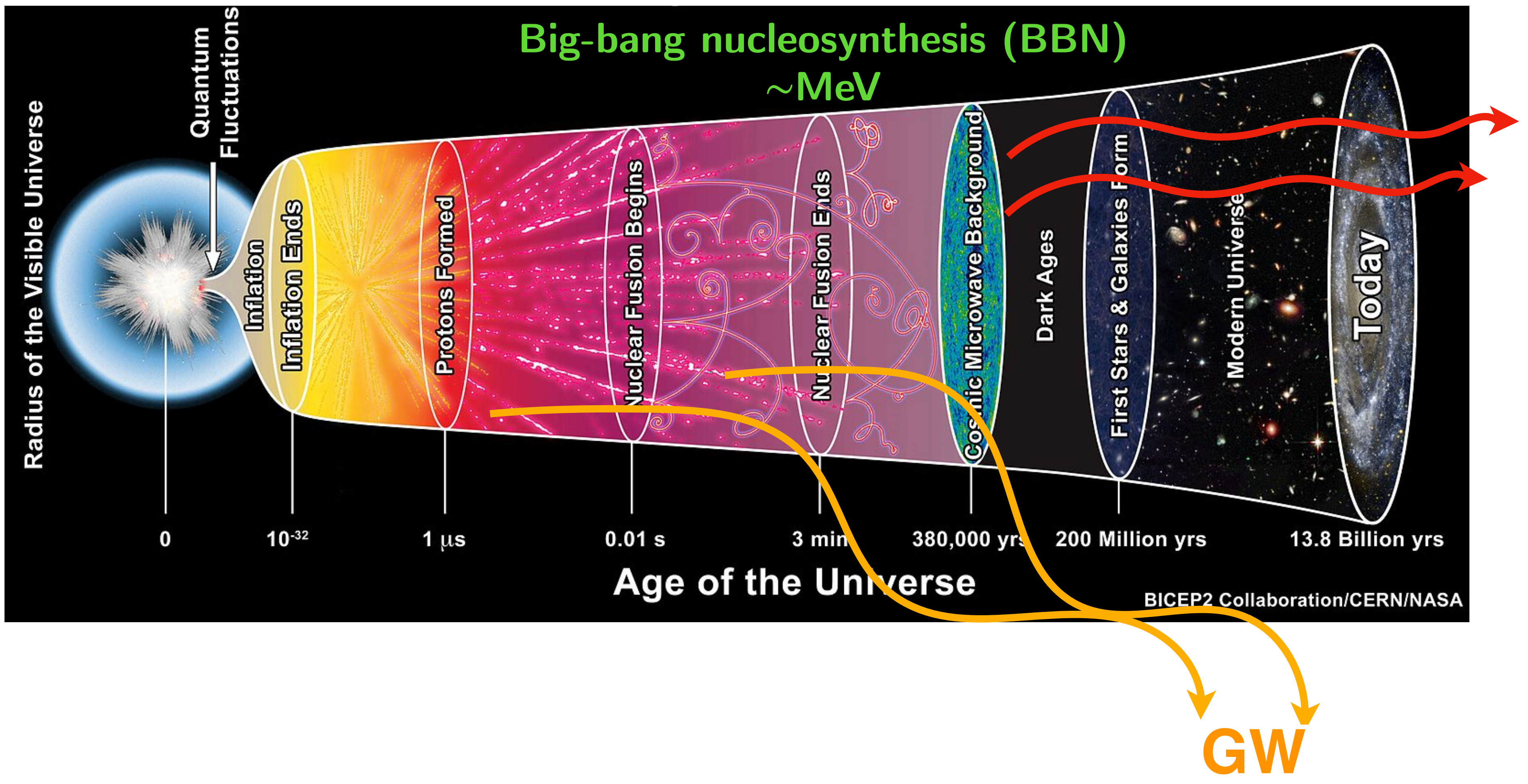
They do not loose memory of conditions when produced.

They retain spectral shape, typical frequency and intensity characteristic of production mechanism, encoding information about particle physics at high-energy scales that cannot be probed by colliders.

Probing high-energy physics with gravitational waves .

High energies **Low energies**

unconstrained ← → **well-tested**



BICEP2 Collaboration/CERN/NASA

Electromagnetic-wave probes

GW

Energy density of GW background:

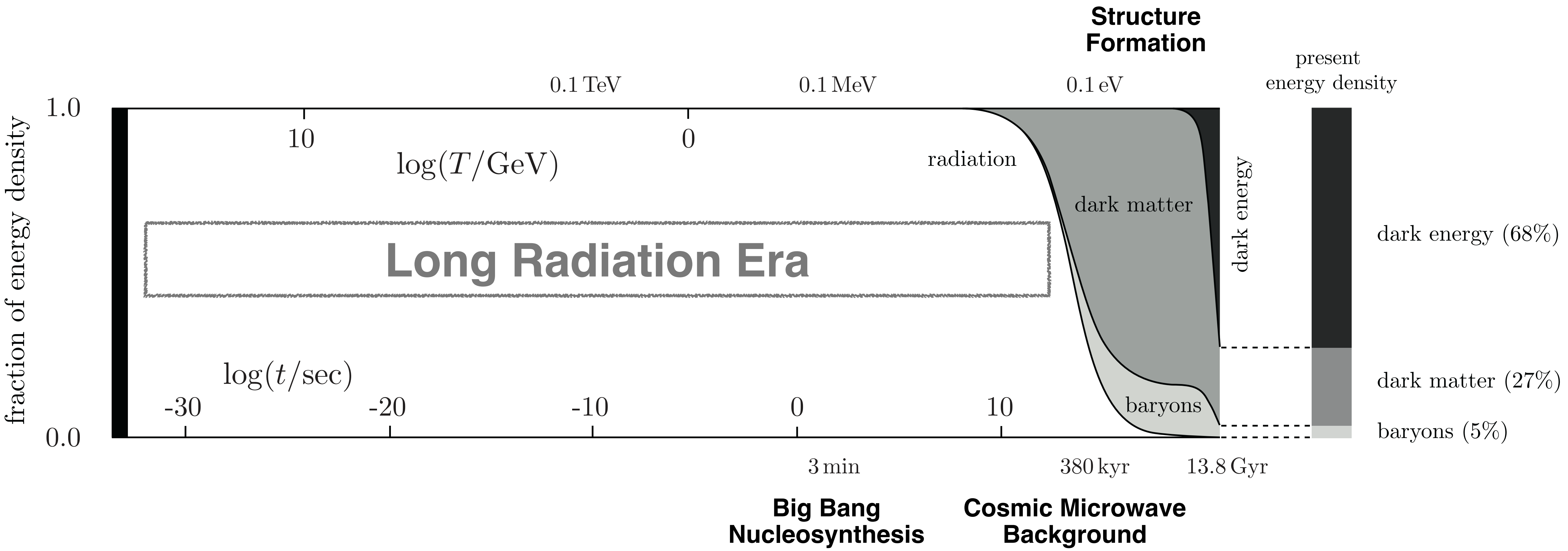
$$\rho^{\mathrm{GW}}_{\mathrm{today}} = \rho^{\mathrm{GW}}_{\mathrm{prod}} \left(\frac{a_{\mathrm{prod}}}{a_{\mathrm{today}}} \right)^4$$

The universe is expanding. ⇨ **BSM of cosmology**

its production mechanism ⇨ particle physics beyond the Standard Model (BSM)

What can we learn on particle physics from early universe cosmology?

Standard Cosmological History



Long Radiation Era

Structure Formation

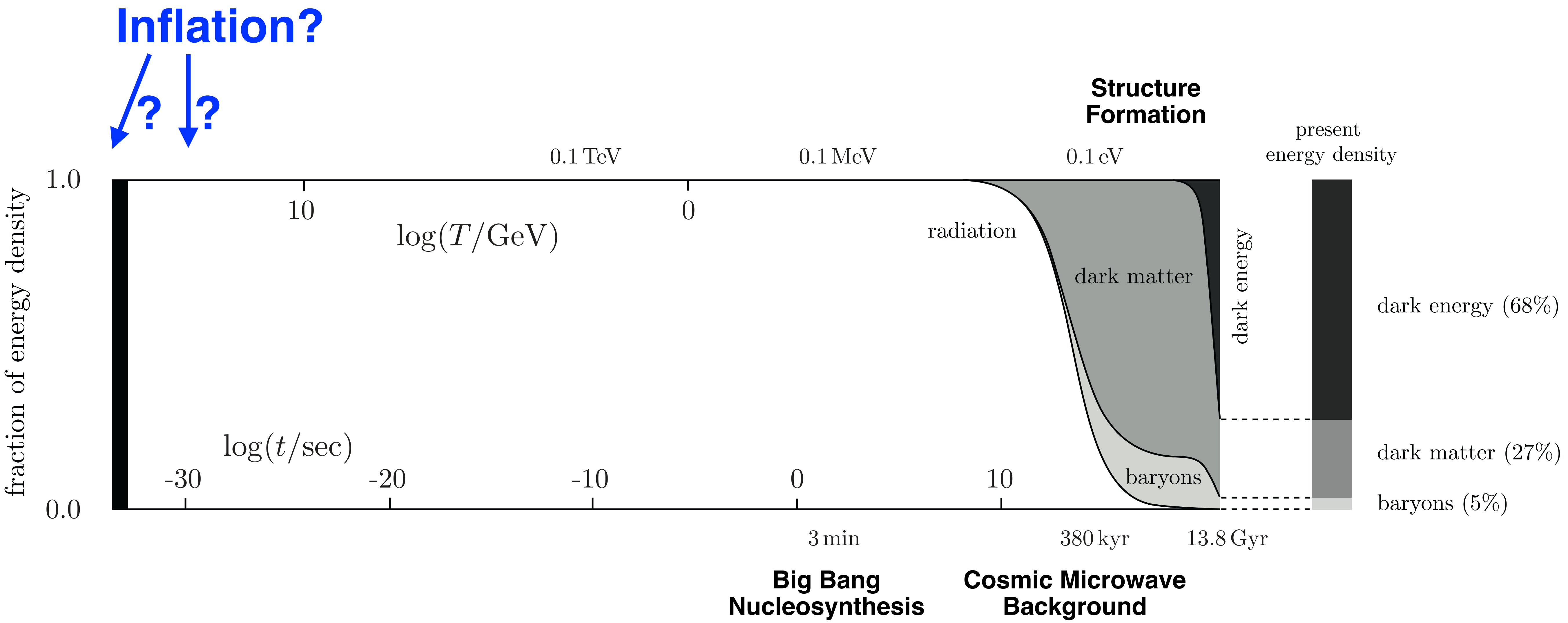
Big Bang Nucleosynthesis

Cosmic Microwave Background

Well-tested

Almost unconstrained

Cosmological History



Inflation?

Structure Formation

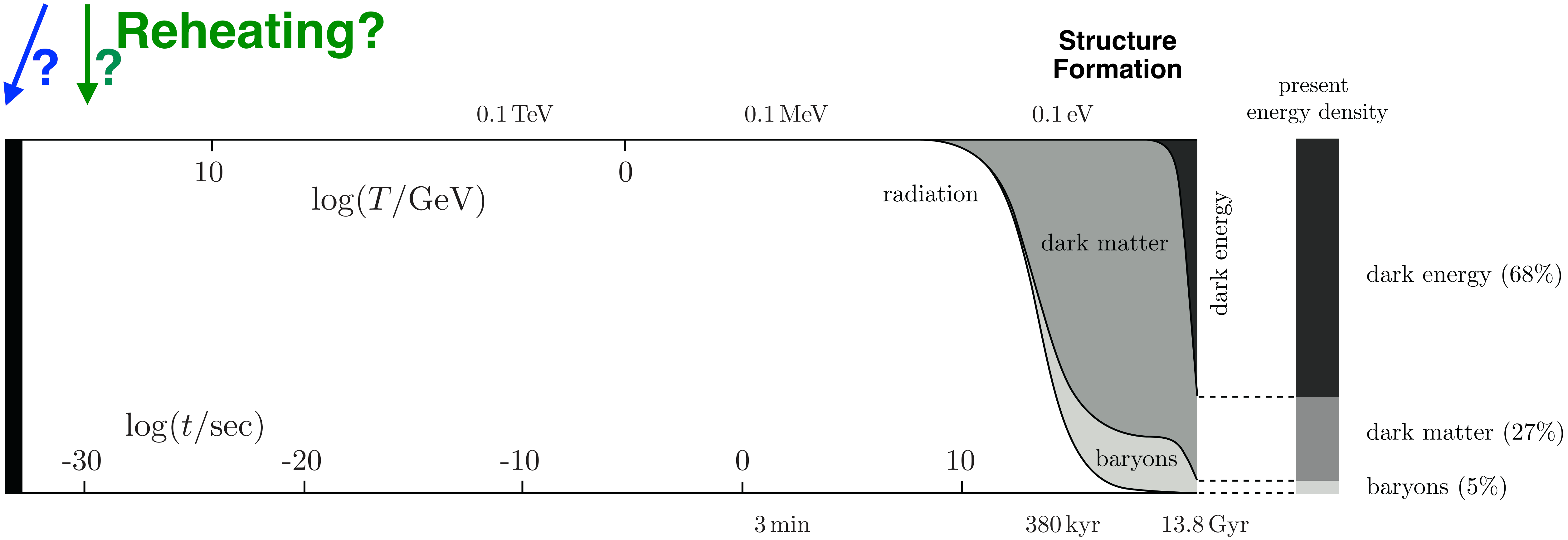
Big Bang Nucleosynthesis

Cosmic Microwave Background

Cosmological History

Inflation?

Reheating?



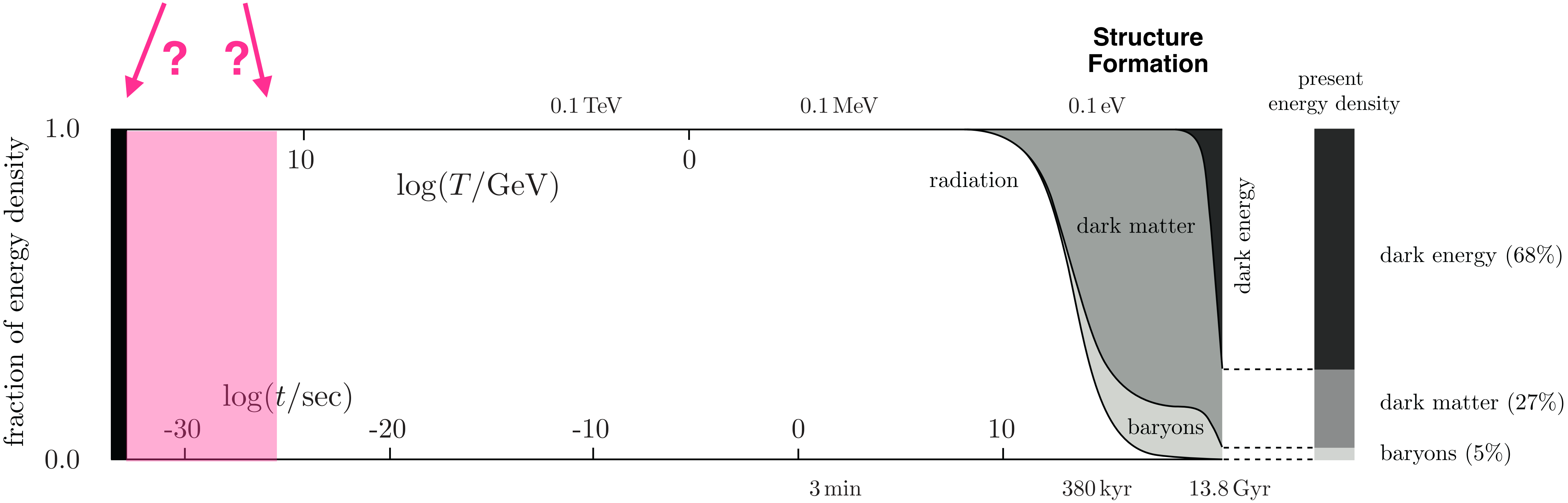
Structure Formation

Big Bang Nucleosynthesis

Cosmic Microwave Background

Cosmological History

Kination after inflation?



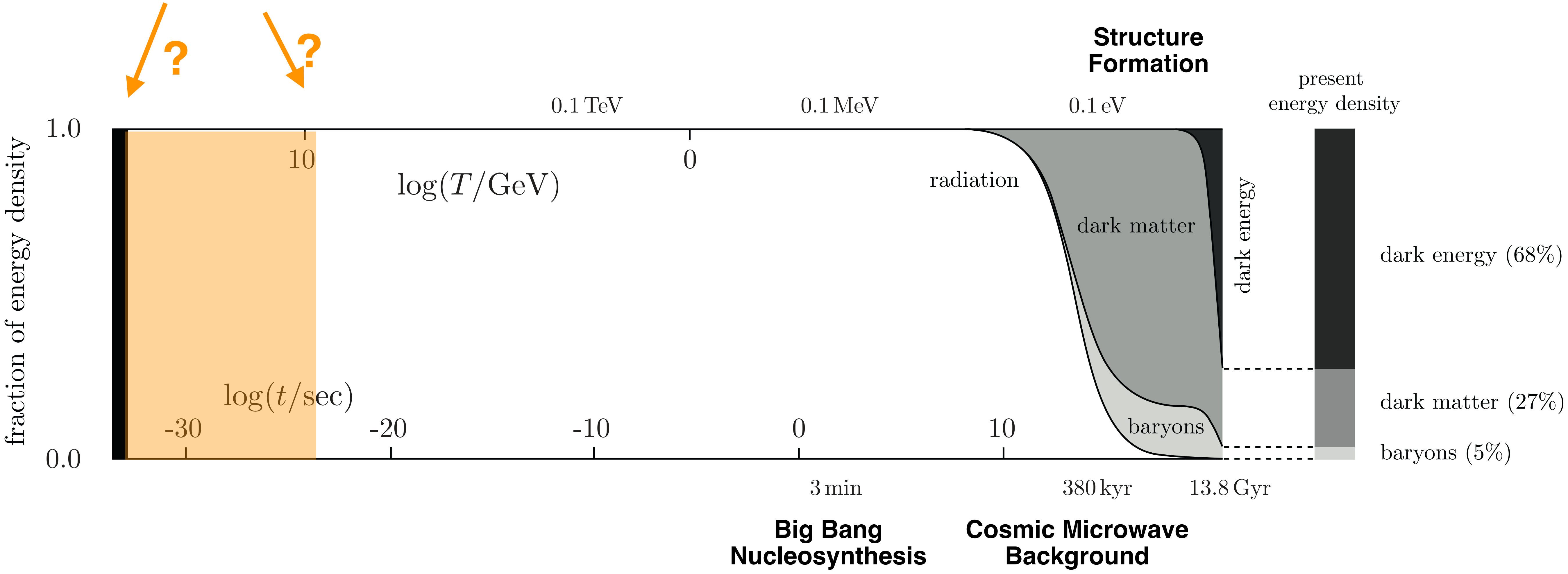
Structure Formation

Big Bang Nucleosynthesis

Cosmic Microwave Background

Cosmological History .

Early Matter era after inflation?



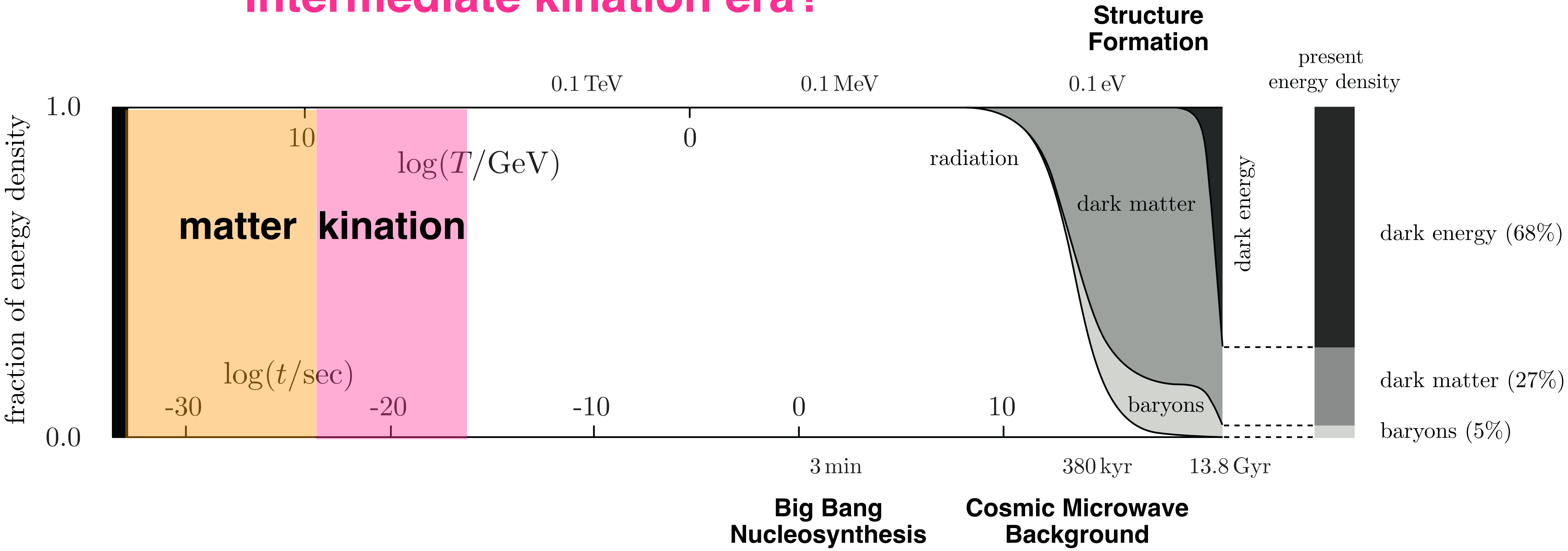
Structure Formation

Big Bang Nucleosynthesis

Cosmic Microwave Background

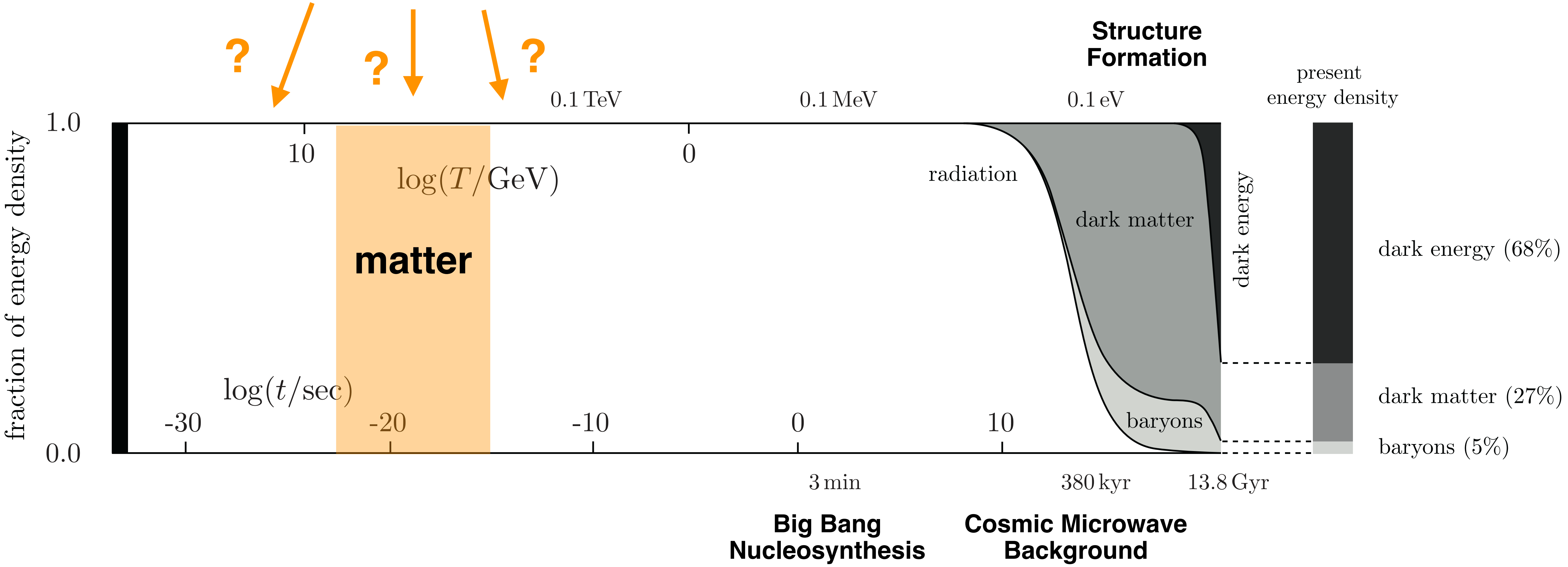
Cosmological History

Intermediate kination era?



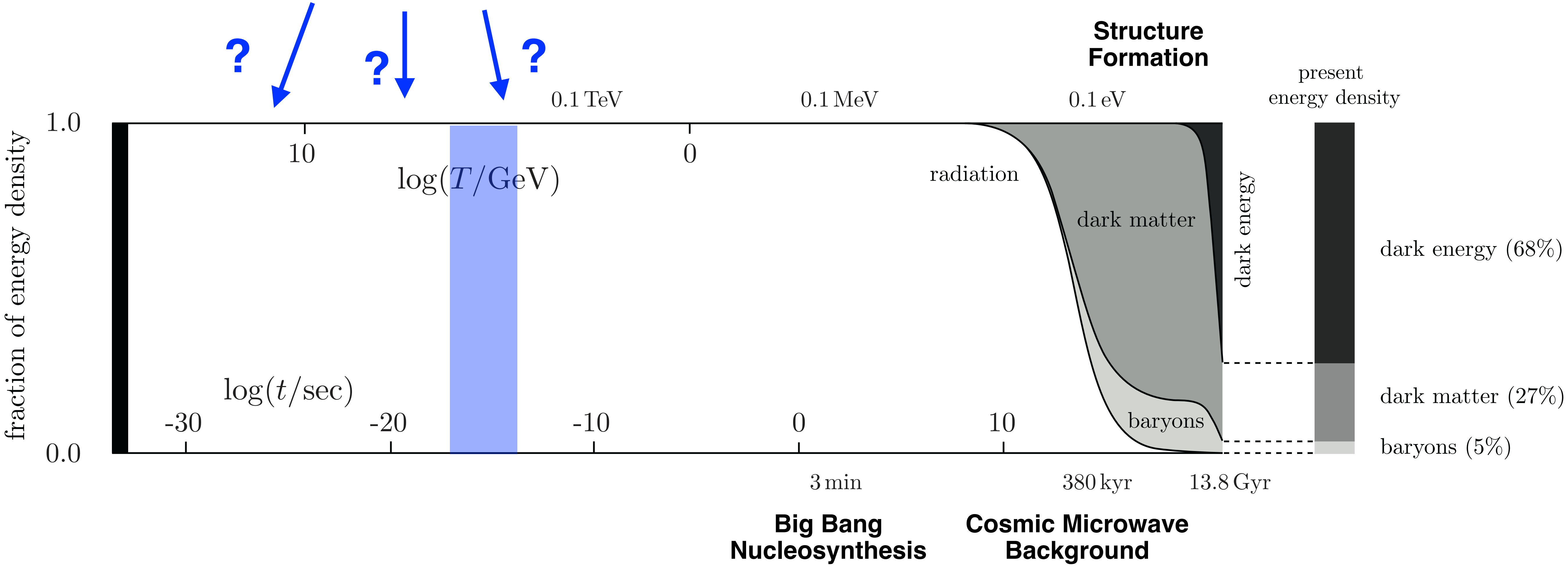
Cosmological History

Intermediate matter eras?



Cosmological History

Secondary intermediate late inflation eras?



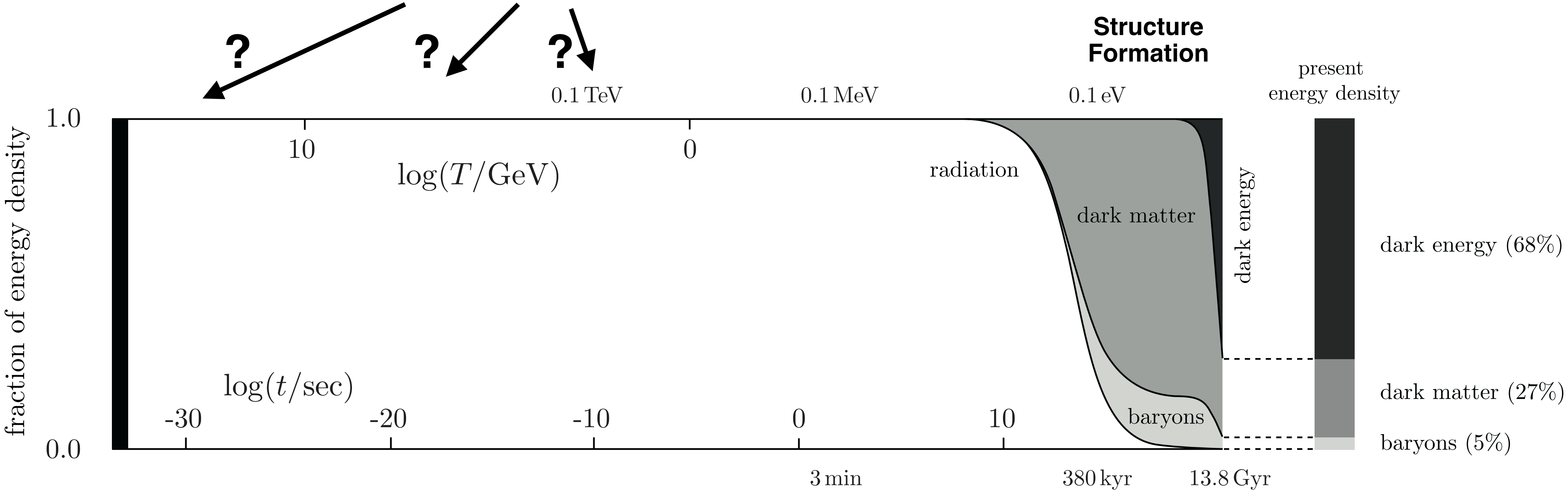
Structure Formation

Big Bang Nucleosynthesis

Cosmic Microwave Background

Cosmological History

Dark Matter Production ?

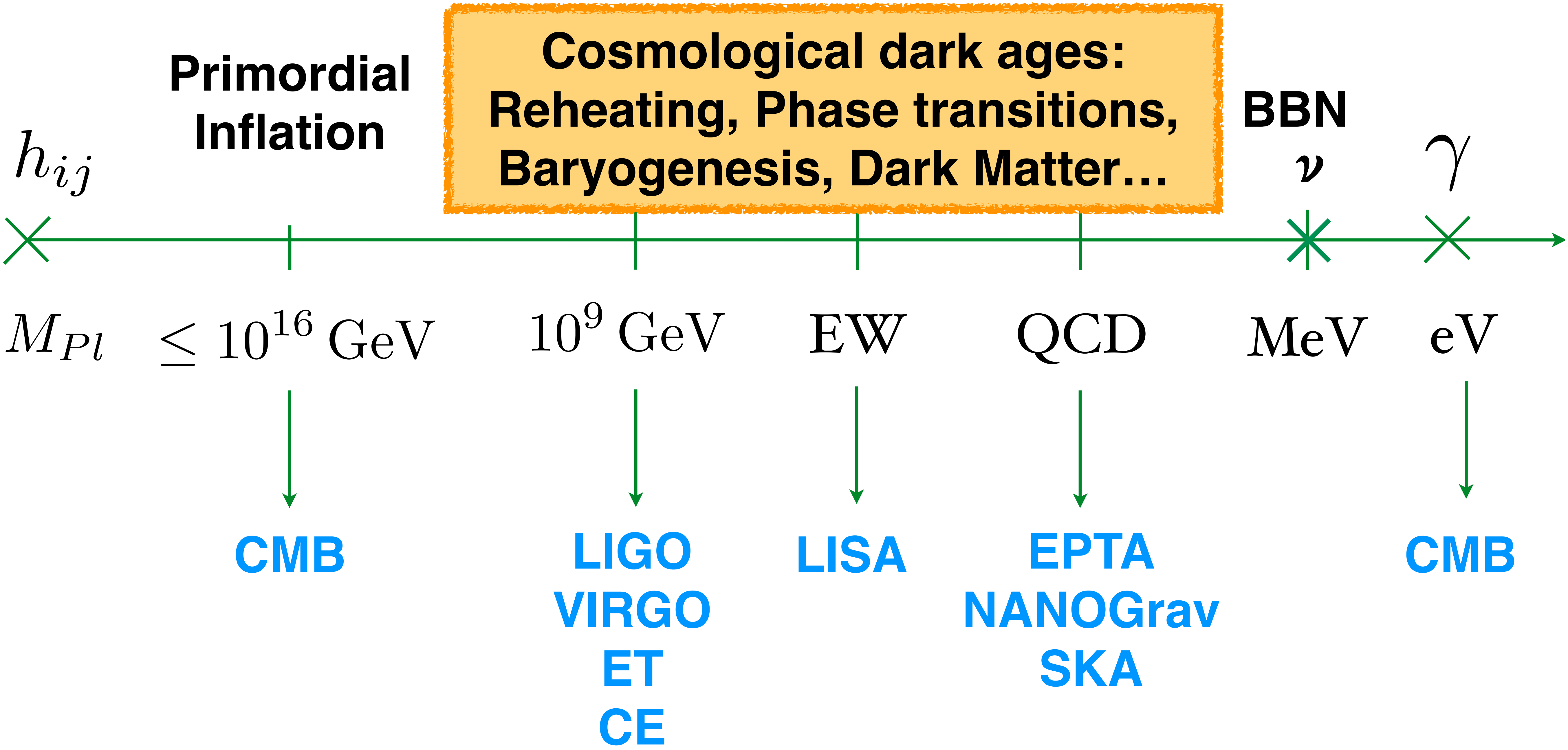


Structure Formation

Big Bang Nucleosynthesis

Cosmic Microwave Background

Probing the cosmological history with Gravitational Waves:



h_{ij}

Primordial Inflation

Cosmological dark ages: Reheating, Phase transitions, Baryogenesis, Dark Matter…

BBN

ν

γ

M_{Pl} $\leq 10^{16}\,\mathrm{GeV}$ $10^{9}\,\mathrm{GeV}$ EW QCD MeV eV

CMB

LIGO
VIRGO
ET
CE

LISA

EPTA
NANOGrav
SKA

CMB

Current and future GW experiments constitute a new avenue of investigation in particle physics and cosmology.

Stochastic GW background of primordial origin.

Consider the cosmological perturbation theory on the isotropic-homogeneous expanding Universe, described by the Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j,$$

GW: tensor perturbation satisfying the transverse traceless condition

The equation-of-motion follows from the linearized Einstein equation,

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\Pi^{\mathrm{TT}}_{ij}(\mathbf{x},t),$$

transverse-traceless part of the

anisotropic stress tensor defined by

$$a^2\Pi_{ij} = T_{ij} - pa^2(\delta_{ij} + h_{ij})$$

Fourier decomposition:

$$\ddot{h}_{ij}(\mathbf{k},t) + 3H\dot{h}_{ij}(\mathbf{k},t) + \frac{k^2}{a^2}h_{ij}(\mathbf{k},t) = 16\pi G\Pi^{\mathrm{TT}}_{ij}(\mathbf{k},t),$$

2 limits:

$$h_\lambda(\mathbf{k},\tau) = \begin{cases} \frac{A_\lambda(\mathbf{k})}{a(\tau)}e^{ik\tau} + \frac{B_\lambda(\mathbf{k})}{a(\tau)}e^{-ik\tau}, & \text{for } k \gg aH \ \text{(sub-horizon)}, \\ A_\lambda(\mathbf{k}) + B_\lambda(\mathbf{k})\int^\tau \frac{d\tau'}{a^2(\tau')}, & \text{for } k \ll aH \ \text{(super-horizon)}, \end{cases}$$

oscillatory behavior redshifted by expansion

stays frozen and later re-enters the horizon and starts oscillating

$d\tau \equiv dt/a$ is the conformal time

Stochastic GW background of primordial origin.

Early-universe production process operates within a causal patch ($\lambda_{\mathrm{GW}} \leq H^{-1}$), much smaller than the horizon size today,

$$\frac{\lambda_{\mathrm{GW},0}}{H_0^{-1}} \leq \frac{H^{-1}_{\mathrm{prod}}}{H_0^{-1}}\left[\frac{a_0}{a_p}\right] \simeq \Omega_{r,0}^{-1/2}\left[\frac{T_0}{T_p}\right] \simeq 2\cdot 10^{-13}\left[\frac{100\ \mathrm{GeV}}{T_p}\right],$$

Primordial GW sources from many uncorrelated patches randomize the amplitude of h_{ij} (x, t) observed today and contribute to the *stochastic GW background*.

For an isotropic, homogeneous, unpolarized, stationary, and gaussian background, the correlation function reads $\langle h_{ij}(\mathbf{x},\tau)\, h_{ij}(\mathbf{x},\tau)\rangle = 2\int d(\log k)\, h_c^2(k,\tau)$

dimensionless
characteristic
strain

$$\rho_{\mathrm{GW}} = \frac{\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle}{32\pi G} = \frac{\left\langle h'_{ij}(\mathbf{x},\tau)h'_{ij}(\mathbf{x},\tau)\right\rangle}{32\pi G a^2}.$$

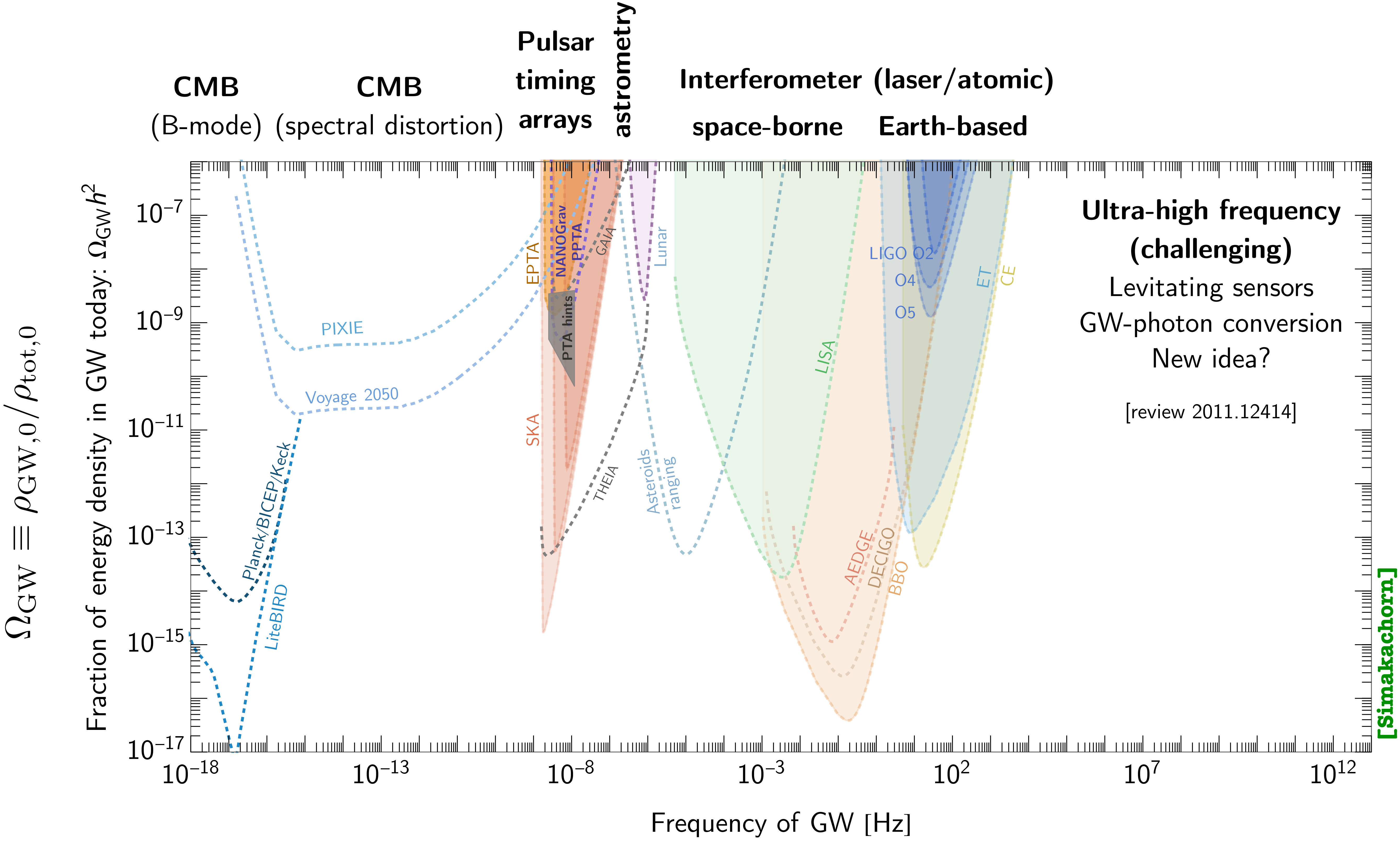
$$\rho_{\mathrm{GW}} = \int d(\log k)\frac{k^2 h_c^2(k,\tau)}{16\pi G a^2(\tau)} \equiv \int d(\log k)\frac{d\rho_{\mathrm{GW}}}{d\log k},$$

$$\Omega_{\mathrm{GW},0}(f) = \frac{k^2 h_c^2(k,\tau_0)}{16\pi G a_0^2} = \frac{\rho^{\mathrm{prod}}_{\mathrm{GW}}(f)}{\rho_{\mathrm{tot},0}}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)^4,$$

$$h_c \simeq 1.26\times 10^{-18}(\mathrm{Hz}/f_{\mathrm{GW}})\sqrt{\Omega_{\mathrm{GW}}h^2}.$$

Due to $h^2_c \sim a^{-2}$ for sub-horizon mode, *the GW energy density of some mode* k *red-shifts as radiation* a^{-4}.

The landscape of current & future GW experiments.



[Simakachorn]

Primordial GW .

Tensor perturbations of Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

Wave equation:

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi^{TT}_{ij}}$$

Source:

Tensor anisotropic stress

=Transverse Traceless component
of the energy-momentum tensor of the source $= (P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})T_{lm}$

$P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Well-known cosmological sources .

-> Cosmological Phase Transitions

-> Cosmic Strings

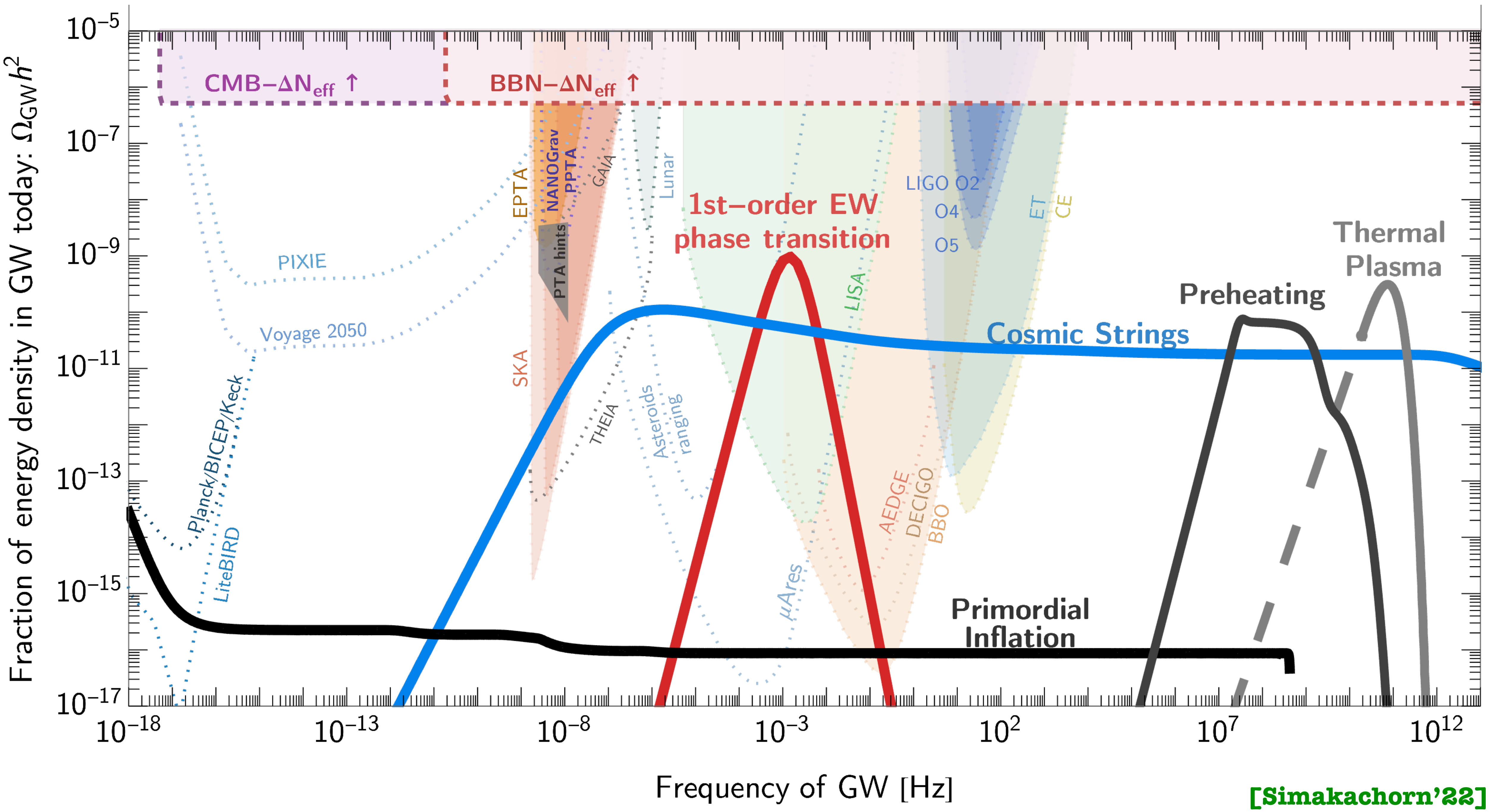
-> Inflation

-> Reheating of the universe

see
-review 1801.04268
-1912.02569 (cosmic strings)
-PhD thesis P. Simakachorn

Beyond-the-Standard Model sources.

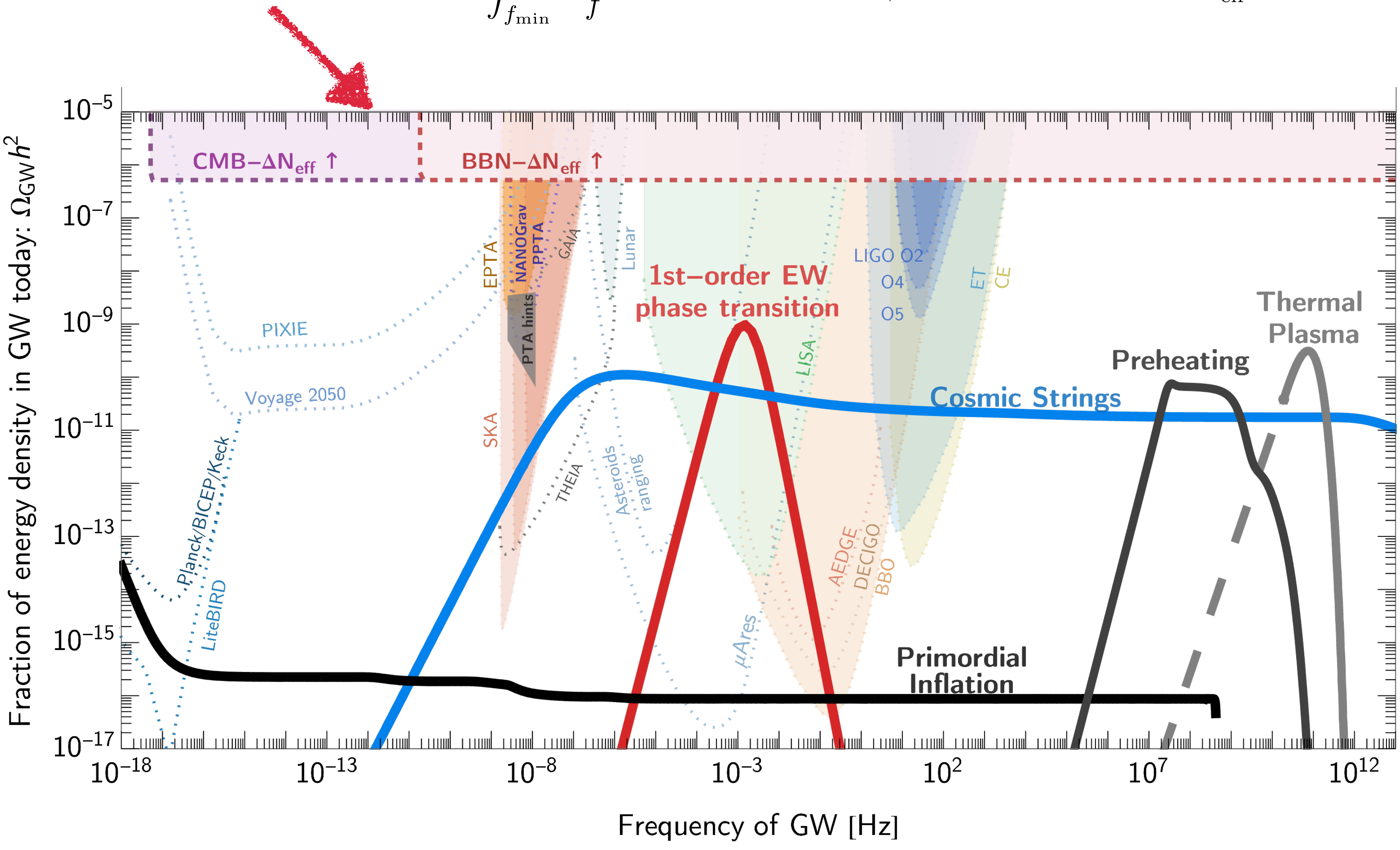
Preheating, first-order phase transitions, cosmic strings



[Simakachorn'22]

Upper theoretical bound.

GW as extra radiation: $$\int_{f_{\rm min}}^{f_{\rm max}} \frac{df}{f}\,\Omega_{\rm GW}(f) \lesssim 0.23\,\Omega_{\rm rad,0}\Delta N_{\rm eff} \qquad \text{where} \quad \Delta N_{\rm eff}^{\rm BBN,CMB} \lesssim 0.2$$



[Simakachorn'22]

Characteristic Frequencies for causal (and short-lasting) sources .

T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*} \sim \frac{T_*}{M_{Pl}} T_0 \sim T_* \times 10^{-13}$ 10^{-19} **GeV**

H_* = Hubble rate at T_*

If T_* **~ 100 GeV:** (Electroweak scale)

$$f \sim 10^{-30}\,\mathrm{GeV} \sim 10^{-30} \times 10^{25}\ \mathrm{Hz} \sim 10^{-5}\ \mathbf{Hz}$$

LISA !

If T_* **~ 10^{10} GeV:** (Peccei-Quinn scale)

f ~ 10^{3} Hz

Frequencies limits .

The lowest frequency of GW is that of those GW produced today with the largest possible source size, i.e., the Hubble horizon today:

$$f_{\mathrm{GW,lowest}} = H_0 \simeq 10^{-18}\ \mathrm{Hz}.$$

The highest frequency of primordial GW arises from the highest energy scale $H_{\mathrm{prod}} \simeq M_P$

$$f_{\mathrm{GW,highest}} \simeq 10^{13}\ \mathrm{Hz}.$$

Largest possible amplitude .

Early produced GW act as an effective number of neutrino relics which is strongly constrained by CMB measurement and by BBN predictions

$$\int_{f_{\mathrm{BBN,CMB}}}^{f_{\mathrm{max}}} \frac{df}{f}\Omega_{\mathrm{GW}}(f) \leq 5.6 \times 10^{-6} \Delta N_\nu$$

$$\Delta N_\nu \leq 0.2$$

$$\Omega_{\mathrm{GW},*} \leq 5.6 \times 10^{-6} \Delta N_\nu \cdot \begin{cases} \log^{-1}\left[\frac{f_{\mathrm{max}}}{\max(f_{\mathrm{ref}}, f_{\mathrm{min}})}\right] & \text{for flat spectrum,} \quad \Omega_{\mathrm{GW}} = \Omega_{\mathrm{GW},*} \text{ for } f_{\mathrm{min}} < f < f_{\mathrm{max}}, \\ \beta\left[1 - \left(\frac{f_{\mathrm{ref}}}{f_{\mathrm{peak}}}\right)^{\beta}\right]^{-1} & \text{for peak,} \quad \Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW},*}(f/f_{\mathrm{peak}})^{\beta} \end{cases}$$

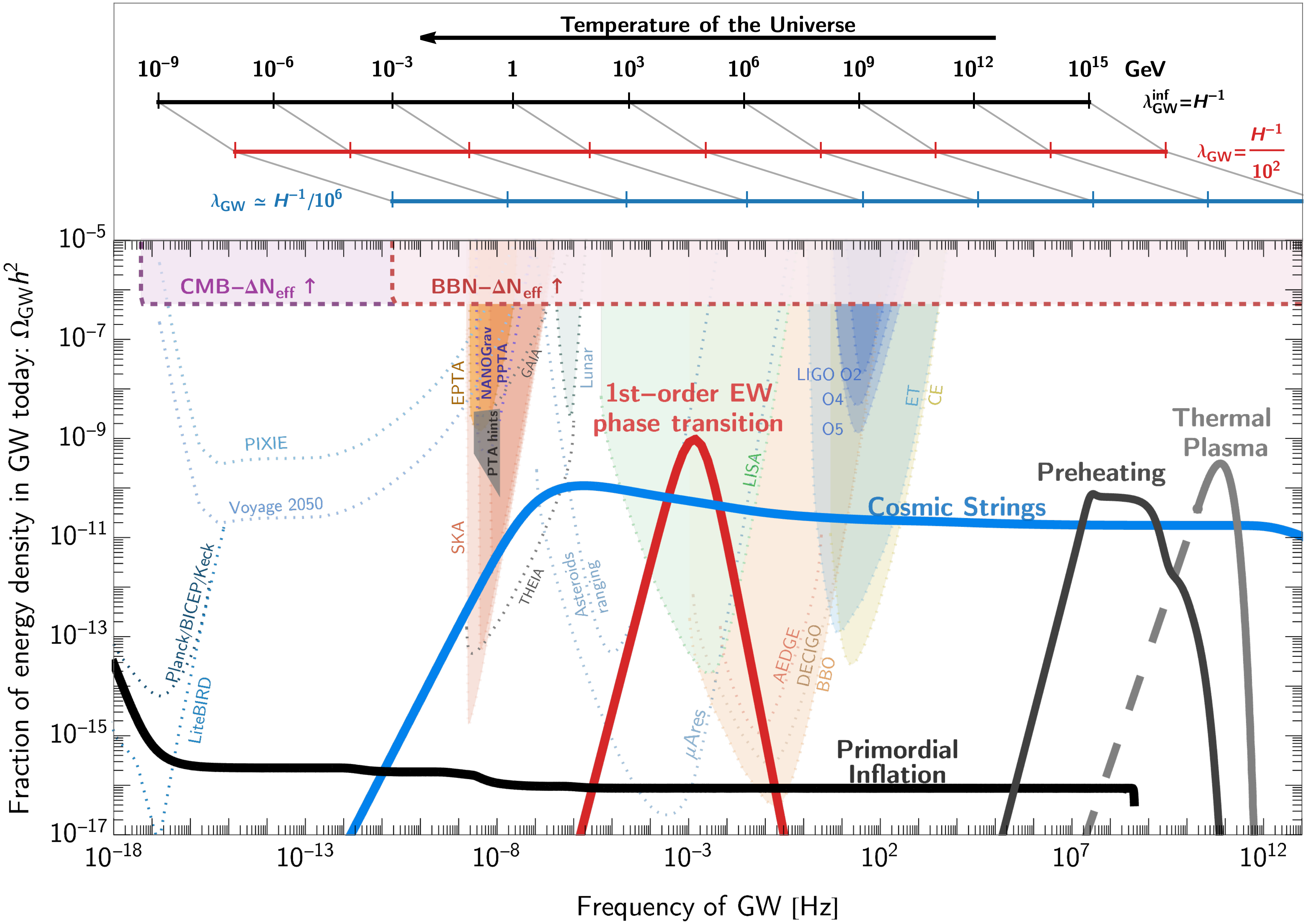
Reading the history of the universe.

GW frequency $$f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1} \left(\frac{a_{\mathrm{prod}}}{a_0} \right)$$

Low-freq. lim

$f_{\mathrm{GW}}^{\mathrm{min}} \simeq H_0^{-1}$

$\simeq 10^{-18}$ Hz



High-freq. limit

$f_{\mathrm{GW}}^{\mathrm{max}} \simeq 10^{13}$ Hz

$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$

[Simakachorn]

Reading the history of the universe.

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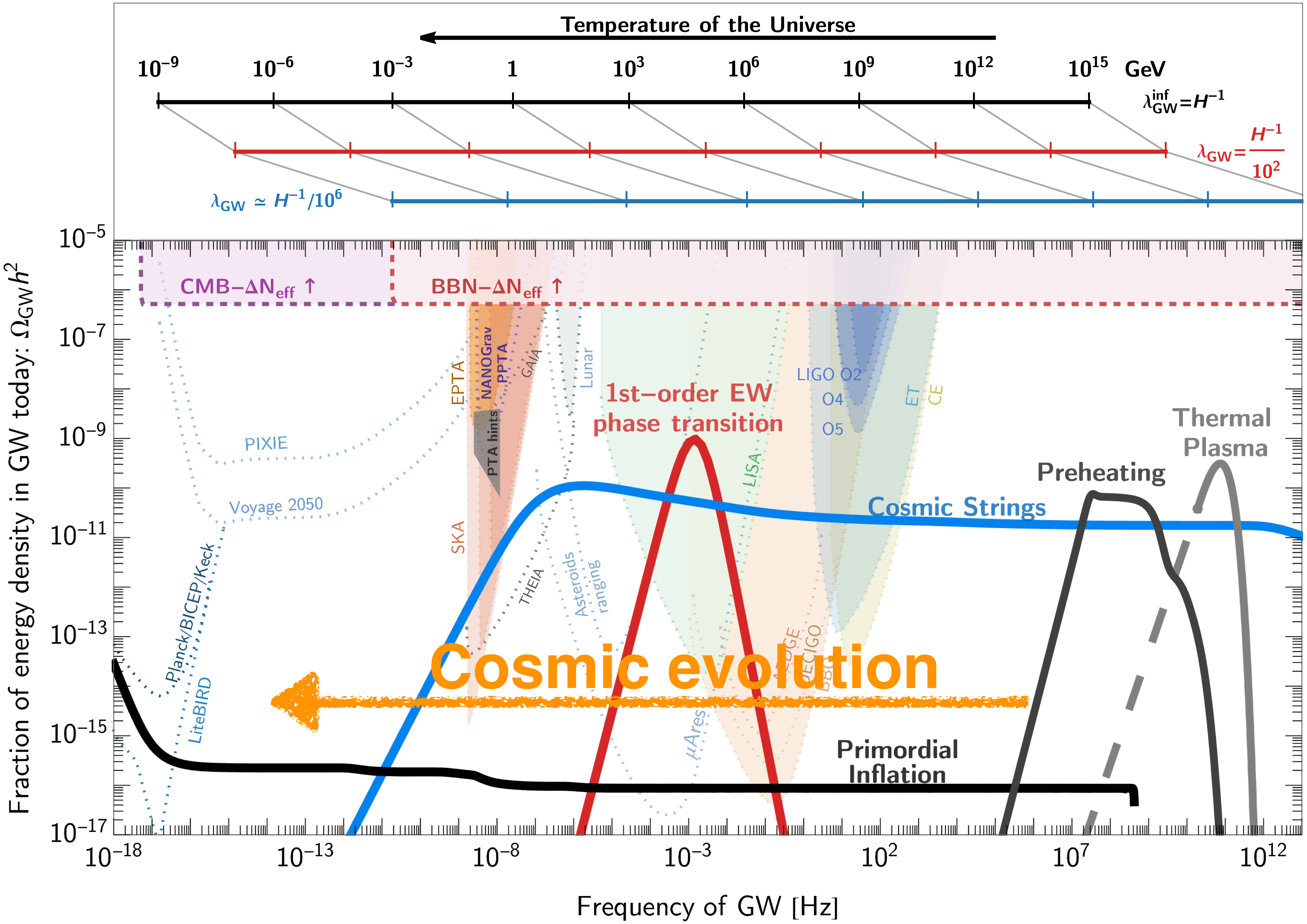


High-freq. limit

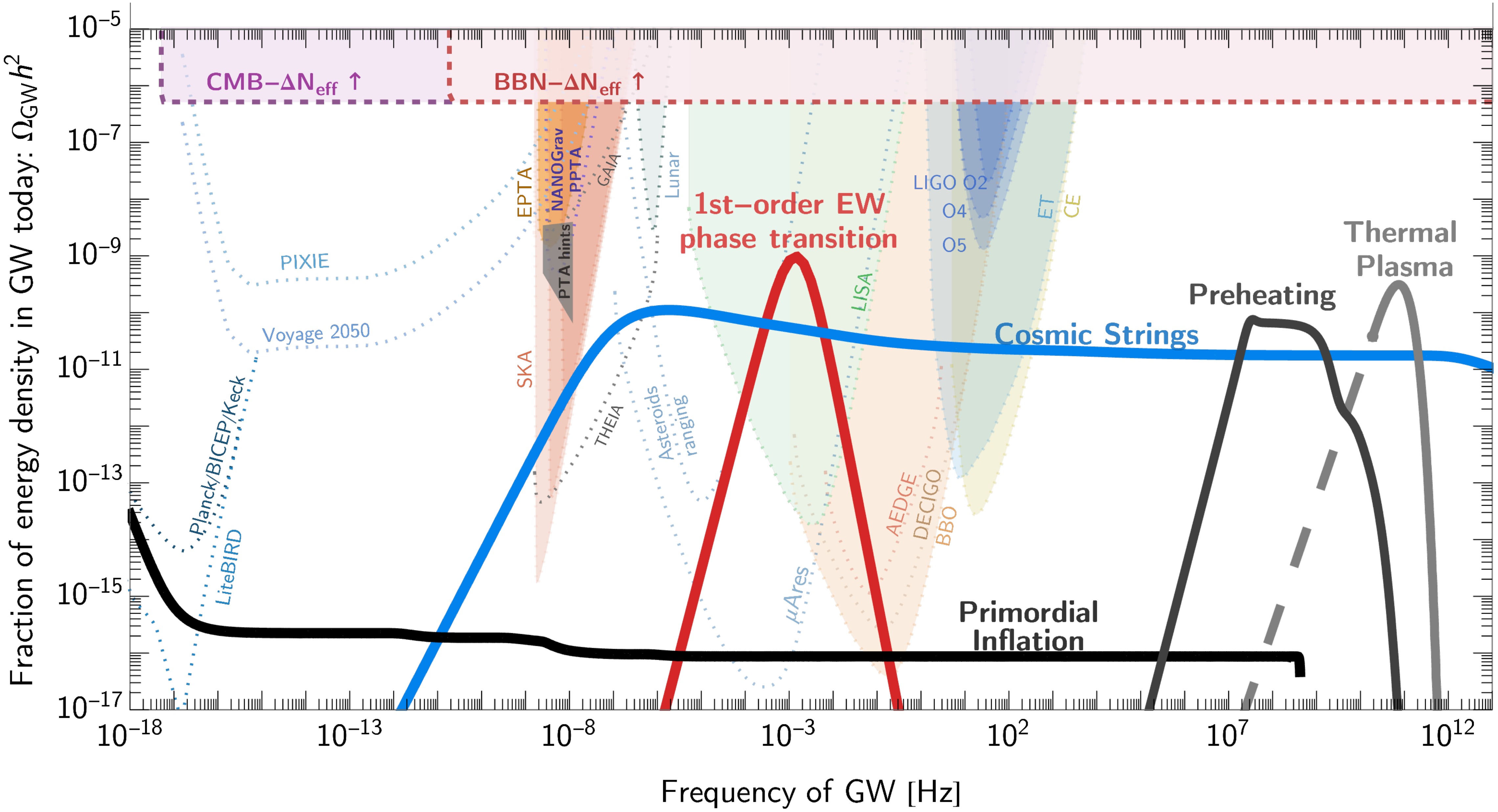
$f_{\mathrm{GW}}^{\mathrm{max}} \simeq 10^{13}$ Hz

$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$

[Simakachorn]

GW spectra are sensitive to the cosmological history.

frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)$ **energy density** $\rho_{\mathrm{GW},0} \simeq \rho_{\mathrm{GW}}^{\mathrm{prod}}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)^4$



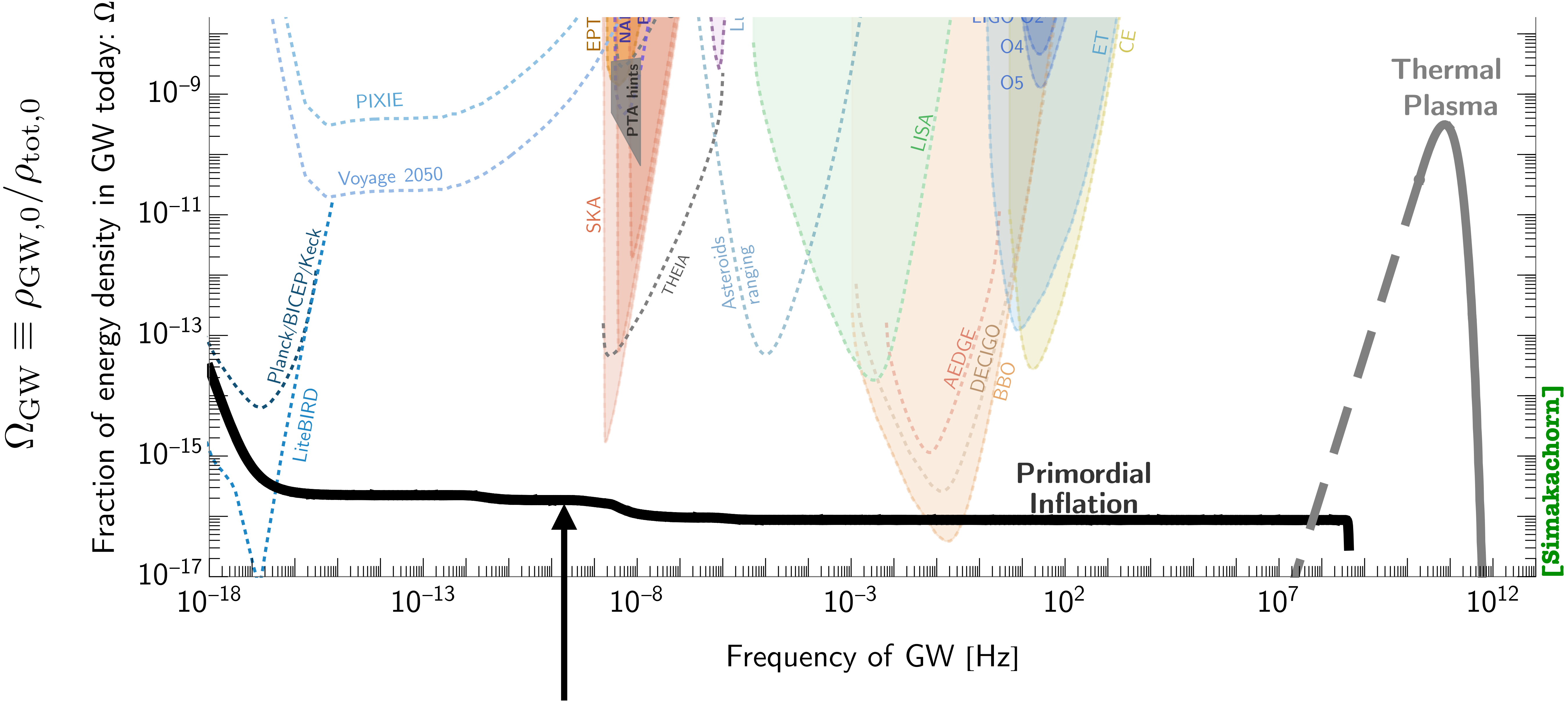
Standard Model radiation era at high energies

[Simakachorn]

What if the universe is not radiation-dominated at high energies?

Standard Model sources of primordial GW.

Primordial inflation & Standard Model thermal plasma



Irreducible GW background from amplification of initial quantum fluctuations of the gravitational field during inflation

GWs from inflation.

Quantum fluctuations of some comoving scale k during inflation classicalize upon horizon exit (k > aH) and stay frozen afterwards.

After the end of the inflation phase, the increasing comoving horizon catches up with these modes and they re-enter (k < aH) the horizon.

Tensor perturbations of comoving wave number k = a$_k$H$_k$ lead to the

the stochastic GW today

$$\Omega_{\rm GW}^{\rm inf}(k) = \frac{k^2 a_k^2}{24 H_0^2}\Omega_T^{\rm inf},$$

$$\Omega_T^{\rm inf} \simeq \frac{2}{\pi^2}\left(\frac{H_{\rm inf}}{M_{\rm pl}}\right)^2 \left(\frac{k}{k_p}\right)^{n_t}$$

~ constant

Pivot scale $k_p/a_0 \simeq 0.002\ {\rm Mpc}^{-1}$

$f_p = 3.1 \times 10^{-18}\ {\rm Hz}$

$n_t \simeq -2\epsilon$

Taking for simplicity $n_t = 0$

GWs from inflation.

$$\Omega \sim (ka)^2 \sim (a^2 H)^2 \sim a^4 \rho \sim a^4 a^{-3(1+\omega)} \sim a^{1-3\omega}$$

$$f \sim aH \sim a\rho^{1/2} a^{-(1+3\omega)/2} \quad \longrightarrow \quad a \sim f^{-2/(1+3\omega)}$$

w: equation of state of the universe

$$\longrightarrow \quad \Omega \sim a^{1-3\omega} \sim f^{-2\frac{(1-3\omega)}{(1+3\omega)}}$$

w=1/3, radiation era —> scale-invariant spectrum

w=0, matter era —> $\Omega \sim f^{-2}$

w=1, kination —> $\Omega \sim f$

GWs from the SM plasma.

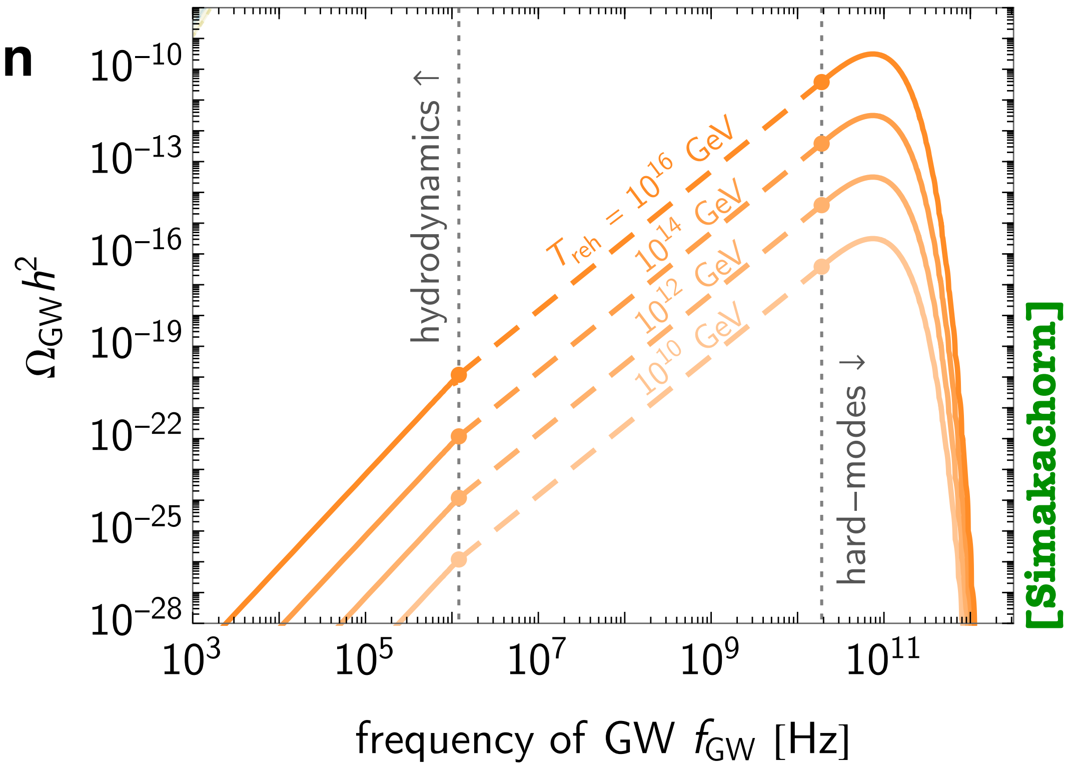
T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \dfrac{T_0}{T_*} \sim \cancel{\mathcal{O}(H_*)} \dfrac{T_0}{T_*}$

Instead $f_* \sim T_*$ —> cancellation

$f_* \sim T_0/2\pi \sim 5.10^{10}$ Hz



[Simakachorn]

- J. Ghiglieria and M. Laine, 1504.02569
- A.Ringwald, J.Schütte-Engeland C.Tamarit, 2011.04731
- A.Ringwalda nd C.Tamarit, 2203.00621

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If T_* ~ 100 GeV: (Electroweak scale)

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LISA !

If T_* ~ 10^{10} GeV: (Peccei-Quinn scale)

f ~ 10^{3} Hz

Gravitational Waves from a 1st-order phase transition .

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G a^2 T_{ij}^{(TT)}(k,t)$$

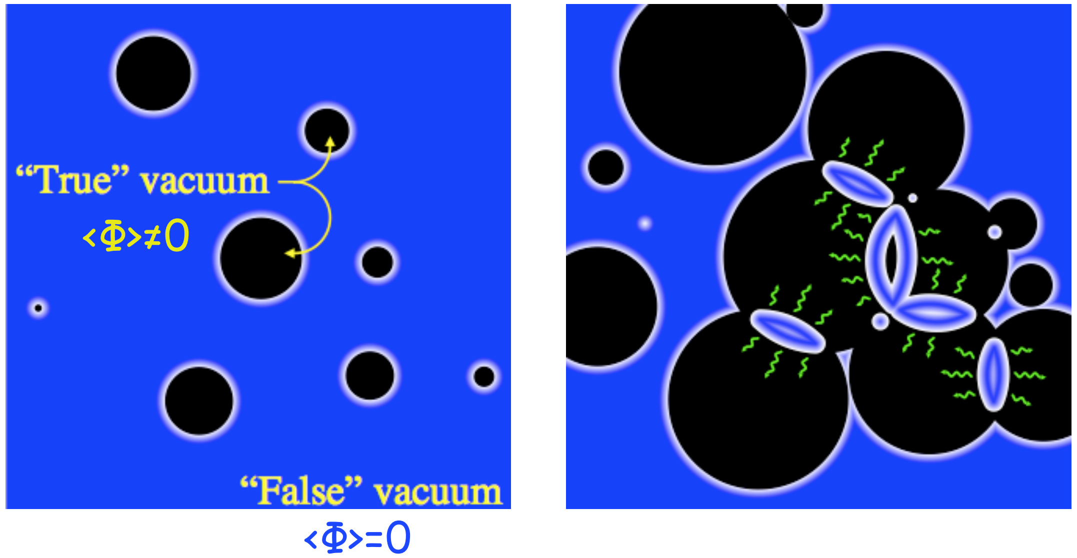
$$T_{ab}(\mathbf{x}) = (\rho + p)\frac{v_a(\mathbf{x})v_b(\mathbf{x})}{1 - v^2(\mathbf{x})}$$

Source of GW:
anisotropic stress

Generated mainly from fluid velocities
in the vicinity of colliding bubble walls

Bubble nucleation

Bubble percolation



violent process if $v_b \sim O(1)$

Fluid flows

Magnetic fields

Turbulence

→

Stochastic bgd of
gravitational radiation

Sources of GWs during a 1st-order phase transition .

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Fluid (sound waves and turbulence)

$$\Pi_{ij} \sim \gamma^2(\rho + p)\,v_i v_j$$

Electromagnetic field (primordial magnetic fields MHD turbulence)

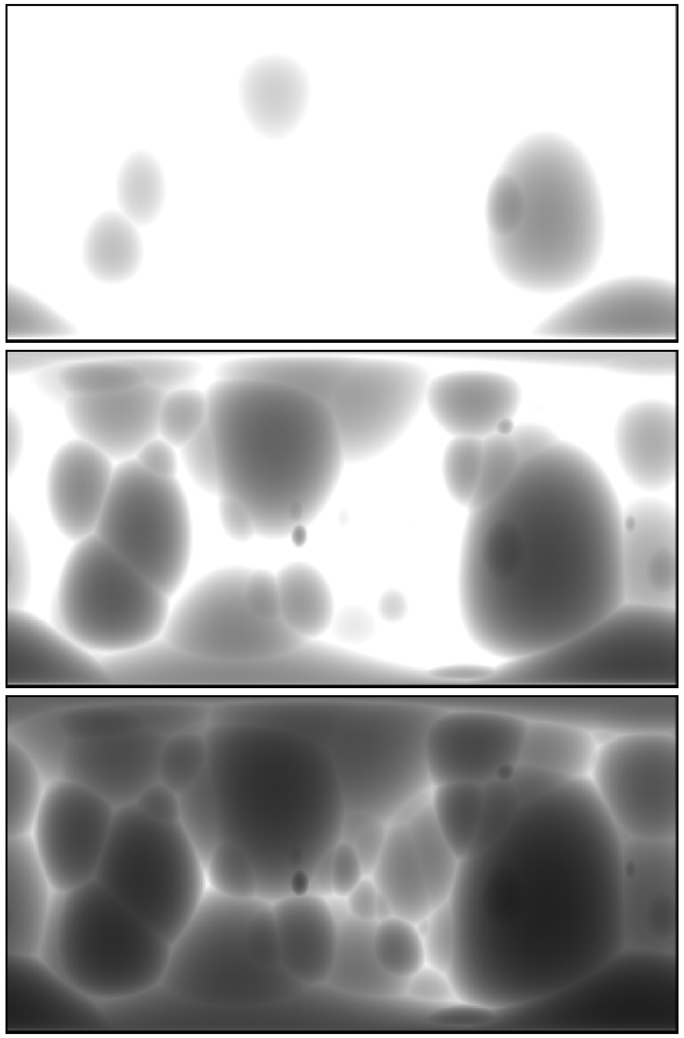
$$\Pi_{ij} \sim (E^2 + B^2)\frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$$

Scalar field (collision of bubble walls)

$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

In principle entirely determined by the Higgs effective potential.

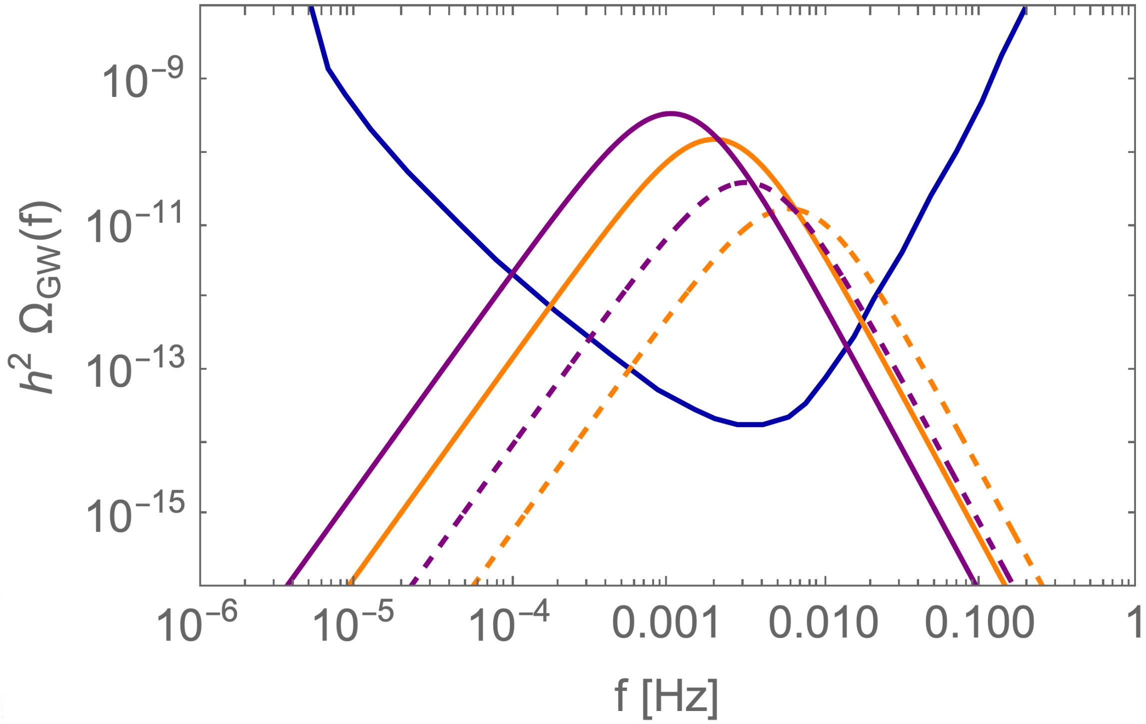
GWs from a 1st-order phase transition .

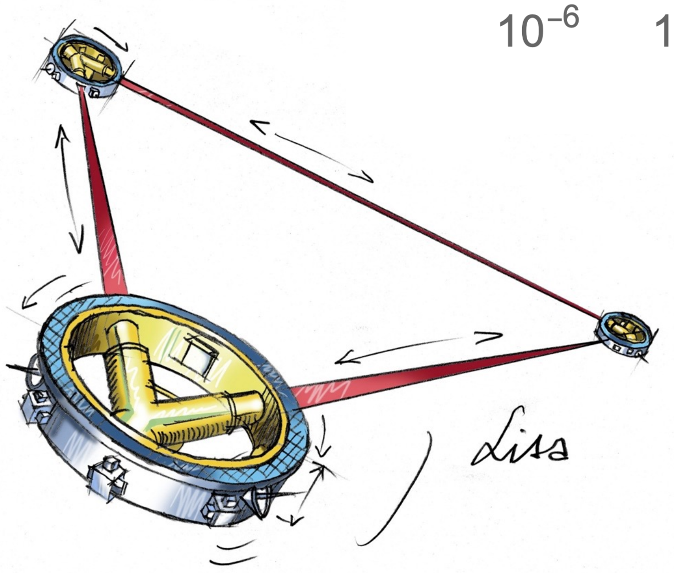


Konstandin'18

Electroweak Phase Transition

-> milli-Hertz —> **LISA !**





[LISA Cosmology Working group, 1512.06239]

+ update 1910.13125

LISA: a new window on the Weak Scale

hep-ph/0607107

complementary to collider informations

Two Characteristic quantities .

- β : **inverse duration of the phase transition**

set by the tunneling probability $P \propto e^{\beta t} \propto \frac{T^4}{H^4} e^{-S_3/T} \sim 1$ ➜ $\frac{S_3}{T}$ ~ 140

$$\beta \equiv \frac{dS}{dt}\Big|_* = -H_* T_* \frac{dS}{dT}\Big|_*$$

typically $\frac{\beta}{H} \sim \mathcal{O}(10^2 - 10^3)$

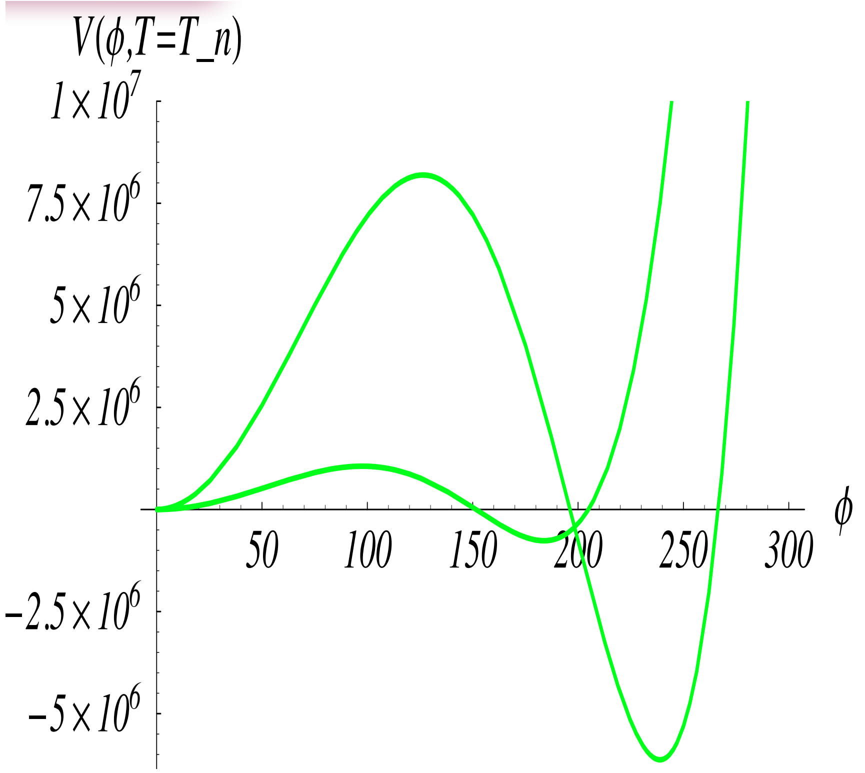
corresponds to the characteristic inverse size of bubbles at time of collisions $R_* = v_w/\beta$

sets the characteristic frequency $f_* \propto \frac{\beta}{v_w}$

$$f_0 = \frac{a_*}{a_0} f_* \approx 20 \frac{1}{v_w} \frac{\beta}{H_*} \frac{T_*}{100GeV} \left(\frac{g_*}{100}\right)^{1/6} \mu\text{Hz}$$

- α : **vacuum energy/radiation energy density**

α and β : entirely determined by the effective scalar potential at high temperature



Estimate of the GW energy density at the emission time .

$$\Omega_{GW,*} = \frac{\rho_{GW,*}}{\rho_{tot,*}} \sim \frac{G}{\beta^2}\Pi^2_{source} \times \frac{1}{\rho_{tot,*}} \sim \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2$$

$$\Pi_{source} \sim \kappa\rho_{vac}$$

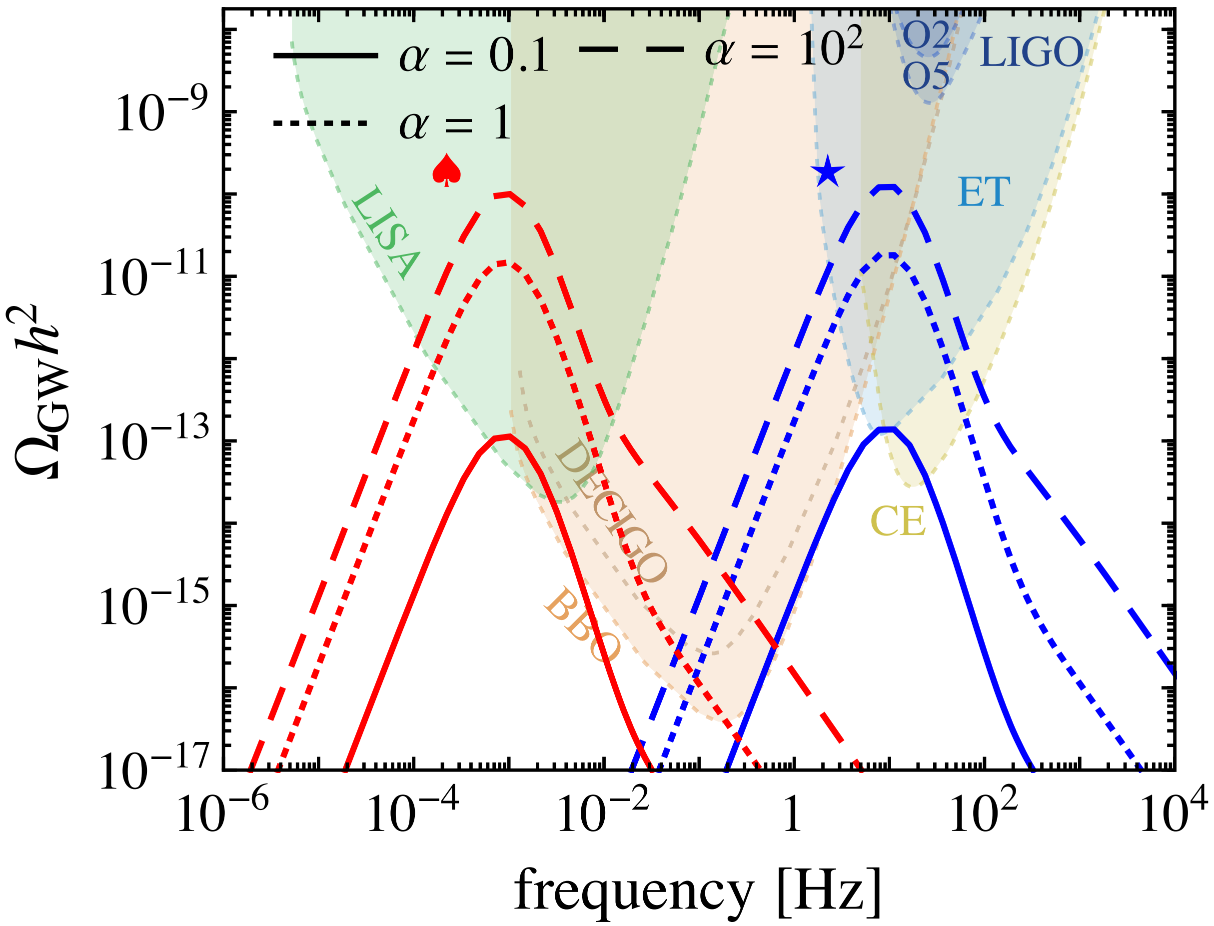
$$\kappa_\phi = \frac{\rho_\phi}{\rho_{\rm vac}} \qquad \kappa_v = \frac{\rho_v}{\rho_{\rm vac}} \qquad \kappa_t = \epsilon\,\kappa_v$$

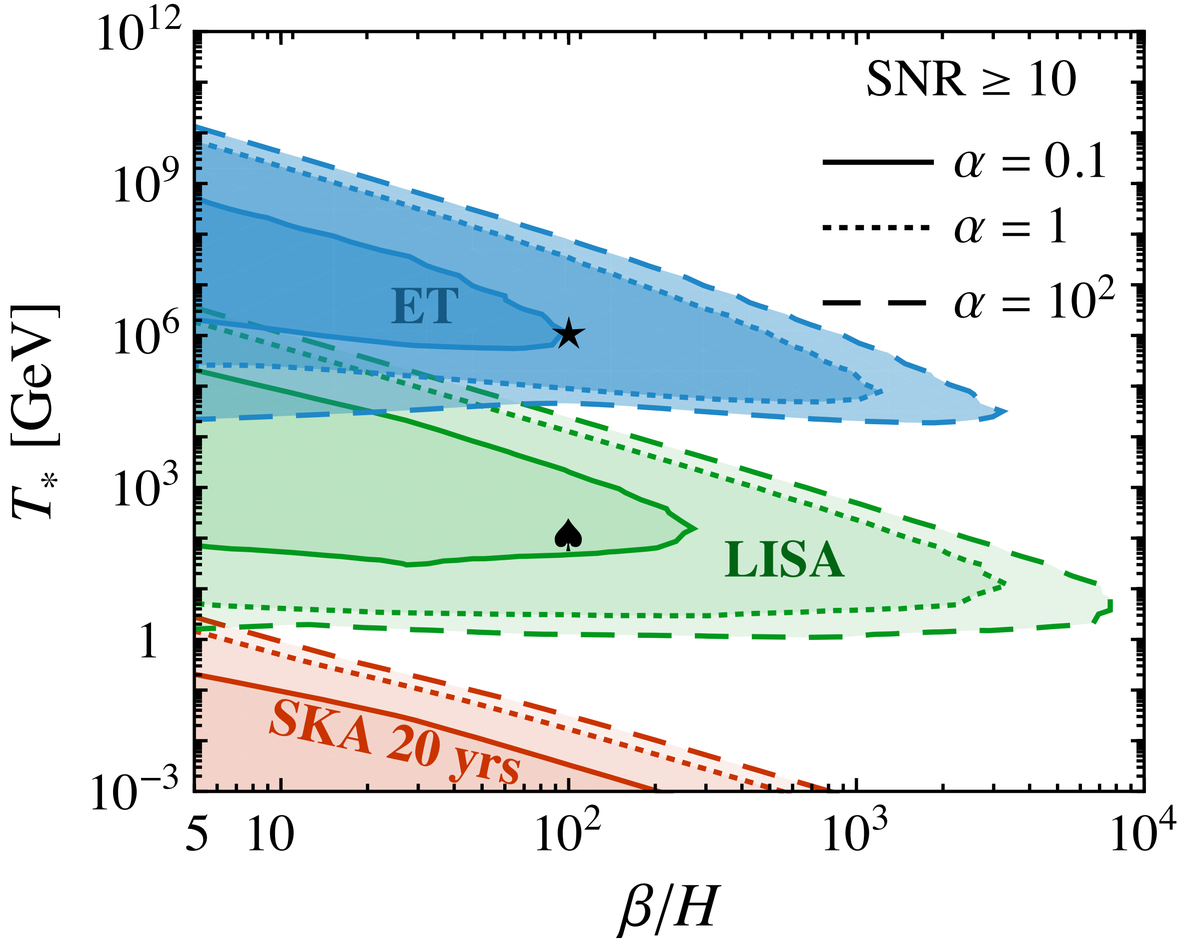
fractions of vacuum energy that goes into either gradient energy in bubble kinetic energy in the fluid o into turbulent motion.

$$\left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2 \sim \frac{\kappa^2\alpha^2}{(1+\alpha)^2}$$

$$\boxed{\Omega_{GW,*} \sim \left(\frac{H_*}{\beta}\right)^2 \times \frac{\kappa^2\alpha^2}{(1+\alpha)^2}}$$

GWs from a 1st-order phase transition: Reach



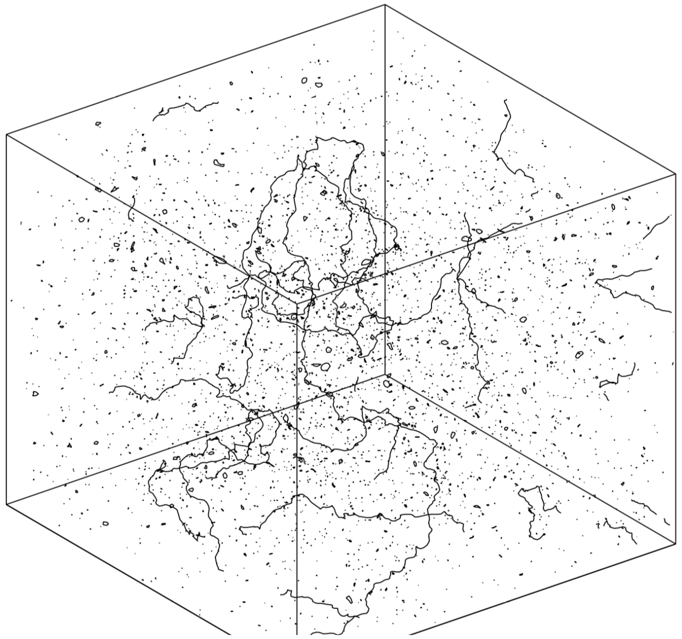


For reviews see e.g.

1512.06239, 1910.13125, 2204.05434

Gravitational Waves from cosmic strings.

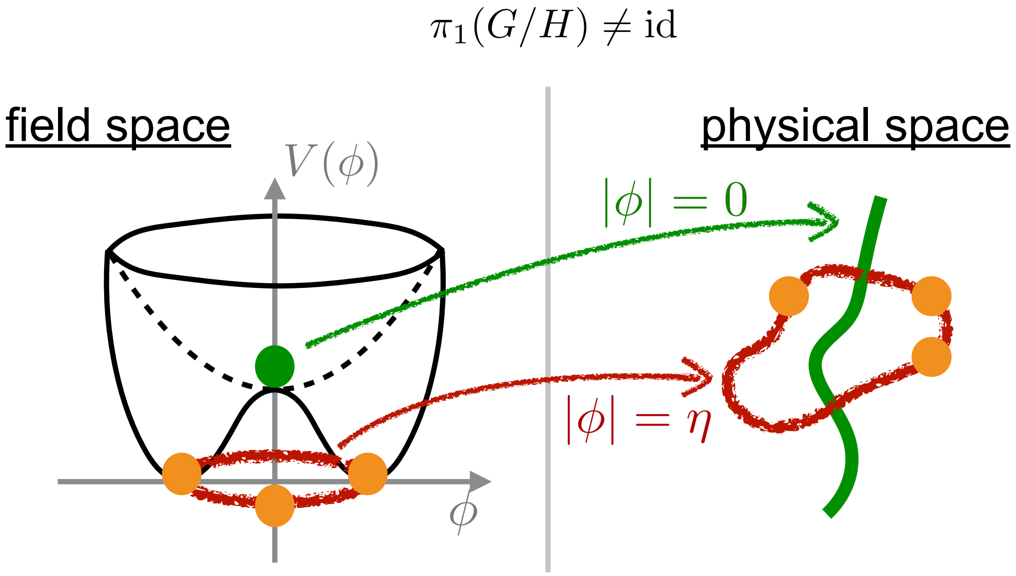
recent reviews: [1909.00819, 1912.02569]



Network of cosmic strings

[Allen & Shellard, 1990]

Gravitational Waves from cosmic strings.



Cosmic string: Line-like topological defect arising after spontaneous U(1) symmetry breaking at some energy scale η.

The broken symmetry can be either local or global;

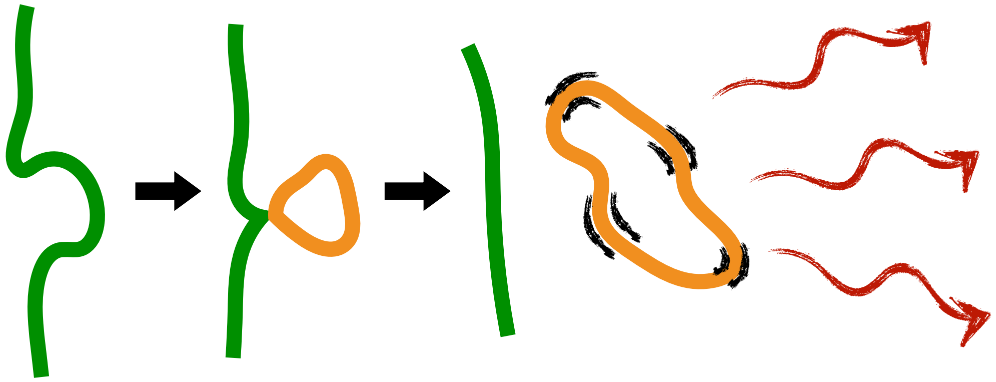
—> local or global (axionic) cosmic strings.

string tension: $\mu \sim \eta^2$

Loop formation & scaling regime.

After the network formation, the string network keeps producing loops.

String intercommutation: loop formation depletes energy from the network.



GW emission
(particle production for global strings)

Cosmic strings do not overclose the universe.

The energy density of the network tracks the total energy density of the Universe

Scaling regime: $\rho_{\rm net}(t) \simeq \mu/t^2 \simeq \; G\mu\rho_{\rm tot}(t)$

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-3} & \text{for matter} \\ a^{-4} & \text{for radiation} \\ a^{-6} & \text{for kination} \end{cases}$$

GW from cosmic strings.

Cosmic strings: Long-lasting source of GW

The produced loops decay into particles and GW.

Local-string loops decay dominantly into GW while global-string loops decay dominantly into Goldstone radiation.

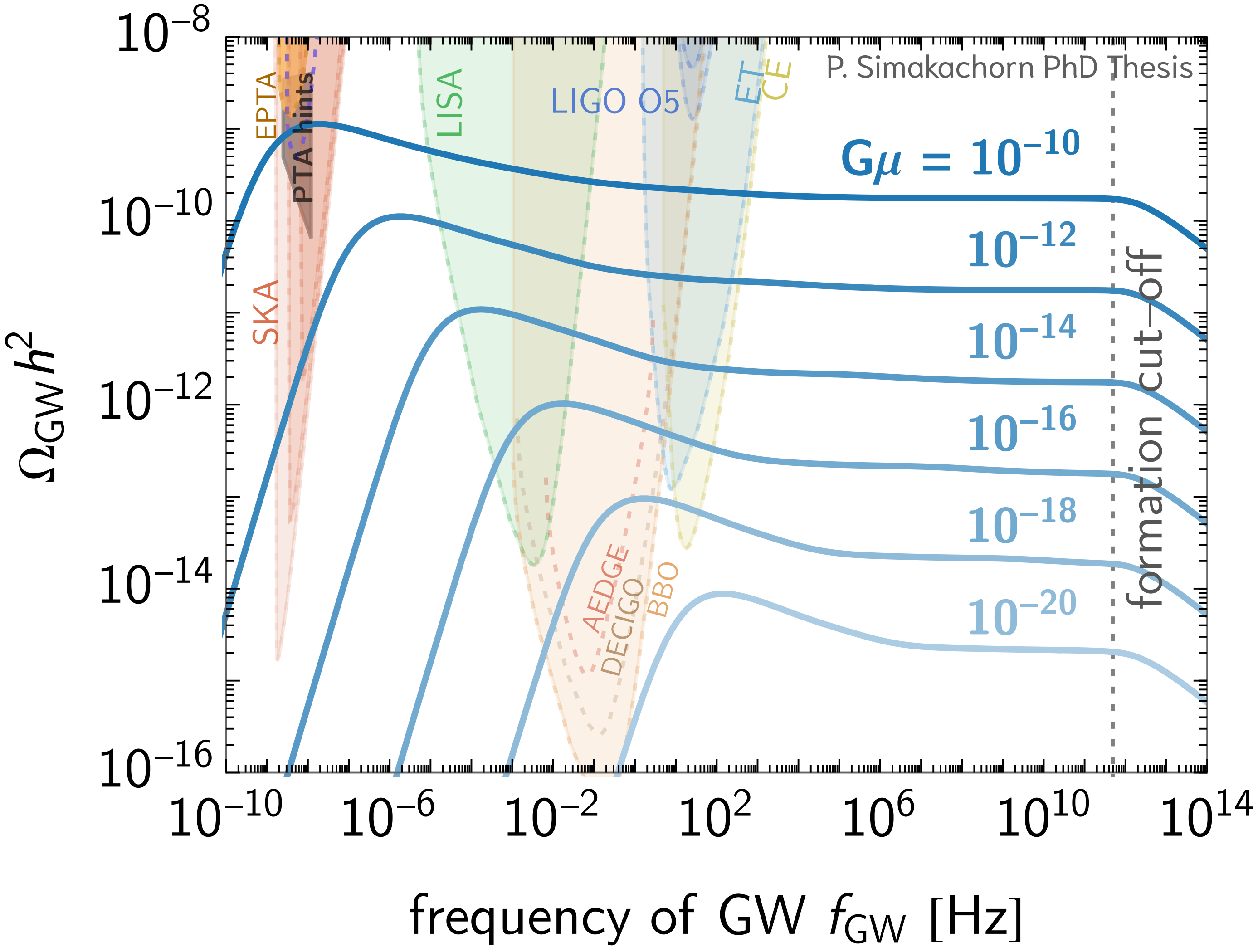
GW from local cosmic strings.

Superposition of many loop populations producing GW at time t̃ and of many oscillation k^{th}-modes,

$$\Omega_{\mathrm{GW}}(f_{\mathrm{GW}}) = \frac{1}{\rho_{c,0}} \boxed{\sum_{k=1}^{k_{\max}} \frac{2k}{f_{\mathrm{GW}}} \cdot \Gamma^{(k)} G\mu^2} \int_{t_{\mathrm{form}}}^{t_0} \boxed{n_{\mathrm{loop}}(\tilde{t})} \left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 d\tilde{t},$$

GW from a loop

# of loops produced along cosmic history
(from production time until today)



GW from cosmic strings.

Cosmic strings: Long-lasting source of GW

GW spectrum ⇨ GW emission from a loop × loop-number density

a^{-3}

$$\Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c}\cdot\frac{2k}{f}\cdot\boxed{\frac{(0.1)\Gamma^{(k)}G\mu^2}{\alpha(\alpha+\Gamma G\mu)}}\int_{t_F}^{t_0} d\tilde{t}\,\boxed{\frac{C_{\rm eff}(t_i)}{t_i^4}}\,\boxed{\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5\left[\frac{a(t_i)}{a(\tilde{t})}\right]^3}\Theta(t_i-t_F)$$

string's nature **loop number** **red-shift**

GW: a^{-4}, loop size: a^{-1}

$t_i \equiv$ loop production, $\tilde{t} \equiv$ loop emission

α = initial loop size as a fraction of Hubble horizon

Γ = GW radiation efficiency

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{1}{\rho_{c,0}}\sum_{k=1}^{k_{\rm max}}\frac{2k}{f_{\rm GW}}\cdot\Gamma^{(k)}G\mu^2 \times \int_{t_{\rm form}}^{t_0} n_{\rm loop}(\tilde{t})\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 d\tilde{t},$$

Relation between observed frequency & Hubble radius at emission.

The broad band GW spectrum is the result of the superposition of GW generated by many populations of loops produced at different temperatures.

Each emits GW at frequency $f_{\mathrm{GW}}^{\mathrm{emit}} \simeq 2k/l$

k : GW mode number of loop oscillation

l: loop's size

The GW contribution at higher frequencies comes from smaller loops produced at higher energy scales.

Time of GW emission for local strings: $\tilde{t} \simeq \alpha/(2\Gamma G\mu)t_i$

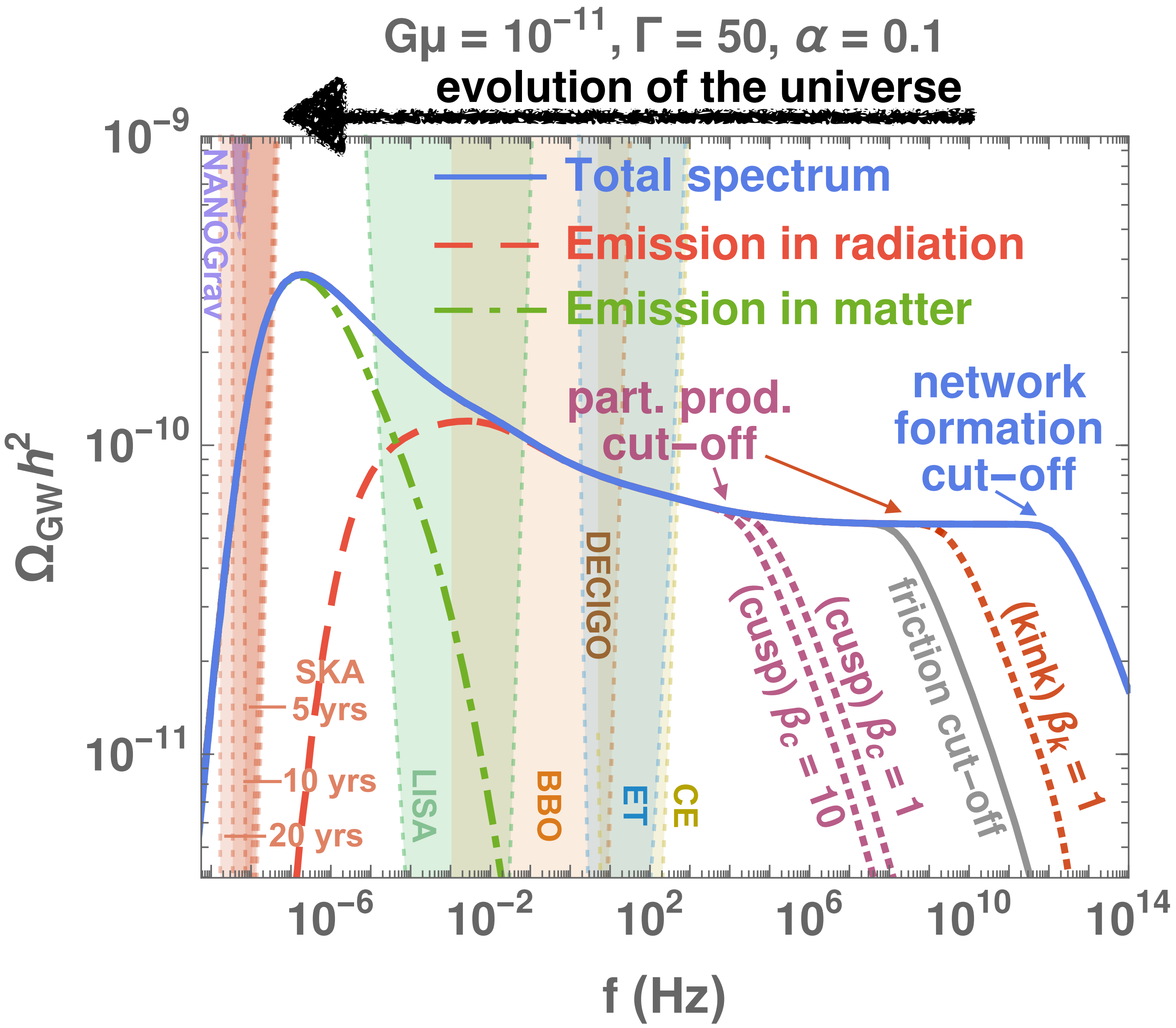
In contrast, global strings quickly emit GW after loop production: $\tilde{t} \simeq t_i$

t_i : time of loop formation

For a loop population created at temperature T, the GW spectrum is sourced maximally at a GW frequency today that is higher for local strings compared to global strings.

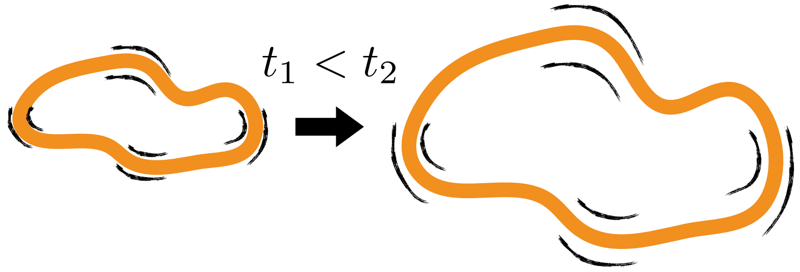
Gravitational Waves from Cosmic strings.

(long-lasting sources).



[1912.02569]

Higher f ⇔ Earlier emission



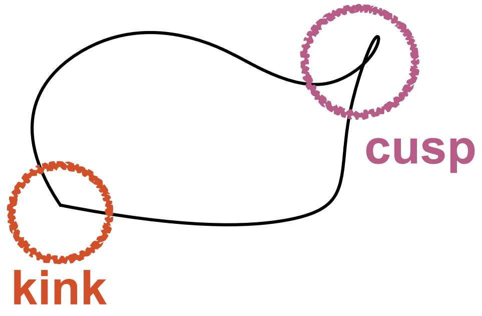
smaller loop ⇔ higher oscillation f

@ earlier t_i

more GW from more loops
but more red-shift

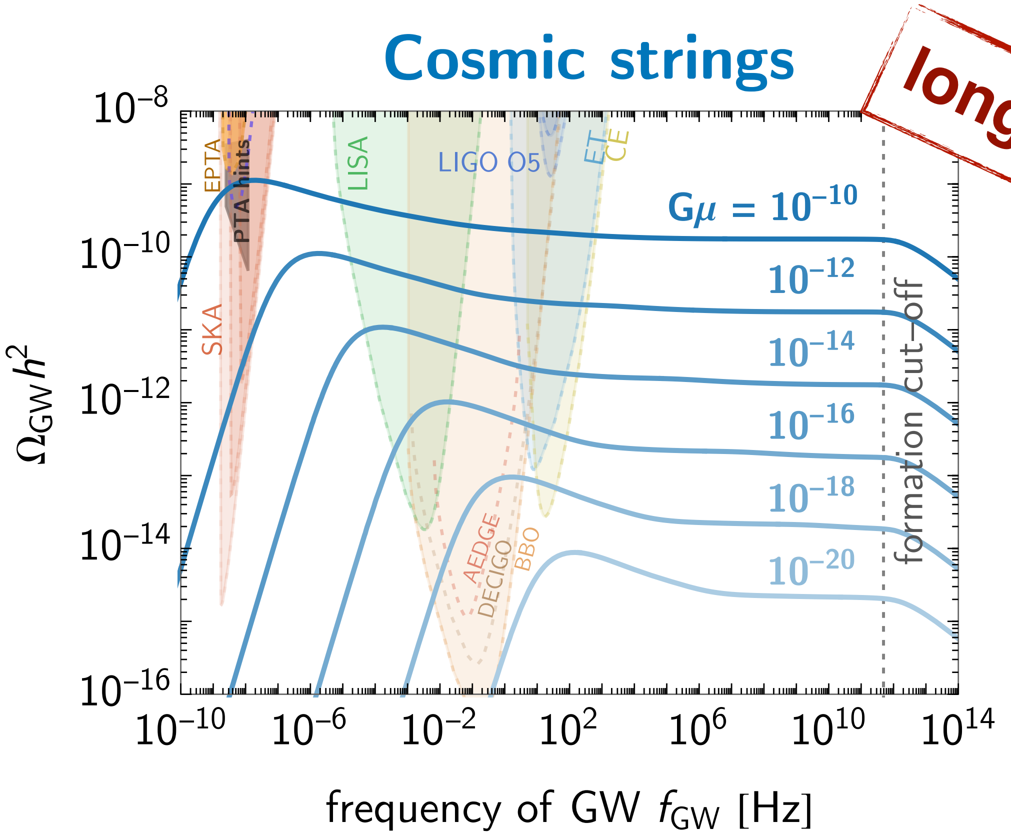
⇨ Flat during radiation

singular structures on loop
(beyond NG approx.)

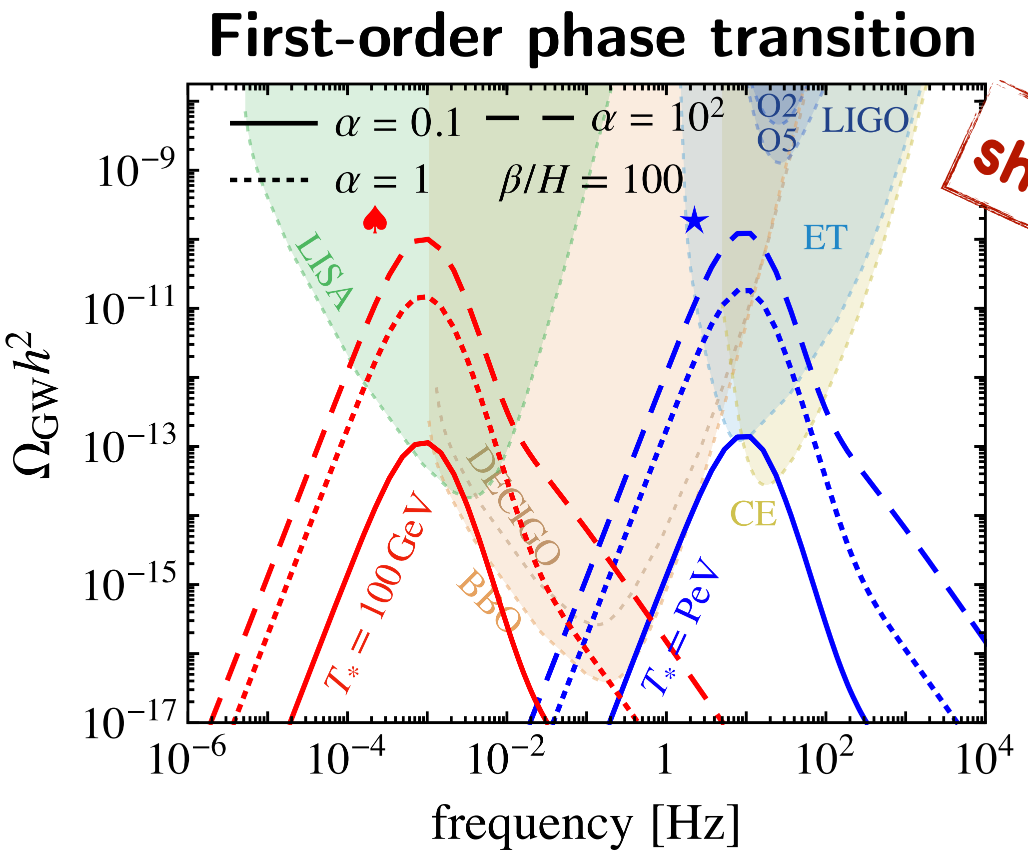


lead to particle emission

Short-lasting vs long-lasting primordial sources.

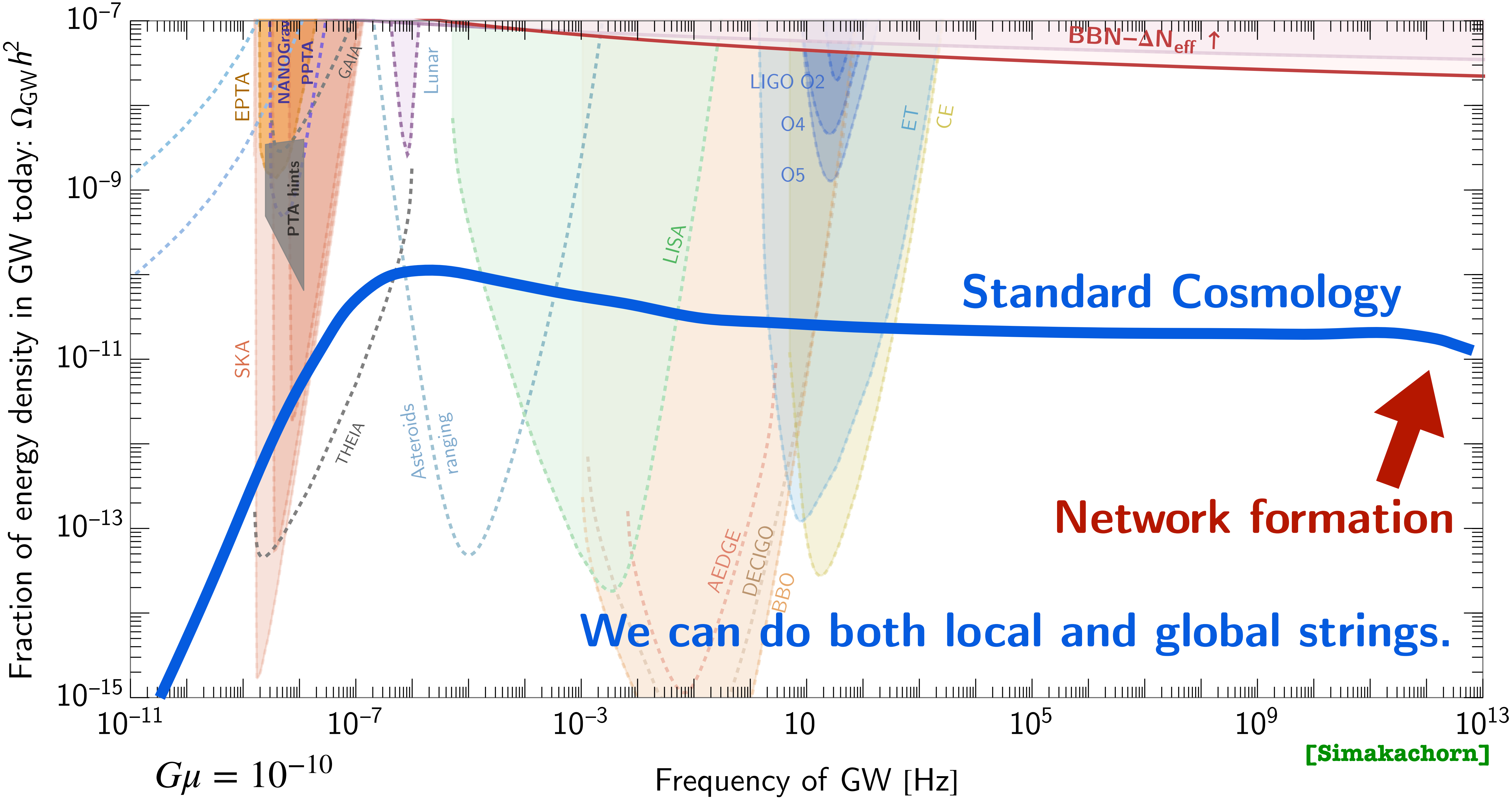


long-lasting!



short-time!

Gravitational Waves from cosmic strings.



Standard Cosmology

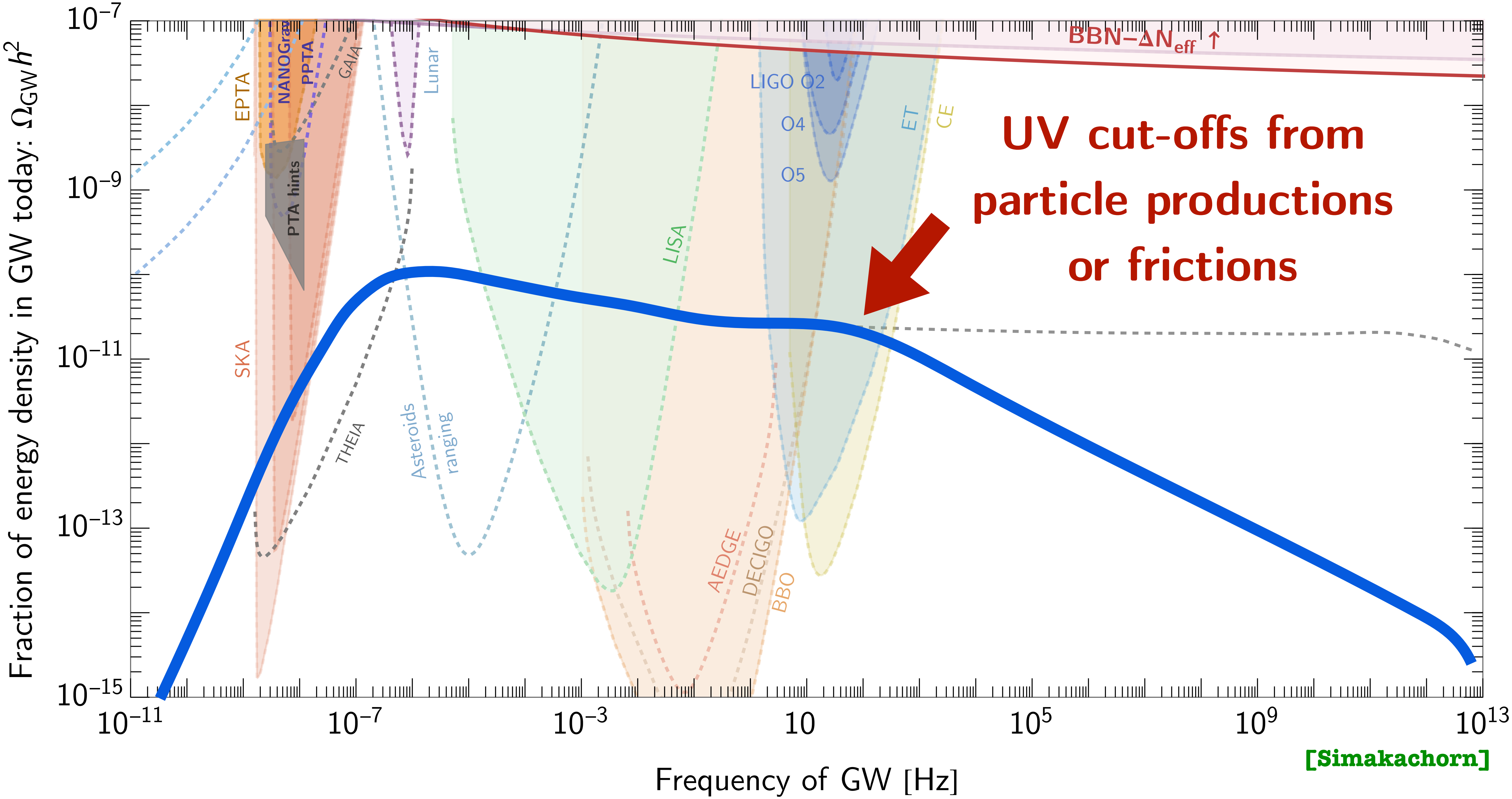
Network formation

We can do both local and global strings.

$G\mu = 10^{-10}$

[Simakachorn]

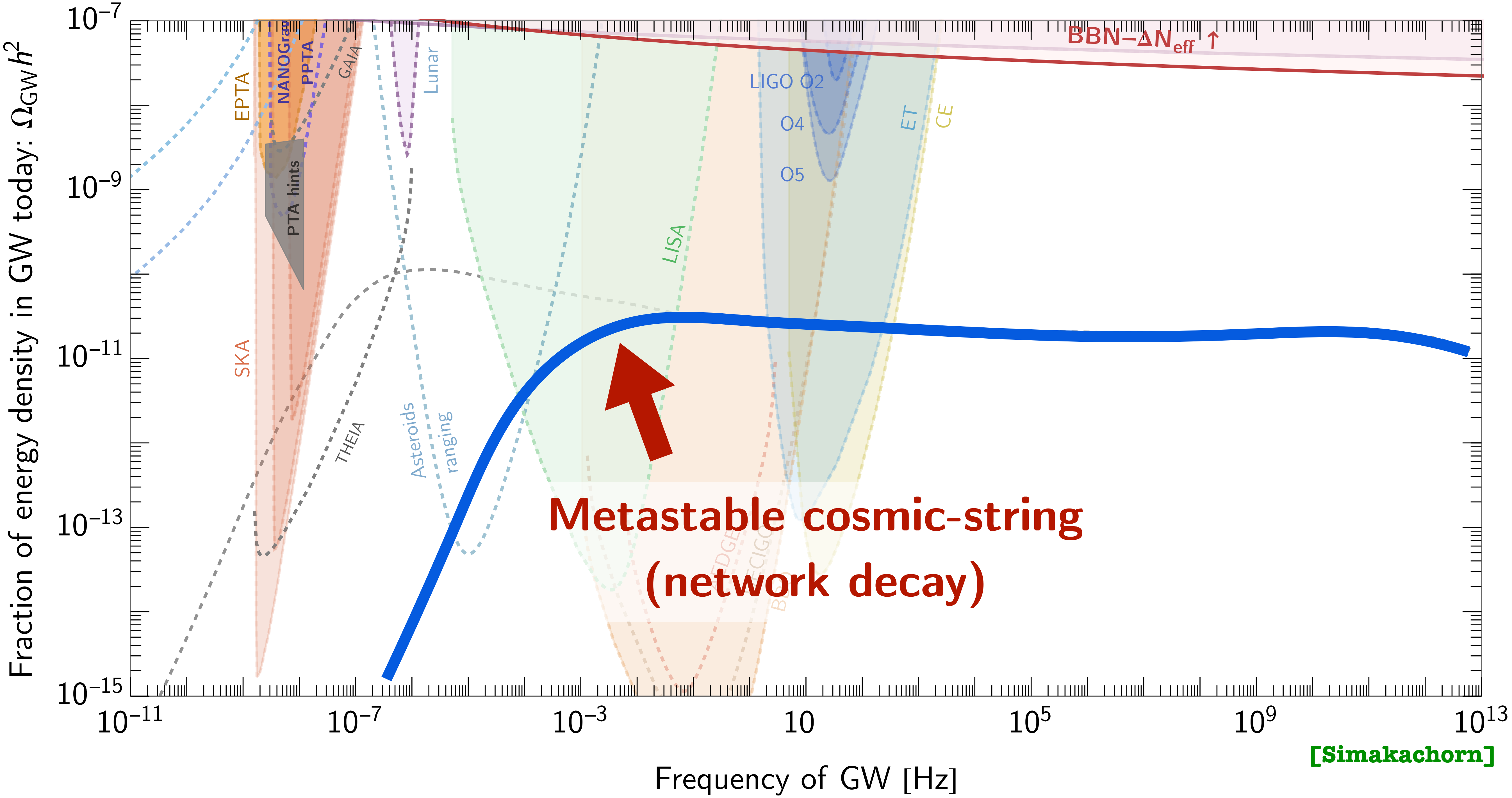
Gravitational Waves from cosmic strings.



UV cut-offs from particle productions or frictions

[Simakachorn]

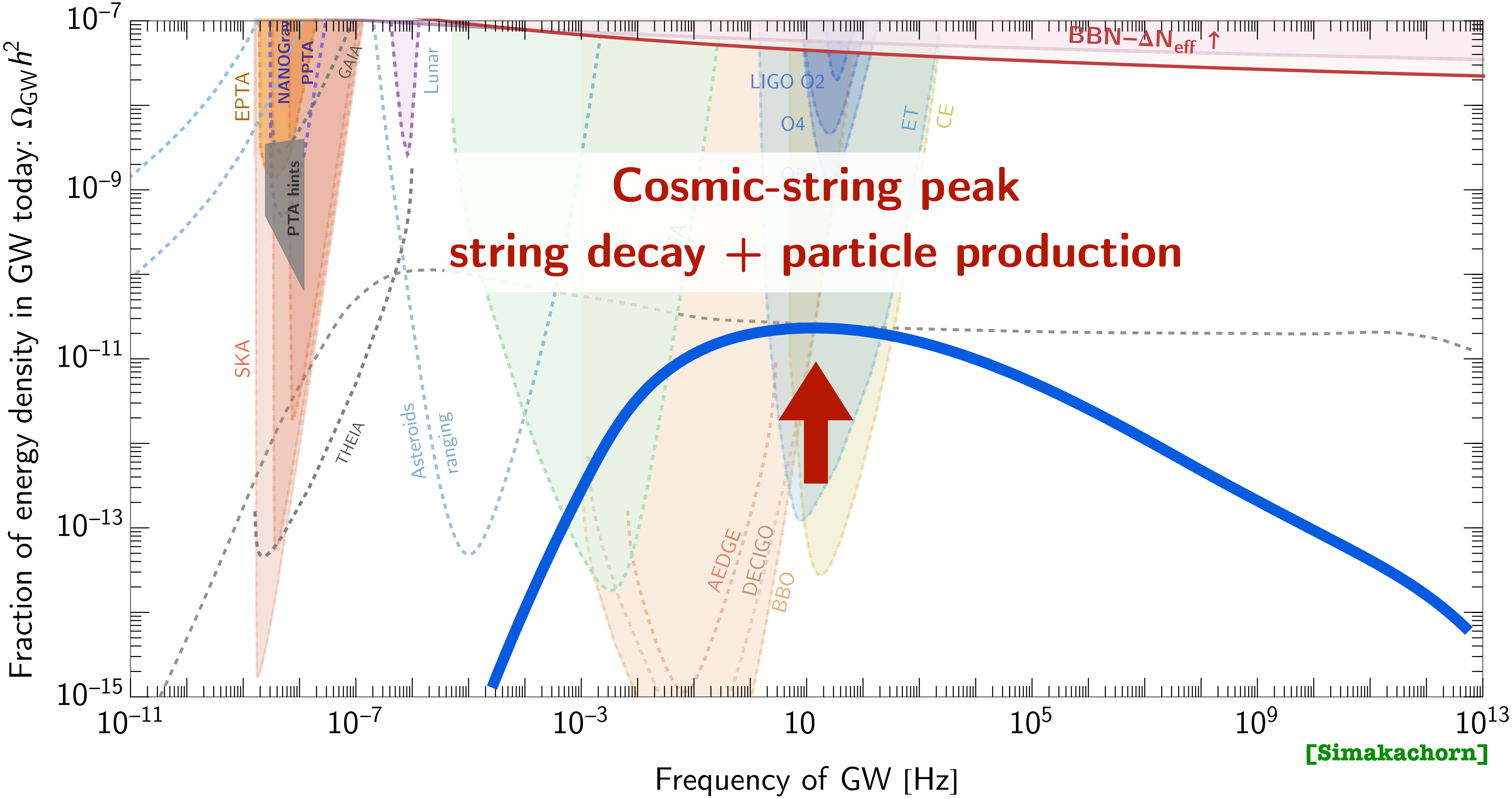
Gravitational Waves from cosmic strings.



Metastable cosmic-string (network decay)

[Simakachorn]

Gravitational Waves from cosmic strings.



[Simakachorn]

Part 2 of lecture 1:
Probing the axion through its cosmology.

Axions

Among the most hunted particles.

Ubiquitous in many extensions of the Standard Model

Axion could arise either as a higher dimensional gauge field, or as a Pseudo- Nambu Goldstone boson (PNGB) from spontaneous breaking of global symmetry which is not exact but broken weakly.

I will assume the second possibility as a simple benchmark. Important for cosmology: Axion is accompanied by its partner, the radial mode of a complex scalar field.

Axion mass is proportional to this breaking.

Very general context.

Historically: QCD axion. Strong dynamics from QCD provides breaking of symmetry.

Axion-like-particles (ALPs): other axions whose mass is not affected by QCD. They get their mass from other sources.

particularly motivated by Strong CP problem

Strong CP pb:

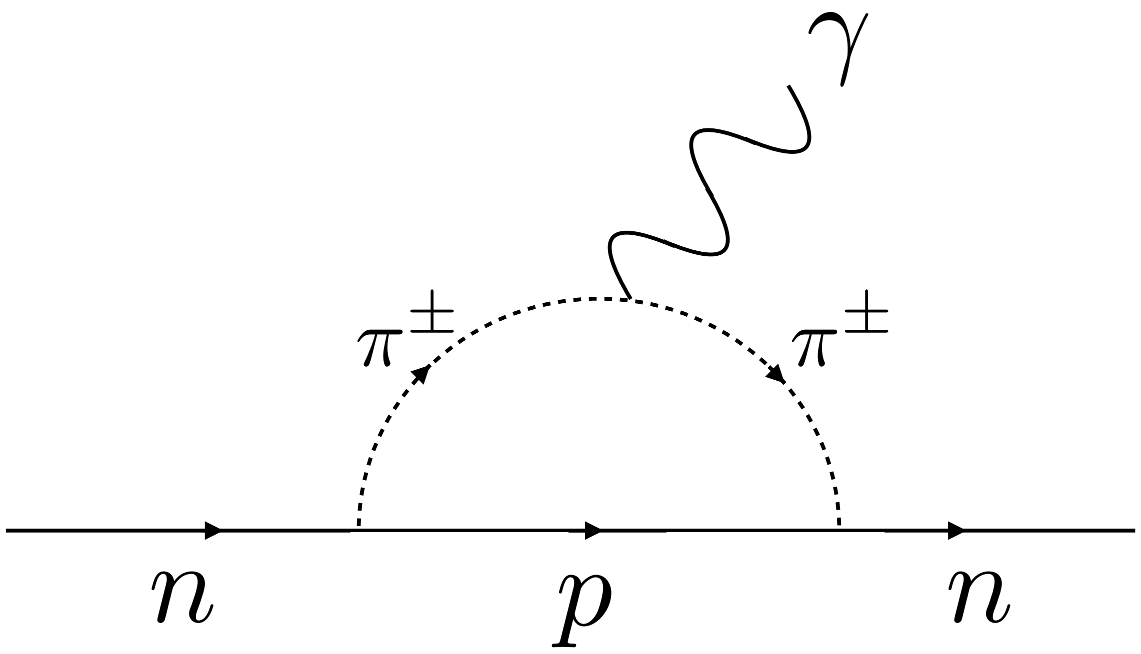
Why is the neutron electric dipole moment (EDM) so small?

$$\mathcal{L} \supset d_n F_{\mu\nu} \overline{n} \gamma^{\mu\nu} i\gamma_5 n.$$

$$iM = 2 d_n \epsilon^*_\mu(q) \overline{u}(p') \gamma^{\mu\nu} q_\nu i\gamma_5 u(p).$$

$$d_n \sim 3 \times 10^{-16}\, \overline{\theta}\, \mathrm{e\ cm}.$$

where $\mathcal{L}_{QCD} \supset -\frac{\overline{\theta}}{32\pi^2} G\tilde{G}$



comparing to the experimental bound leads to

$$\overline{\theta} \lesssim 10^{-10}$$

can be beautifully solved by introducing an axion, see L. Alvarez-Gaume’s lectures.

References on axions

Some recent references for reviews

-TASI Lectures on the Strong CP Problem and Axions,
Anson Hook, https://arxiv.org/abs/1812.02669

- ICTP summer school 2015, 3 lectures by Surjeet Rajendran
http://indico.ictp.it/event/a14276/session/27/contribution/110/material/slides/0.pdf
http://indico.ictp.it/event/a14276/session/28/contribution/115/material/slides/0.pdf
http://indico.ictp.it/event/a14276/session/29/contribution/119/material/slides/0.pdf

- 2015 GGI lectures by G. Villadoro:
https://www.ggi.infn.it/ggilectures/ggilectures2015/program.html
https://www.youtube.com/watch?v=Bpund1fndCg&list=PLDxsZU4NC6Z4kL18PhWTeHicRP13OfHYI&index=1

-Review “The landscape of QCD axion models“, Di Luzio et al.
https://arxiv.org/pdf/2003.01100.pdf

- **Review by Redondo and Irastorza**
  “New experimental approaches in the search for axion-like particles”
  https://arxiv.org/pdf/1801.08127.pdf

- A. Pich on chiral perturbation theory:
https://arxiv.org/pdf/hep-ph/9502366.pdf
(useful to compute the scalar potential as a function of theta angle)

Axion-Like-Particles (ALPs).

Consider complex scalar field

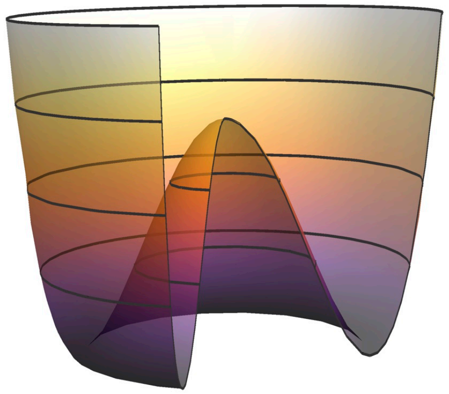
$$\Phi = \phi e^{i\theta}$$

charged under anomalous U(1) global symmmetry (Peccei-Quinn symmetry)

Spontaneously broken at scale f_a

$$V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2$$

$$\langle \varphi \rangle = f_a/\sqrt{2}$$



Axion as Goldstone boson

$$\theta \rightarrow \theta + \mathrm{const.}$$

$$\boxed{\theta = a / f_a}$$

ALPs.

Non-perturbative effects at energy $\Lambda_b \ll f_a$ break the shift symmetry and generate a potential/mass for the axion

$$V = m_a^2(T) f_a^2 \left[1 - \cos(\theta)\right]$$

$$m_a = \Lambda_b^2 / f_a$$

QCD axion

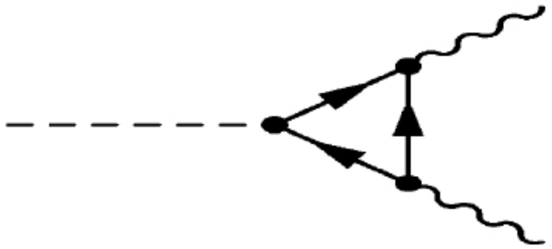
$$m_a^2 f_a^2 \approx (76\ \mathrm{MeV})^4$$

Generic ALP

m_a and f_a : free parameters

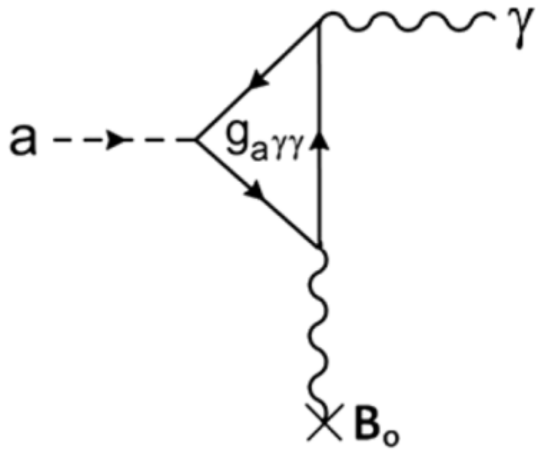
The hunt for axions.

Mainly through Axion-photon coupling



$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

In a background magnetic field:
axion<->photon conversion

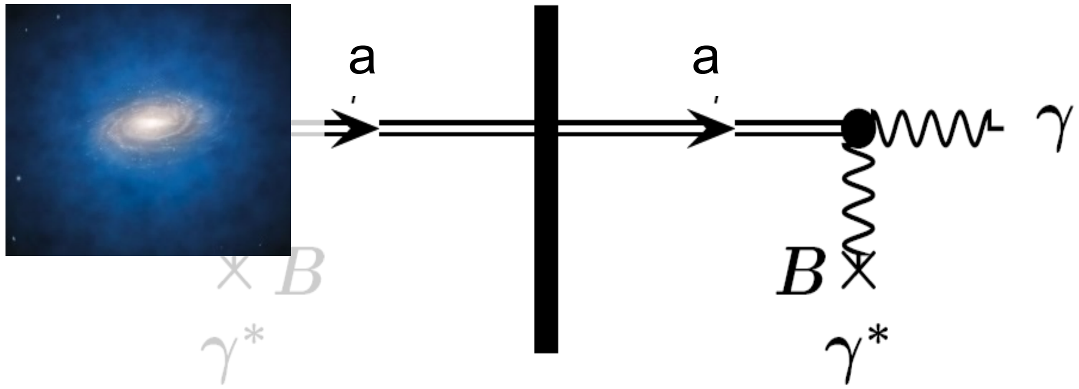


If long-lived: Dark Matter candidate

Lifetime depends on axion-photon coupling.
However, relic abundance only depends on f_a

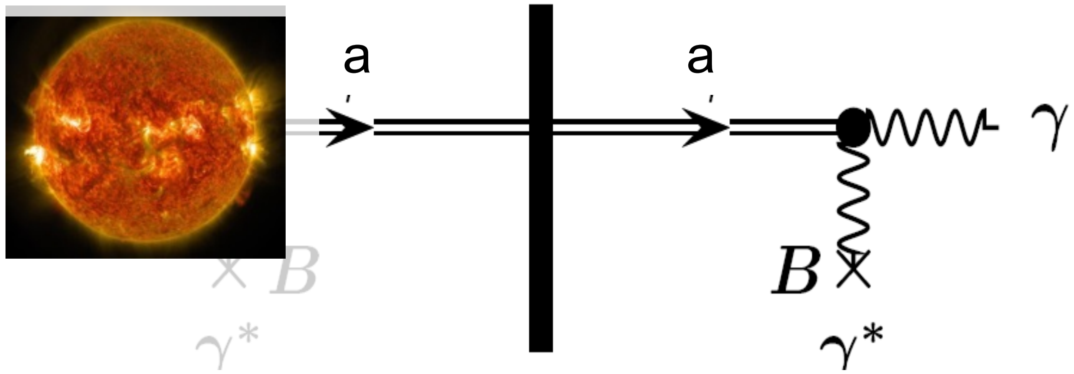
Three main ways to search for ALPs.

All rely on ALP-photon mixing in magnetic field

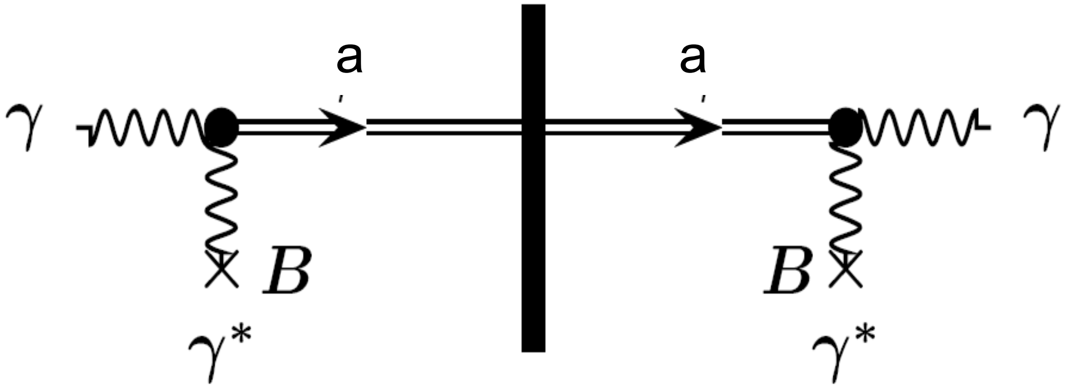


Haloscopes

looking for dark matter constituents, microwaves



Helioscopes
Axions emitted by the sun, X-rays



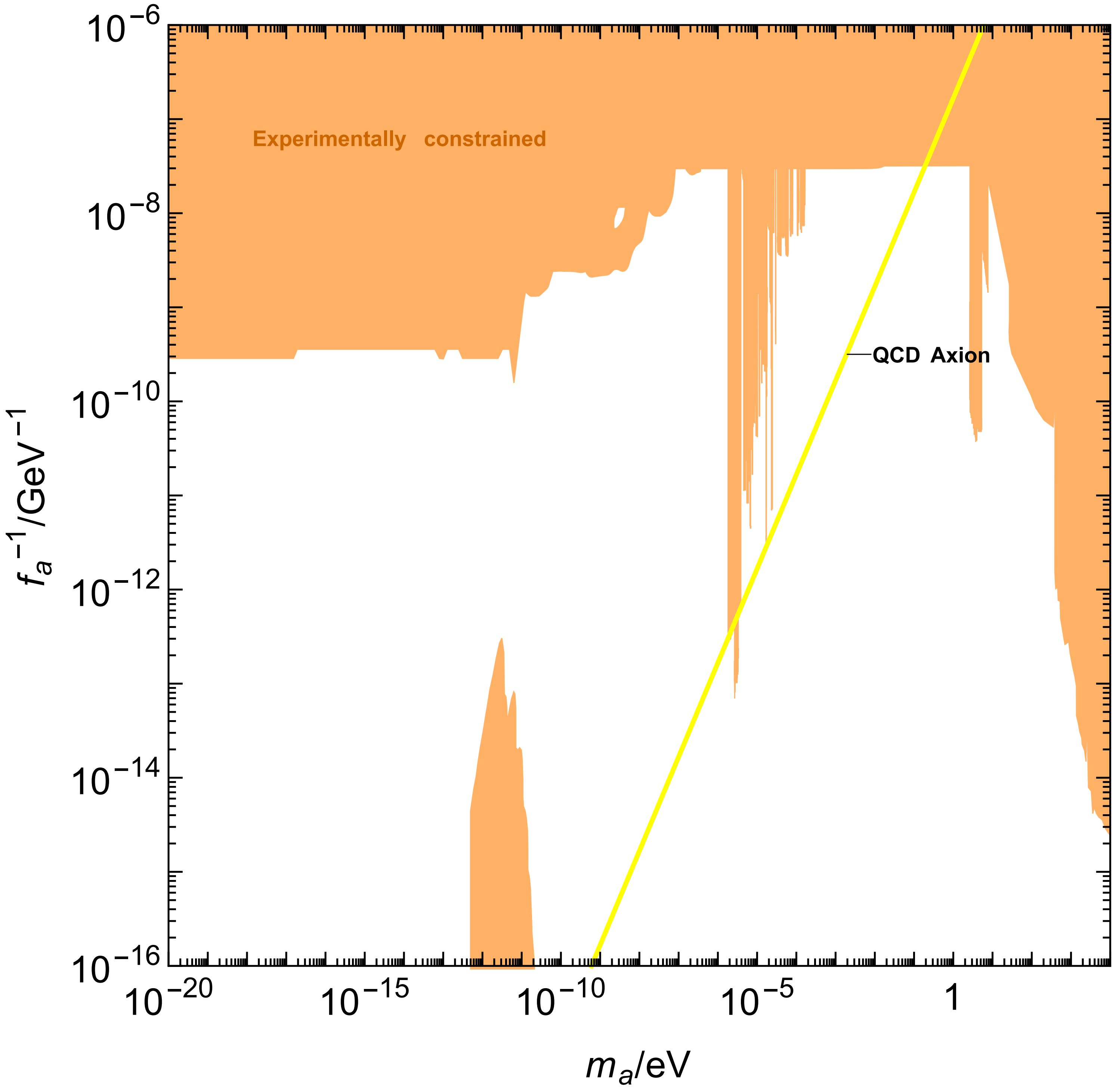
Purely laboratory experiments

“light-shining-through-walls”,

microwaves, optical photons

The Axion-Like-Particle (ALP) parameter space.

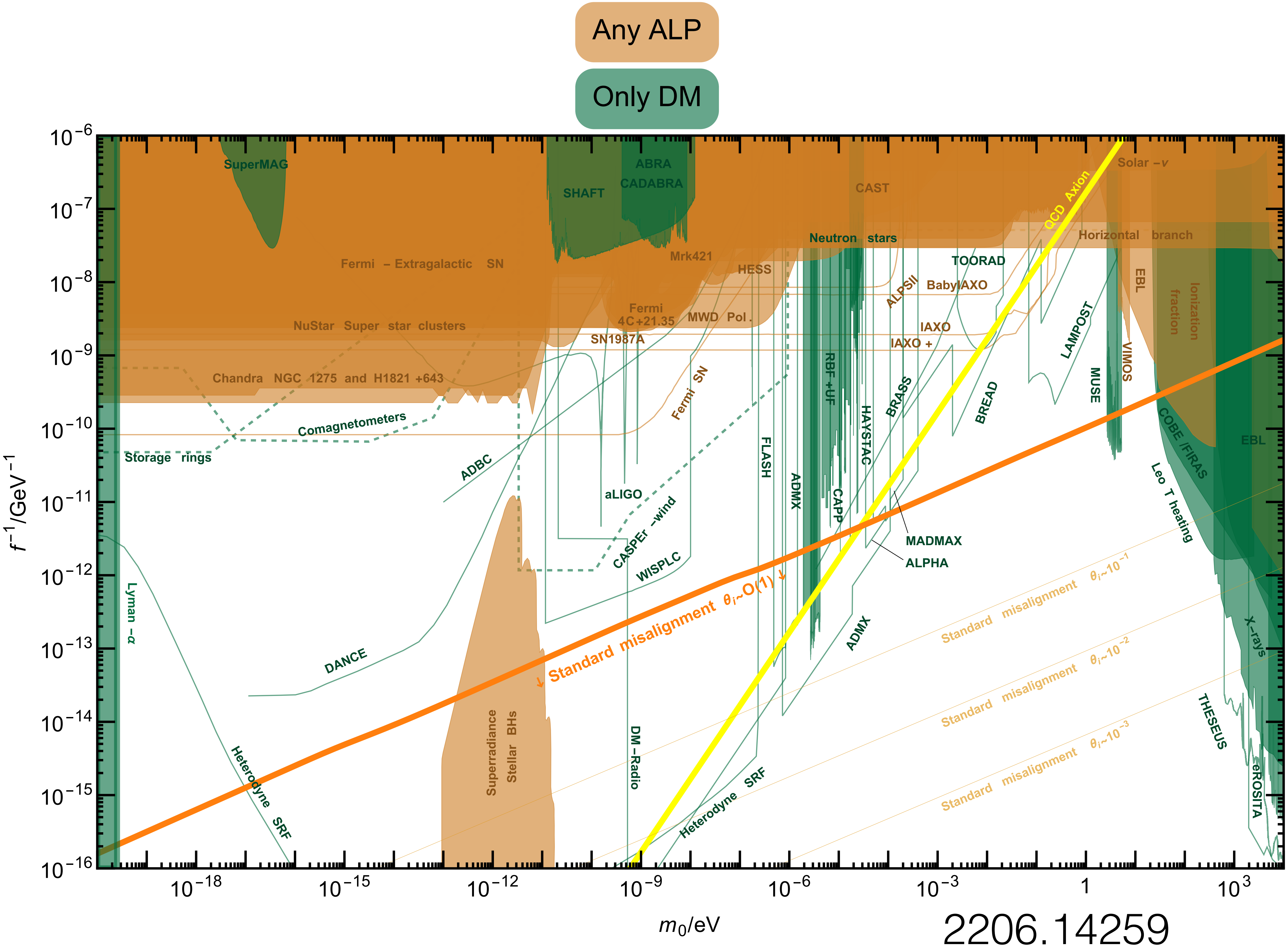
If axions are given an interaction to photons then a long list of constraints from ALP searches apply



$$\frac{f_\gamma}{f_a} \approx 0.5 \times 10^3$$

assuming KSVZ-like coupling

The hunt for axions.



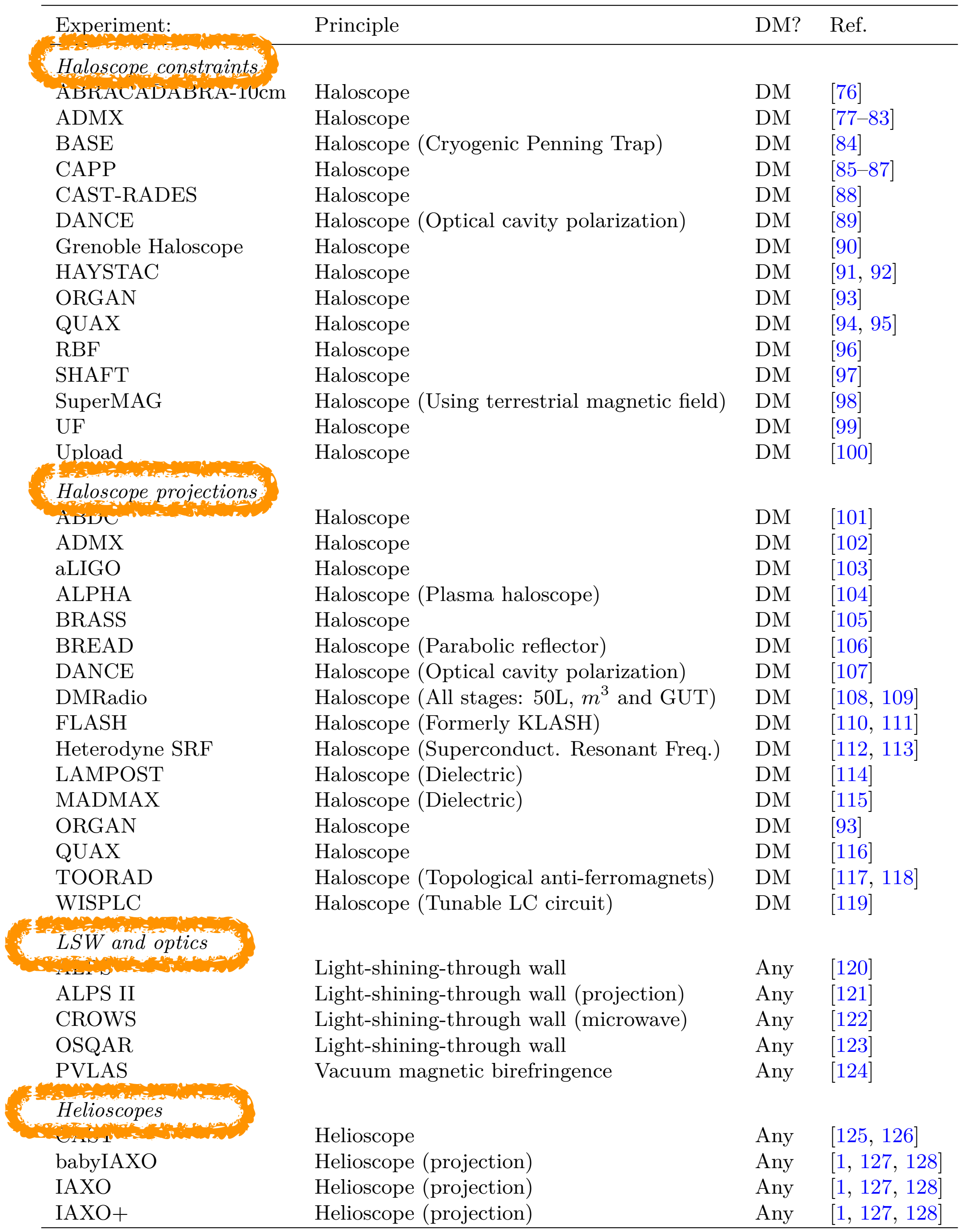
2206.14259

A whole set of experiment constraints.

All data can be found here:

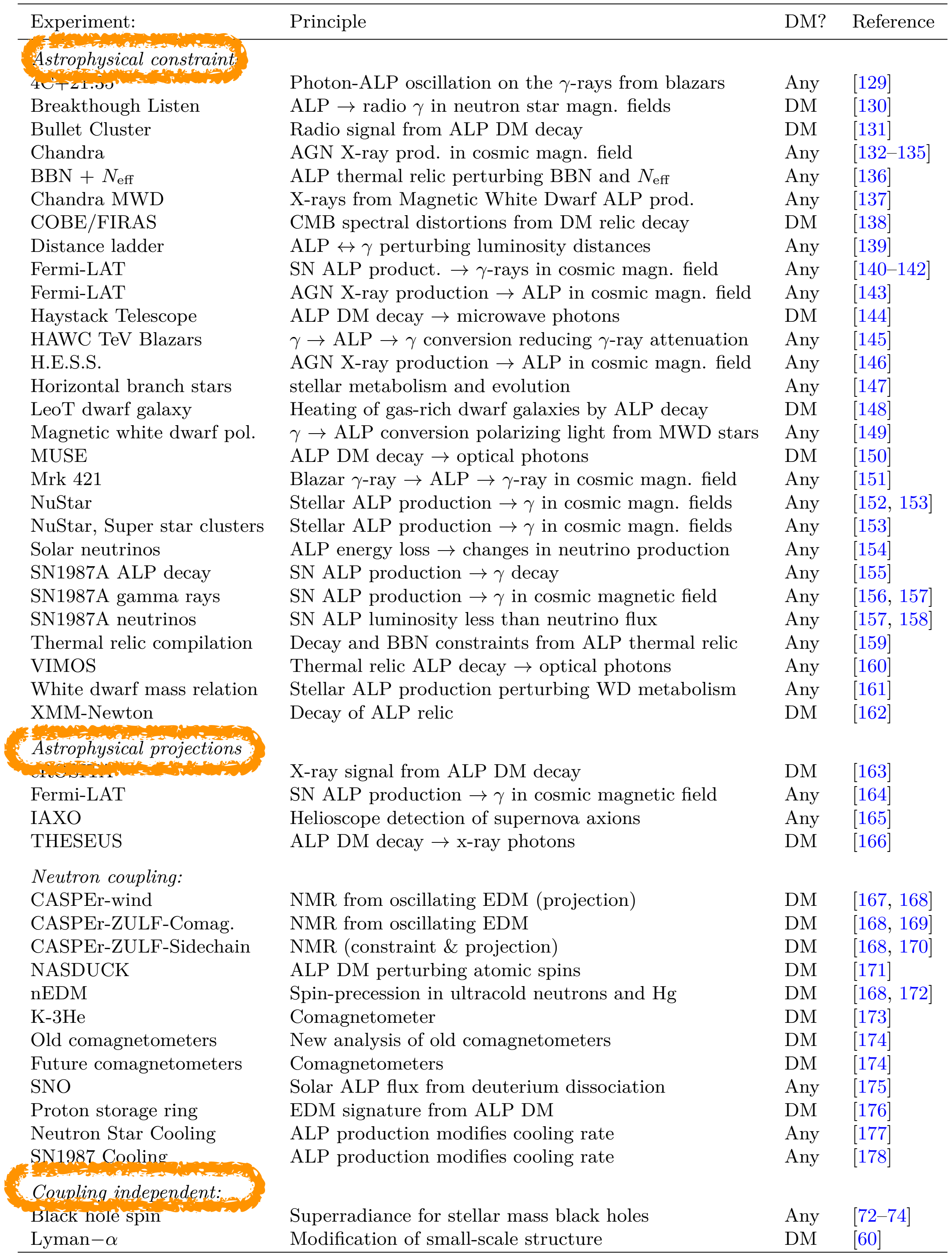
C. O'Hare, *cajohare/axionlimits: Axionlimits*, https://cajohare.github.io/AxionLimits/ (2020) [10.5281/zenodo.3932430].

All experiments also listed in tables 1 and 2 of 2206.14259:



Experiment:	Principle	DM?	Ref.
Haloscope constraints			
ABRACADABRA-10cm	Haloscope	DM	[76]
ADMX	Haloscope	DM	[77–83]
BASE	Haloscope (Cryogenic Penning Trap)	DM	[84]
CAPP	Haloscope	DM	[85–87]
CAST-RADES	Haloscope	DM	[88]
DANCE	Haloscope (Optical cavity polarization)	DM	[89]
Grenoble Haloscope	Haloscope	DM	[90]
HAYSTAC	Haloscope	DM	[91, 92]
ORGAN	Haloscope	DM	[93]
QUAX	Haloscope	DM	[94, 95]
RBF	Haloscope	DM	[96]
SHAFT	Haloscope	DM	[97]
SuperMAG	Haloscope (Using terrestrial magnetic field)	DM	[98]
UF	Haloscope	DM	[99]
Upload	Haloscope	DM	[100]
Haloscope projections			
ABDC	Haloscope	DM	[101]
ADMX	Haloscope	DM	[102]
aLIGO	Haloscope	DM	[103]
ALPHA	Haloscope (Plasma haloscope)	DM	[104]
BRASS	Haloscope	DM	[105]
BREAD	Haloscope (Parabolic reflector)	DM	[106]
DANCE	Haloscope (Optical cavity polarization)	DM	[107]
DMRadio	Haloscope (All stages: 50L, m^3 and GUT)	DM	[108, 109]
FLASH	Haloscope (Formerly KLASH)	DM	[110, 111]
Heterodyne SRF	Haloscope (Superconduct. Resonant Freq.)	DM	[112, 113]
LAMPOST	Haloscope (Dielectric)	DM	[114]
MADMAX	Haloscope (Dielectric)	DM	[115]
ORGAN	Haloscope	DM	[93]
QUAX	Haloscope	DM	[116]
TOORAD	Haloscope (Topological anti-ferromagnets)	DM	[117, 118]
WISPLC	Haloscope (Tunable LC circuit)	DM	[119]
LSW and optics			
ALPS	Light-shining-through wall	Any	[120]
ALPS II	Light-shining-through wall (projection)	Any	[121]
CROWS	Light-shining-through wall (microwave)	Any	[122]
OSQAR	Light-shining-through wall	Any	[123]
PVLAS	Vacuum magnetic birefringence	Any	[124]
Helioscopes			
CAST	Helioscope	Any	[125, 126]
babyIAXO	Helioscope (projection)	Any	[1, 127, 128]
IAXO	Helioscope (projection)	Any	[1, 127, 128]
IAXO+	Helioscope (projection)	Any	[1, 127, 128]

Table 1. List of experimental searches for axions and ALPs. The table is continued in table 2. All experiments here rely on the axion-photon coupling.



Experiment:	Principle	DM?	Reference
Astrophysical constraints			
4C+21.35	Photon-ALP oscillation on the γ-rays from blazars	Any	[129]
Breakthough Listen	ALP → radio γ in neutron star magn. fields	DM	[130]
Bullet Cluster	Radio signal from ALP DM decay	DM	[131]
Chandra	AGN X-ray prod. in cosmic magn. field	Any	[132–135]
BBN + $N_{\rm eff}$	ALP thermal relic perturbing BBN and $N_{\rm eff}$	Any	[136]
Chandra MWD	X-rays from Magnetic White Dwarf ALP prod.	Any	[137]
COBE/FIRAS	CMB spectral distortions from DM relic decay	DM	[138]
Distance ladder	ALP ↔ γ perturbing luminosity distances	Any	[139]
Fermi-LAT	SN ALP product. → γ-rays in cosmic magn. field	Any	[140–142]
Fermi-LAT	AGN X-ray production → ALP in cosmic magn. field	Any	[143]
Haystack Telescope	ALP DM decay → microwave photons	DM	[144]
HAWC TeV Blazars	γ → ALP → γ conversion reducing γ-ray attenuation	Any	[145]
H.E.S.S.	AGN X-ray production → ALP in cosmic magn. field	Any	[146]
Horizontal branch stars	stellar metabolism and evolution	Any	[147]
LeoT dwarf galaxy	Heating of gas-rich dwarf galaxies by ALP decay	DM	[148]
Magnetic white dwarf pol.	γ → ALP conversion polarizing light from MWD stars	Any	[149]
MUSE	ALP DM decay → optical photons	DM	[150]
Mrk 421	Blazar γ-ray → ALP → γ-ray in cosmic magn. field	Any	[151]
NuStar	Stellar ALP production → γ in cosmic magn. fields	Any	[152, 153]
NuStar, Super star clusters	Stellar ALP production → γ in cosmic magn. fields	Any	[153]
Solar neutrinos	ALP energy loss → changes in neutrino production	Any	[154]
SN1987A ALP decay	SN ALP production → γ decay	Any	[155]
SN1987A gamma rays	SN ALP production → γ in cosmic magnetic field	Any	[156, 157]
SN1987A neutrinos	SN ALP luminosity less than neutrino flux	Any	[157, 158]
Thermal relic compilation	Decay and BBN constraints from ALP thermal relic	Any	[159]
VIMOS	Thermal relic ALP decay → optical photons	Any	[160]
White dwarf mass relation	Stellar ALP production perturbing WD metabolism	Any	[161]
XMM-Newton	Decay of ALP relic	DM	[162]
Astrophysical projections			
eROSITA	X-ray signal from ALP DM decay	DM	[163]
Fermi-LAT	SN ALP production → γ in cosmic magnetic field	Any	[164]
IAXO	Helioscope detection of supernova axions	Any	[165]
THESEUS	ALP DM decay → x-ray photons	DM	[166]
Neutron coupling:			
CASPEr-wind	NMR from oscillating EDM (projection)	DM	[167, 168]
CASPEr-ZULF-Comag.	NMR from oscillating EDM	DM	[168, 169]
CASPEr-ZULF-Sidechain	NMR (constraint & projection)	DM	[168, 170]
NASDUCK	ALP DM perturbing atomic spins	DM	[171]
nEDM	Spin-precession in ultracold neutrons and Hg	DM	[168, 172]
K-3He	Comagnetometer	DM	[173]
Old comagnetometers	New analysis of old comagnetometers	DM	[174]
Future comagnetometers	Comagnetometers	DM	[174]
SNO	Solar ALP flux from deuterium dissociation	Any	[175]
Proton storage ring	EDM signature from ALP DM	DM	[176]
Neutron Star Cooling	ALP production modifies cooling rate	Any	[177]
SN1987 Cooling	ALP production modifies cooling rate	Any	[178]
Coupling independent:			
Black hole spin	Superradiance for stellar mass black holes	Any	[72–74]
Lyman$-\alpha$	Modification of small-scale structure	DM	[60]

Table 2. List of experimental searches for axions and ALPs.

Which of these axions can make Dark Matter ?

First, let us ask the question:

How light can the dark matter particle be?

Lower bound on Dark Matter Mass

Dark Matter must behave classically to be confined on galaxy scales.
DM with De Broglie wavelength > size of dwarf galaxies ~ kpc
will prevent their formation

We demand $\lambda <$ kpc -> $m\, v > 1/$ kpc

$1\ \mathrm{pc} = 3\times10^{18}\ \mathrm{cm} = 3\times10^{18}\ /\ (2\times10^{-14}\ \mathrm{GeV}) = 10^{32}\ \mathrm{GeV}^{-1} = (10^{-32}\ \mathrm{GeV})^{-1}$

$1\ \mathrm{kpc}^{-1} = 10^{-35}\ \mathrm{GeV} = 10^{-26}\ \mathrm{eV}$

$v \sim 10^{-3}$

$mv \sim m\ 10^{-3}$

$m_{DM} \gtrsim 10^{-23}\ \mathrm{eV}$

More stringent bound for fermionic Dark Matter

Pauli exclusion principle.
Phase space density for fermions has a maximum value,

$$M_{\rm halo} = mV \int d^3p f(p) < mV \int d^3p < mV(mv)^3$$

$$v \sim \sqrt{\frac{GM_{\rm halo}}{r_{\rm halo}}}$$

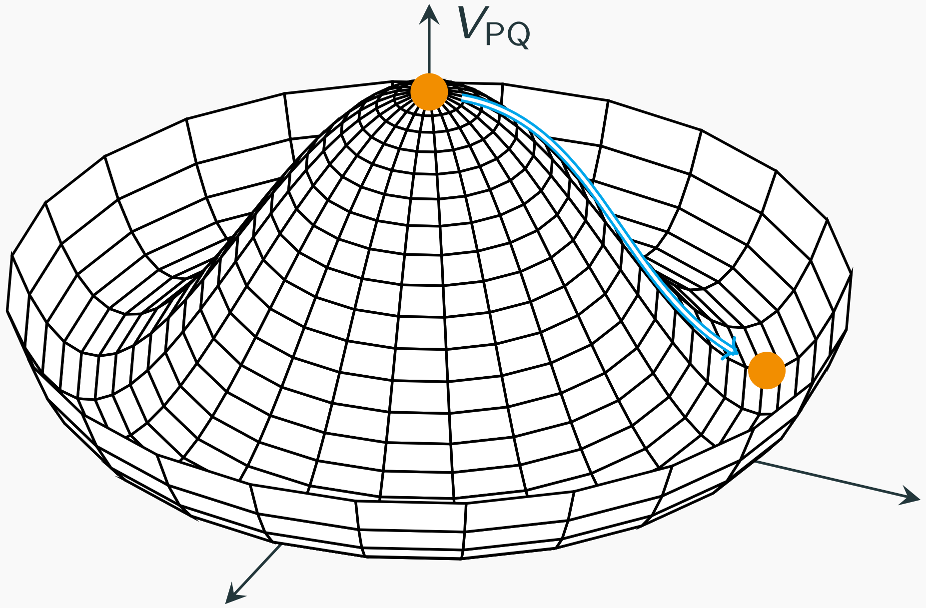
$$M_{\rm halo} < R_{\rm halo}^3 m^4 \left(\frac{GM_{\rm halo}}{R_{\rm halo}}\right)^{3/2}$$

$$m > \frac{1}{(G^3 R_{\rm halo}^3 M_{\rm halo})^{1/8}}$$

for dwarf galaxies: m> 0.7 keV

Pre- and post-inflationary scenarios.

Potential of full complex PQ scalar field



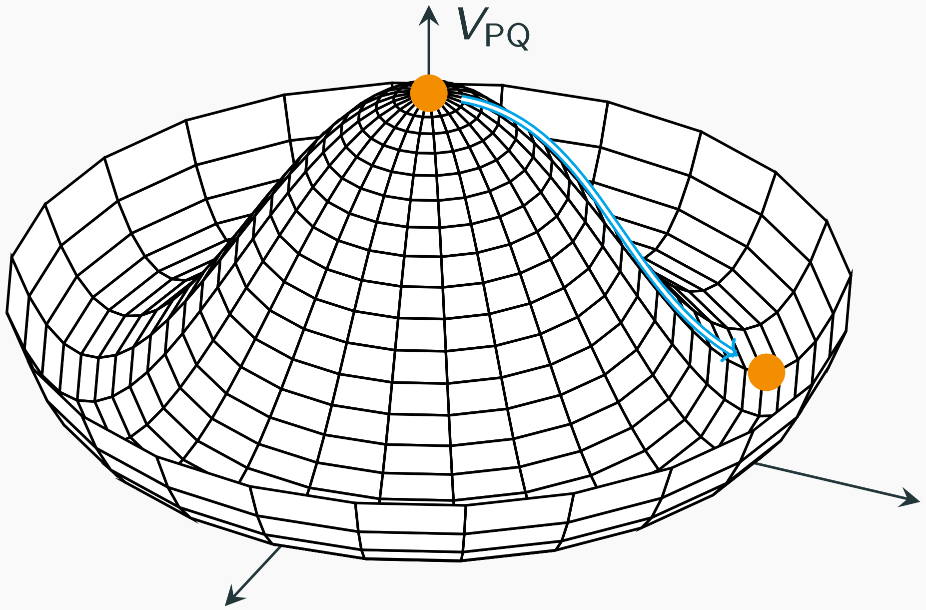
Post-inflationary scenario

- Different initial angle in each Hubble patch.
- Inhomogeneous including topological defects.

Pre-inflationary scenario

- Random initial angle in the observable universe.
- Initially homogeneous w/o topological defects.

Pre- and post-inflationary scenarios.



Post-inflationary scenario

- Different initial angle in each Hubble patch.
- Inhomogeneous including topological defects.

Pre-inflationary scenario

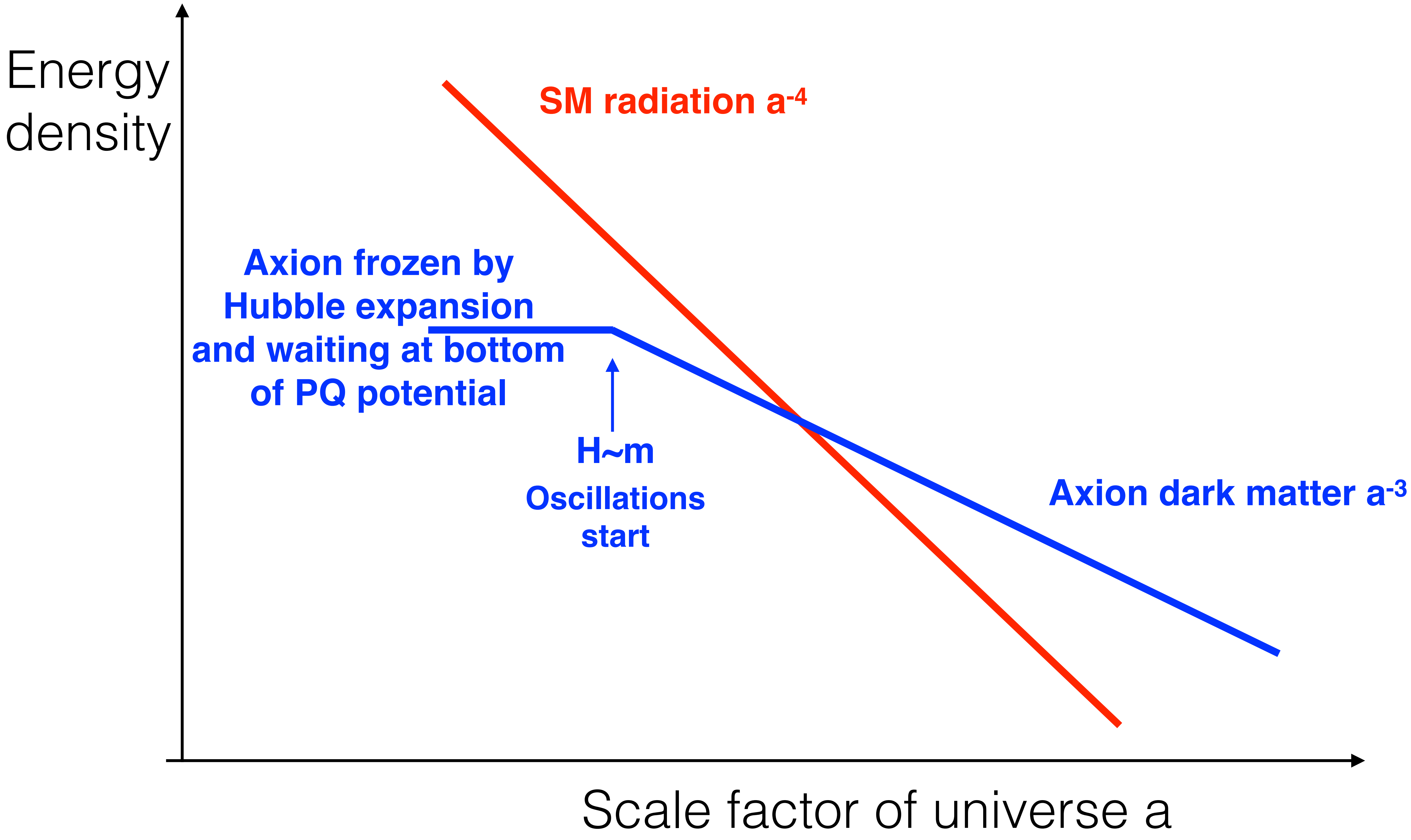
- Random initial angle in the observable universe.
- Initially homogeneous w/o topological defects.

GLOBAL (axionic) COSMIC STRINGS

primordial GW bgd

Usual story.

(Most axion cosmology literature is about the rather late cosmology from moment axion gets a mass)



Axions from the misalignment mechanism.

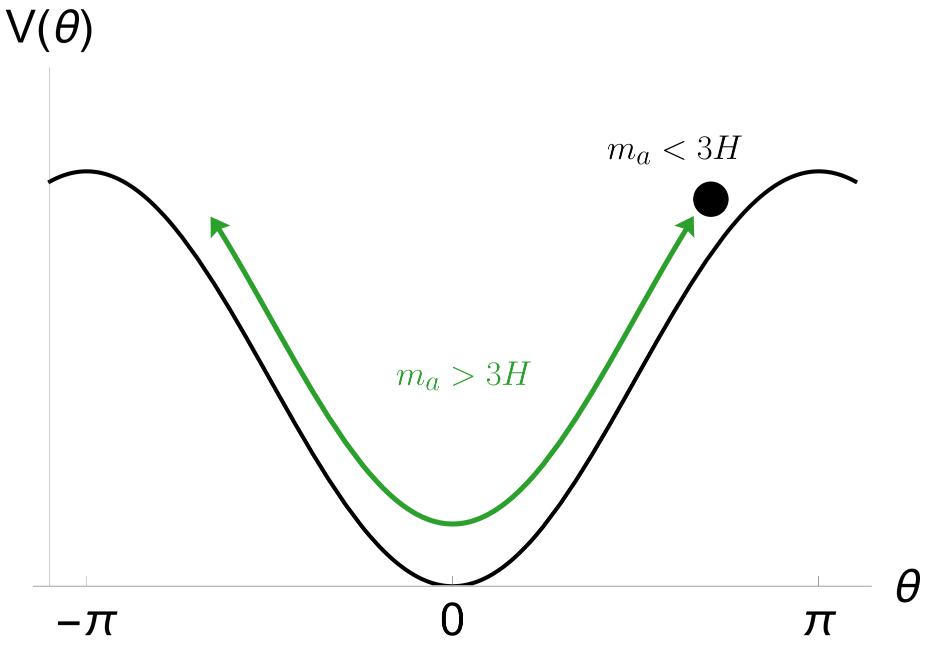
Axion late cosmology

Neglecting fluctuations, the homogeneous zero-mode satisfies

$$\ddot{\Theta} + 3H\dot{\Theta} + m_{\mathrm{a}}^2(T)\sin(\Theta) = 0,$$

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

With initial conditions: $\Theta(t_i) = \Theta_i,\ \dot{\Theta}(t_i) = 0.$ ***standard assumption***



- $m_a \ll 3H \iff \rho_a \propto a^0$ (Frozen)
- $m_a \gg 3H \iff \rho_a \propto a^{-3}$ (Oscillating)

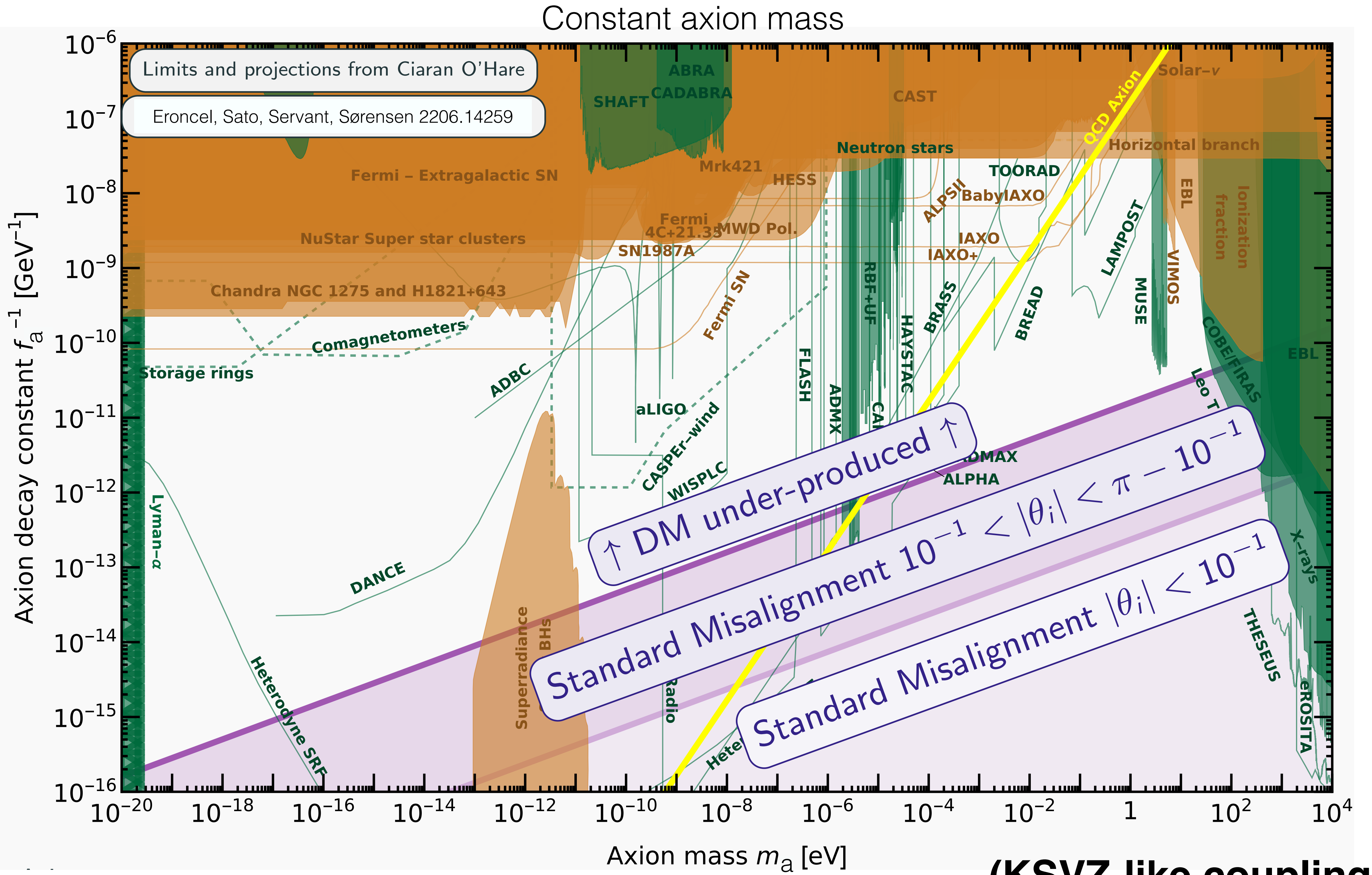
—> standard misalignment mechanism

For $\Theta_i \sim 1$ $\rho_{\mathrm{DM}} \sim \rho_{osc}\left(\frac{a_{osc}}{a_0}\right)^3 \sim m_a^2 f_a^2 \left(\frac{T_0}{T_{osc}}\right)^3$

$$T_{osc} \sim \sqrt{m_a M_{Pl}}$$

ρ_{DM} grows with f_a —> Axion Dark Matter overabundance for too large f_a

Conventional misalignement makes too little DM for low f_a



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\rm em}/2\pi)(1.92/f)$$

A way out: switch on initial velocity for the axion

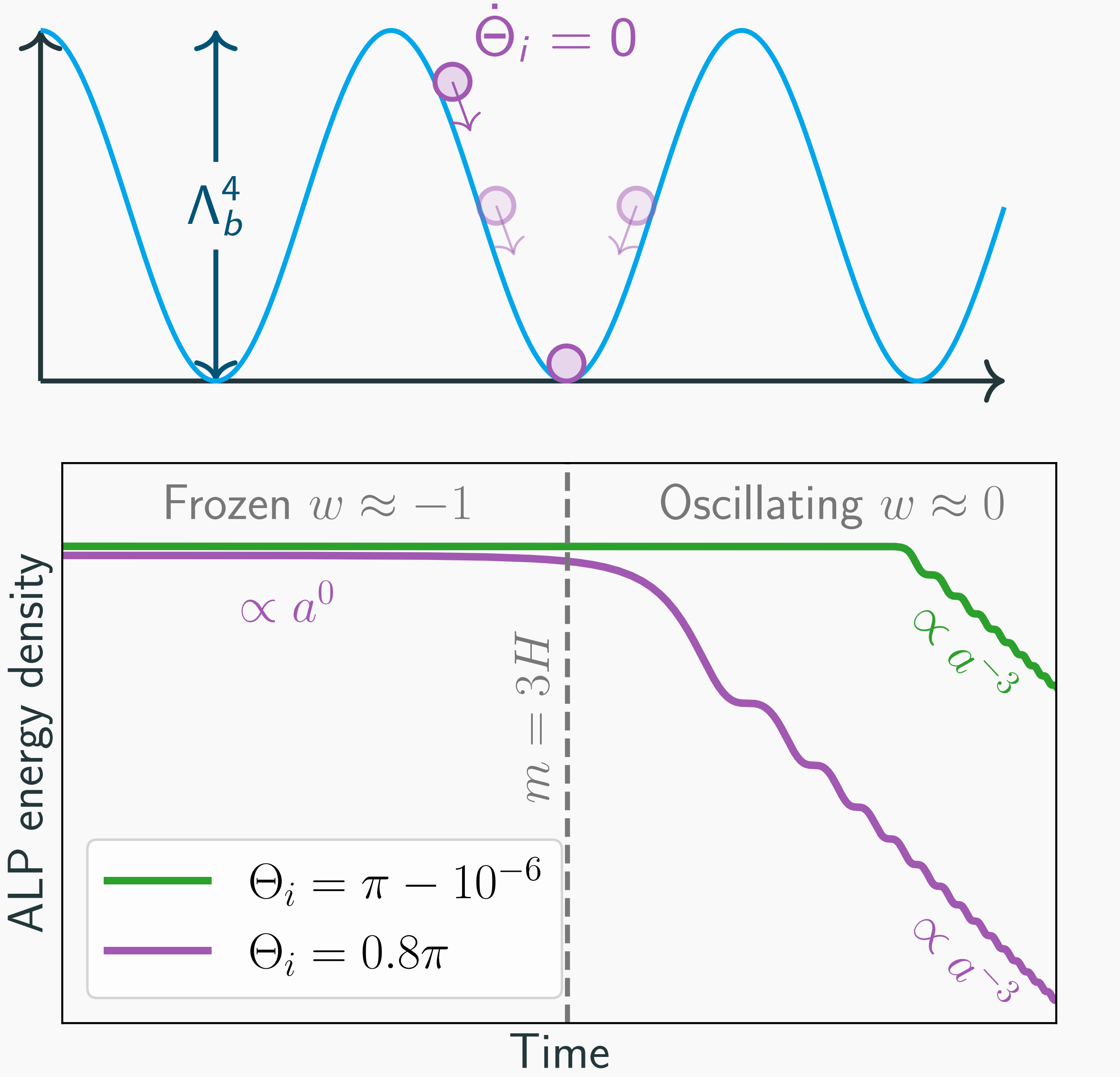
Standard versus kinetic Misalignment.

Two ways to delay the onset of oscillations

Initial field value tuned to top of potential:

Standard (Large) misalignment

Zhang,Chiueh 1705.01439; Arvanitaki et al. 1909.11665



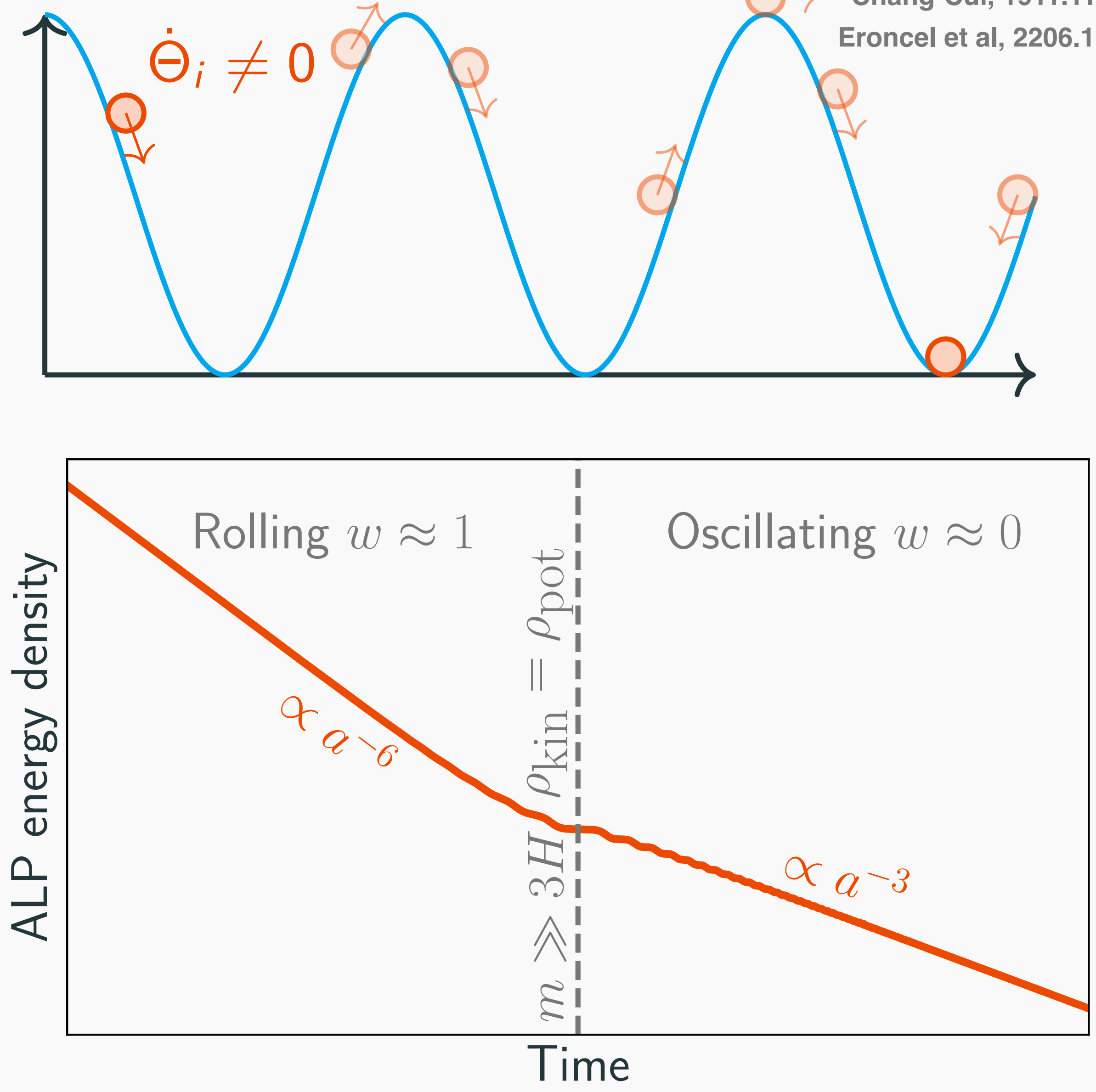
Large initial velocity

Kinetic misalignment

Co et al. 1910.14152

Chang-Cui, 1911.11885

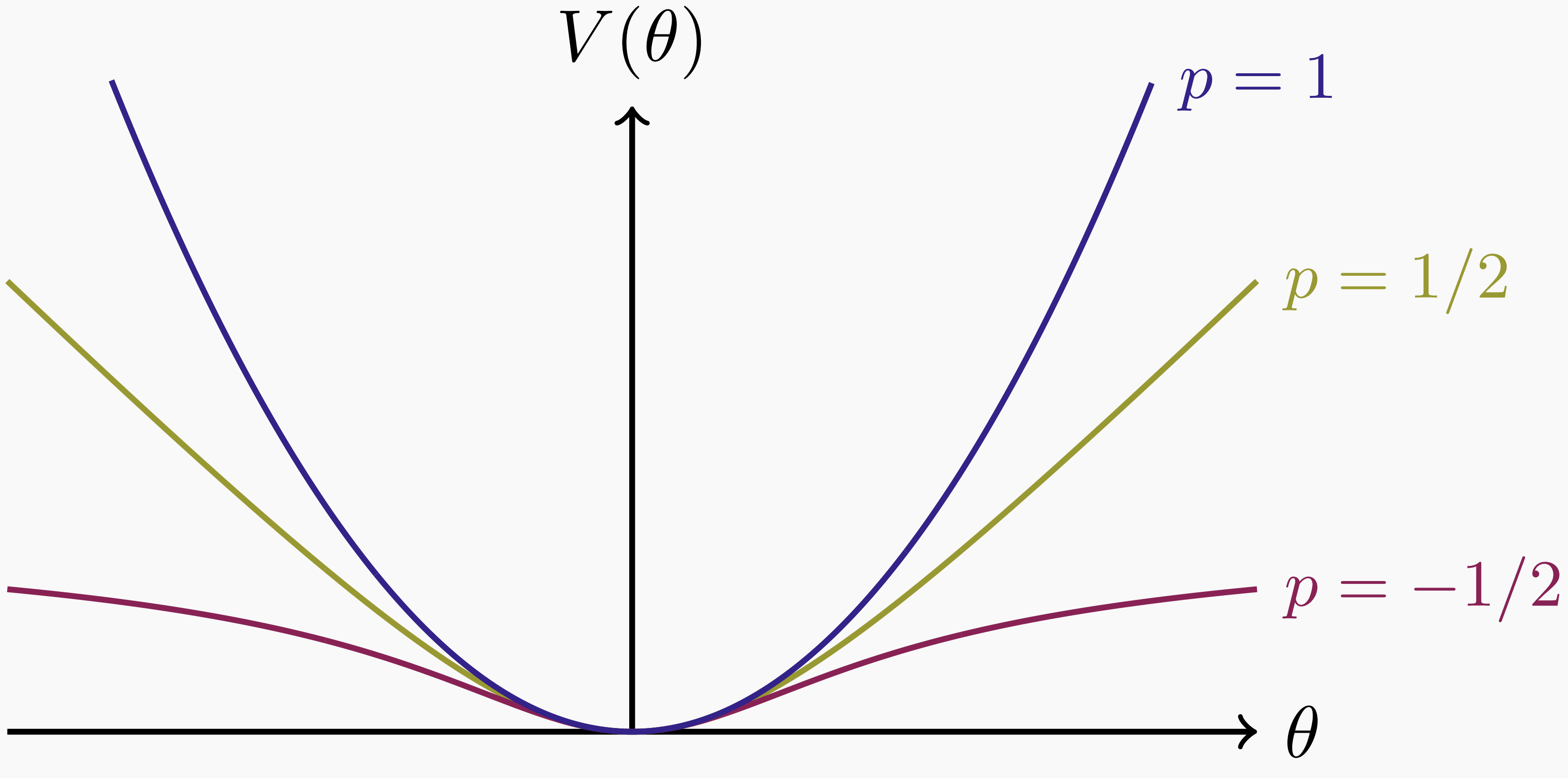
Eroncel et al, 2206.14259



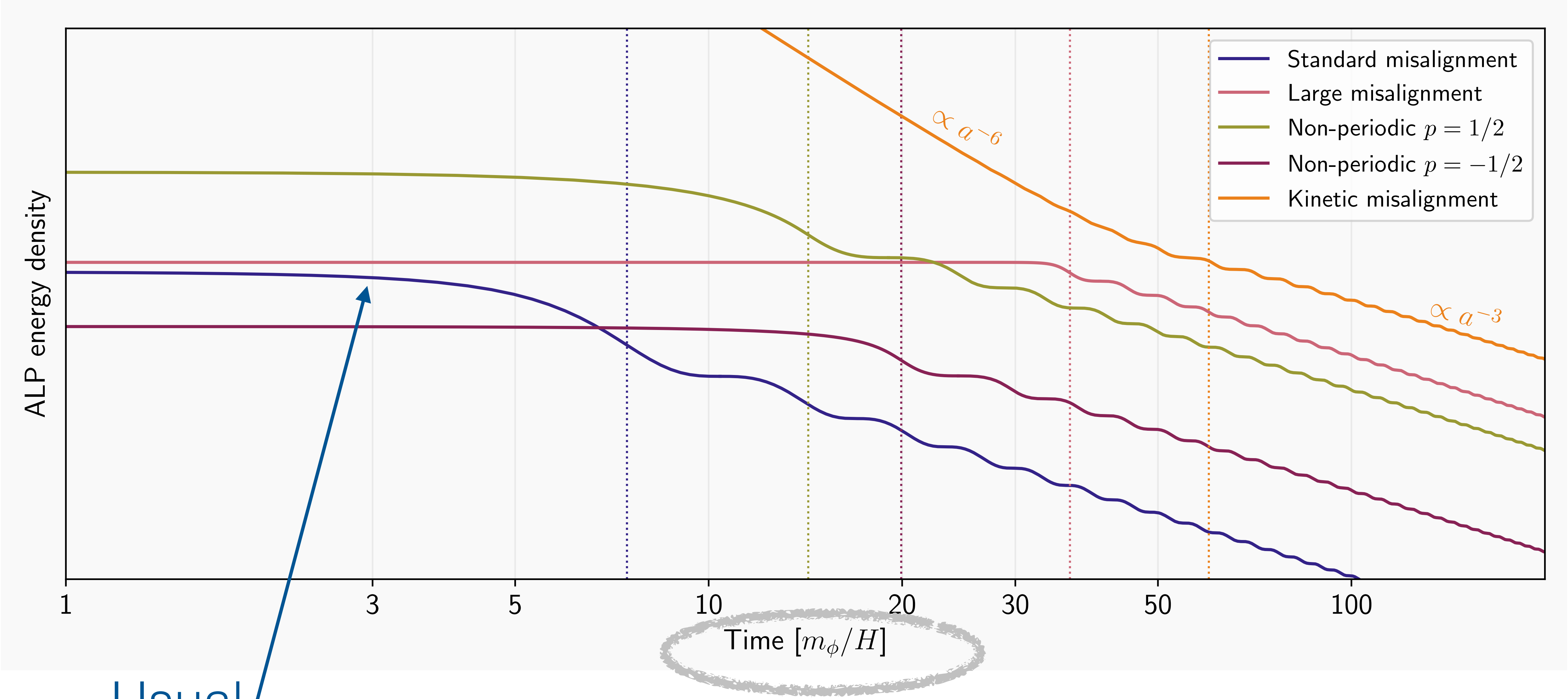
A third way to delay the onset of oscillations: a non-periodic potential.

1906.06352, 2305.03756

$$V(\theta) = \frac{m_\phi^2 f_\phi^2}{2p}\left[\left(1+\theta^2\right)^p - 1\right], \quad p < 1.$$

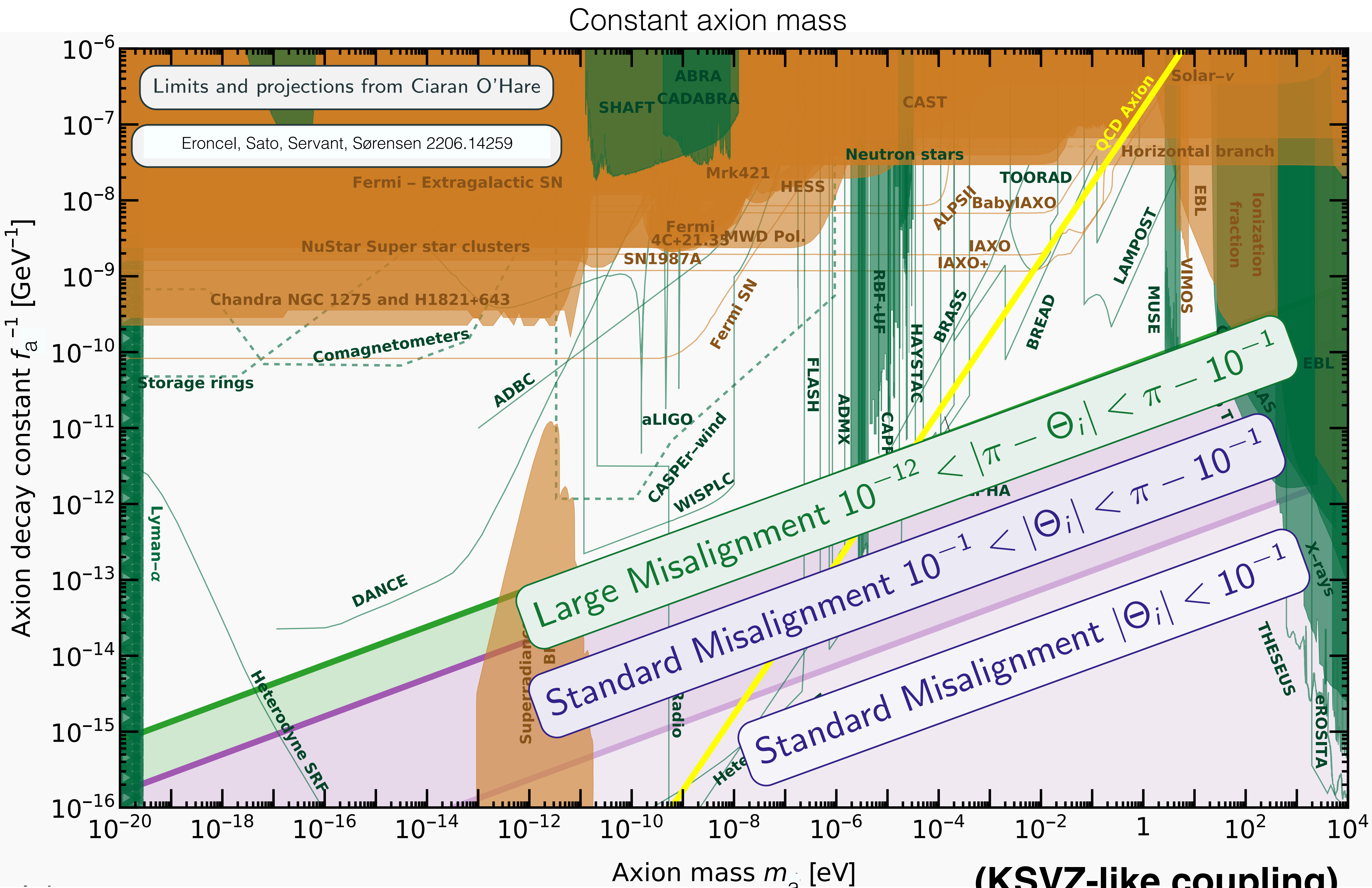


Common property of all these cases: onset of oscillations is delayed which boosts the dark matter abundance, and extends the ALP dark matter parameter space to lower decay constants.



Usual story

ALP DM parameter space.

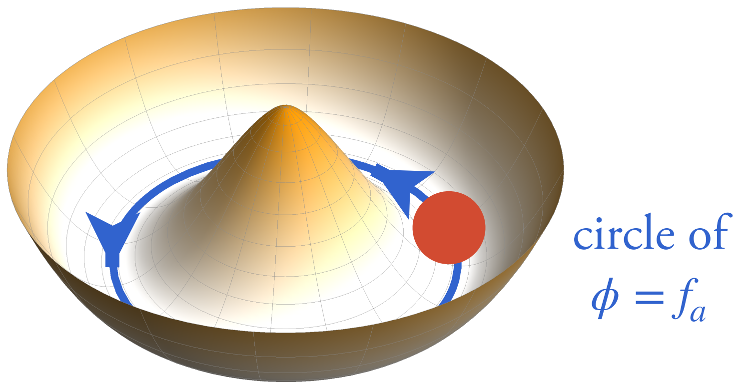


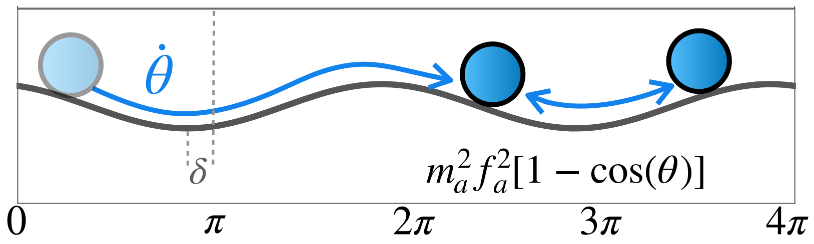
(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

Kinetic misalignment.

Add kinetic energy to delay onset of oscillations



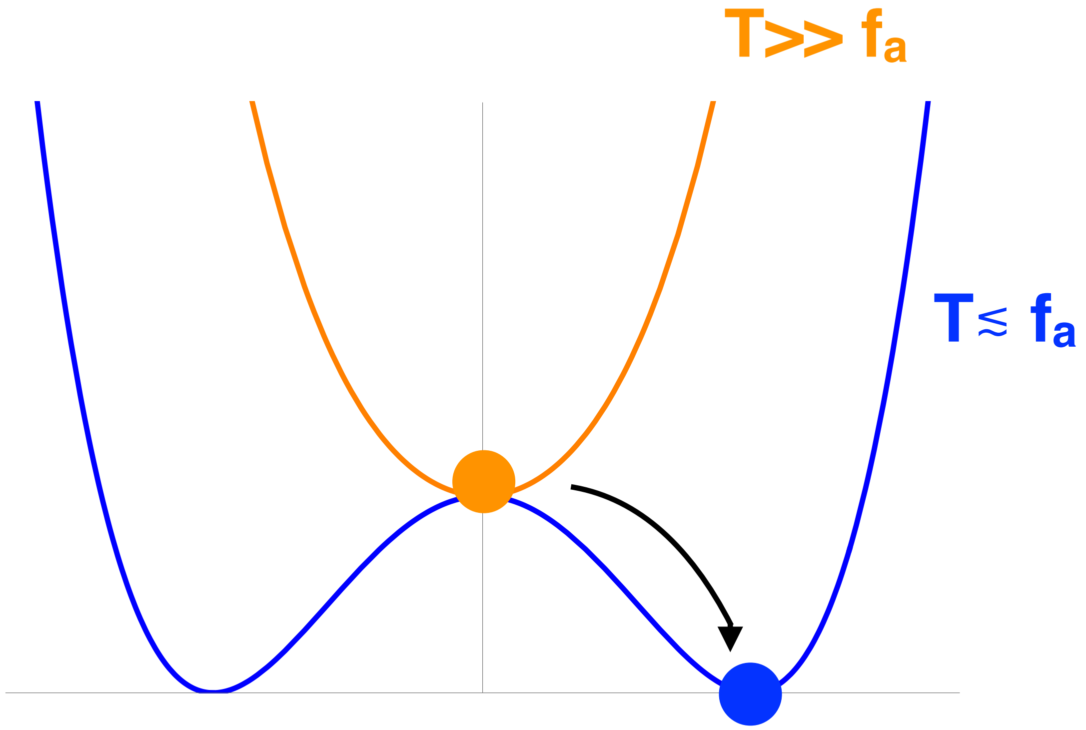


> Delay oscillations
$\Rightarrow$ less redshift
$\Rightarrow$ more DM
$\Rightarrow$ lower f_a

-> ALP can be DM for low f_a

Co, Hall, Harigaya et al '19'20
Chang, Cui'19
Eröncel et al, '22

Axion cosmology.

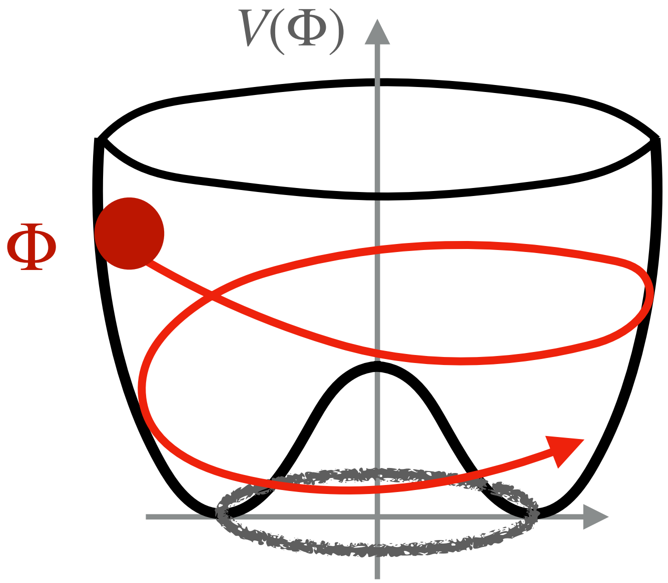


“Common” story:

Starts at <ϕ>=0

Studies axion cosmology ignoring the radial mode

Alternative:



Starts at <ϕ> >> f$_a$

(field can be driven naturally to these large field values during inflation due to a negative Hubble-induced mass term)

Radial mode /axion interplay

How did the axion acquire a kick?

If PQ symmetry is broken explicitly at high energies
—> mexican hat potential is tilted

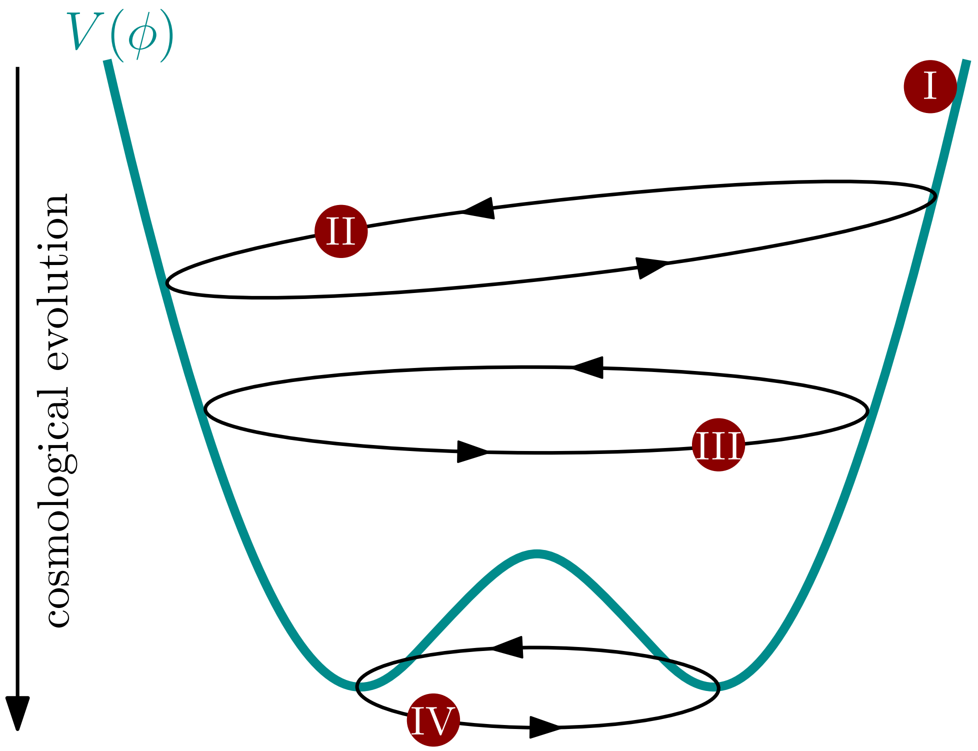


Figure by P. Simakachorn

If radial mode of PQ field starts at large VEV, the angular mode gets a large kick in the early universe

With initial conditions:

$$\frac{1}{2}\dot{\Theta}_i^2 \gg 2m^2(T_i)$$

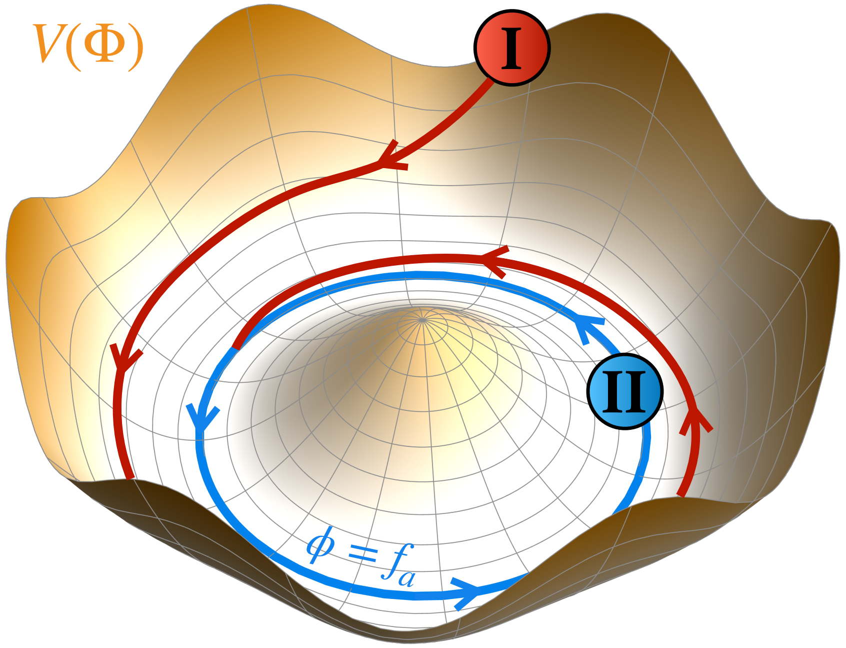
Delayed axion oscillations !

-> kinetic misalignment mechanism

[Co, Harigaya, Hall’19] **1910.14152**

2004.00629

Initial conditions.



Similar to Affleck-Dine ’85 scenario

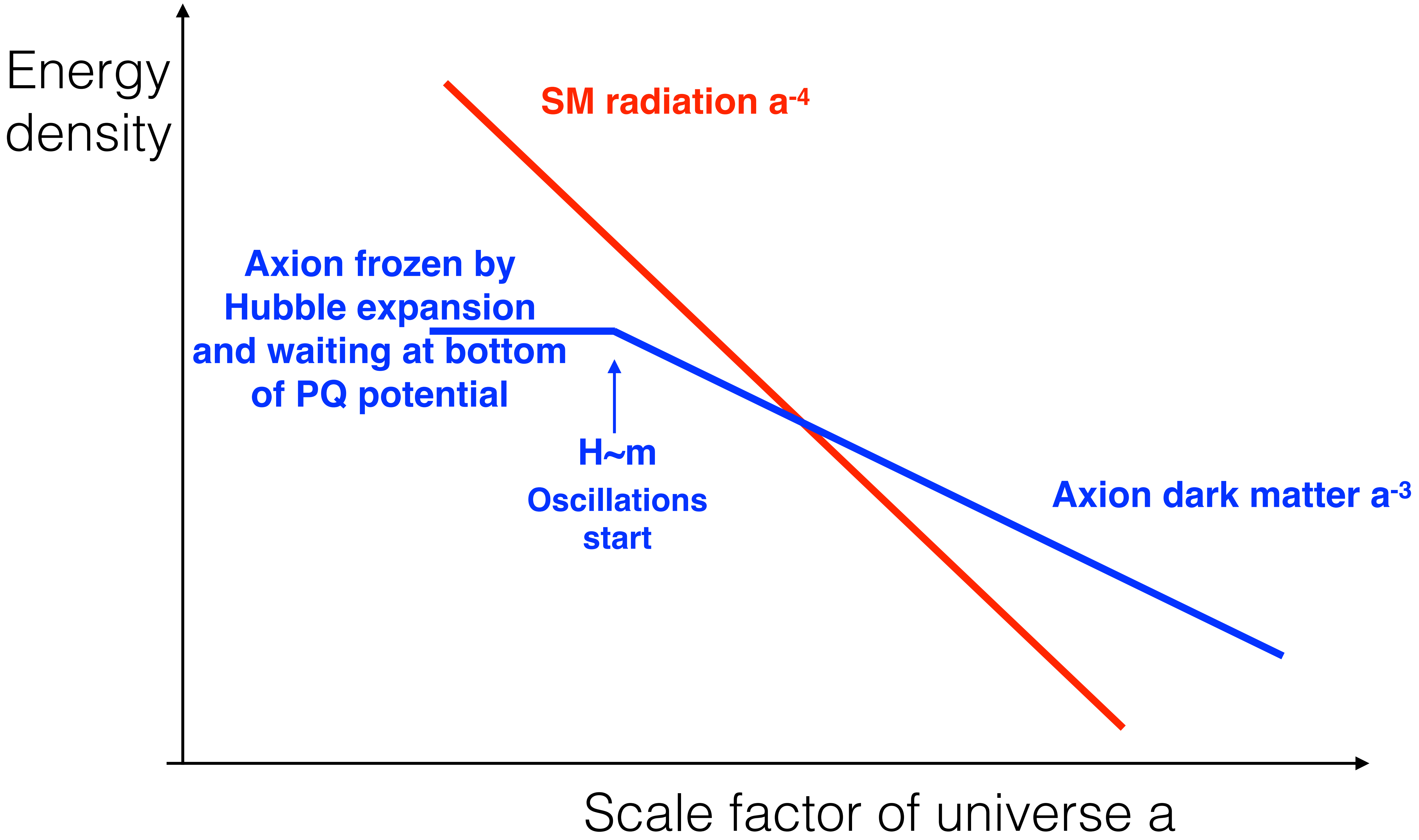
At early times, ϕ is driven away from ϕ =0,
towards $\langle\phi\rangle \gg f_a$
by negative Hubble-induced mass term $H \gg m_\phi$

$$V_H(\Phi, H) \supset -cH^2|\Phi|^2$$

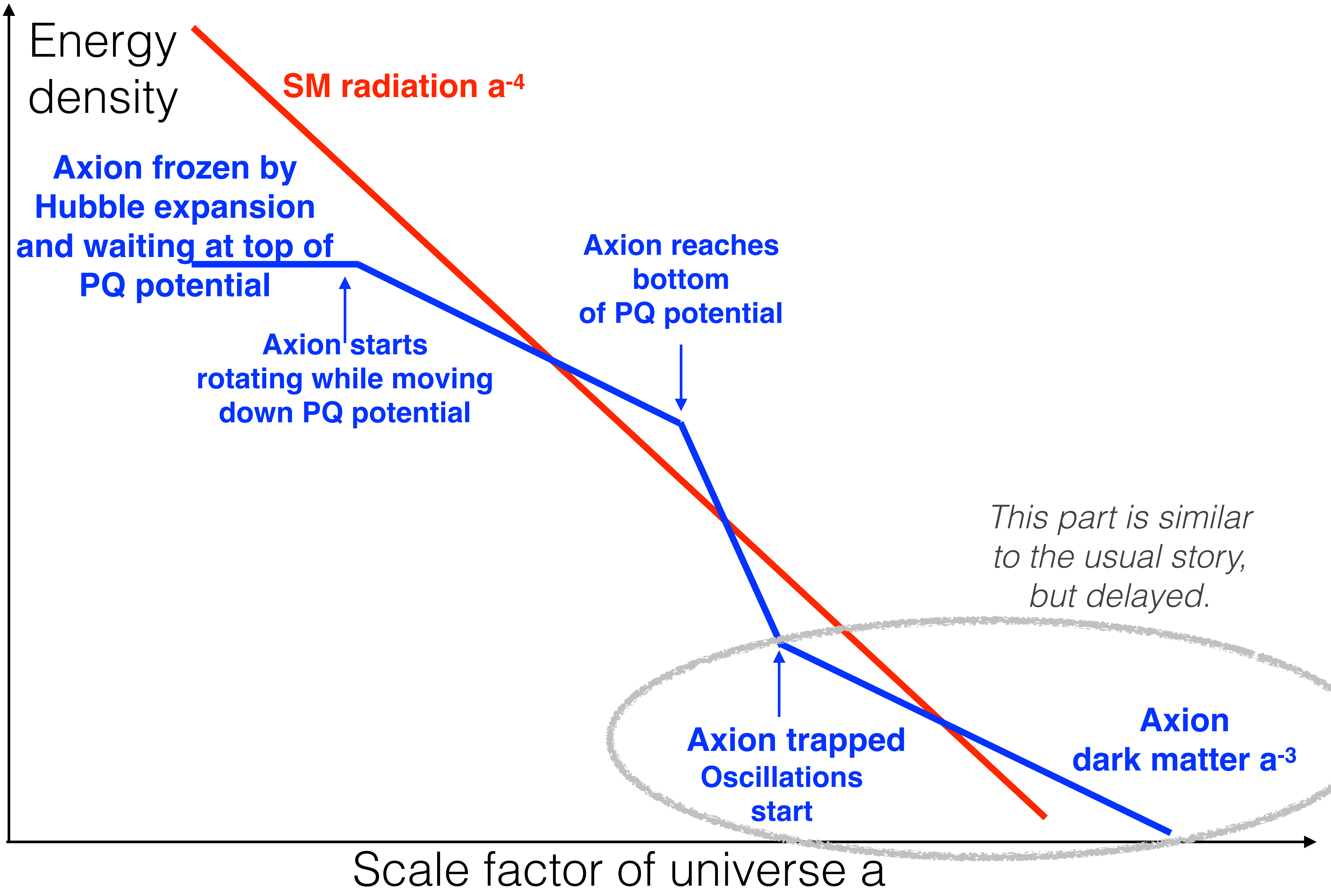
Dine, Randall, Thomas ‘95

+ explicit U(1) breaking term transfers radial mode motion into kick for the axion

Usual story.


Energy density
SM radiation a^{-4}
Axion frozen by Hubble expansion and waiting at bottom of PQ potential
H~m
Oscillations start
Axion dark matter a^{-3}
Scale factor of universe a

New story.



Energy density

SM radiation a^{-4}

Axion frozen by Hubble expansion and waiting at top of PQ potential

Axion starts rotating while moving down PQ potential

Axion reaches bottom of PQ potential

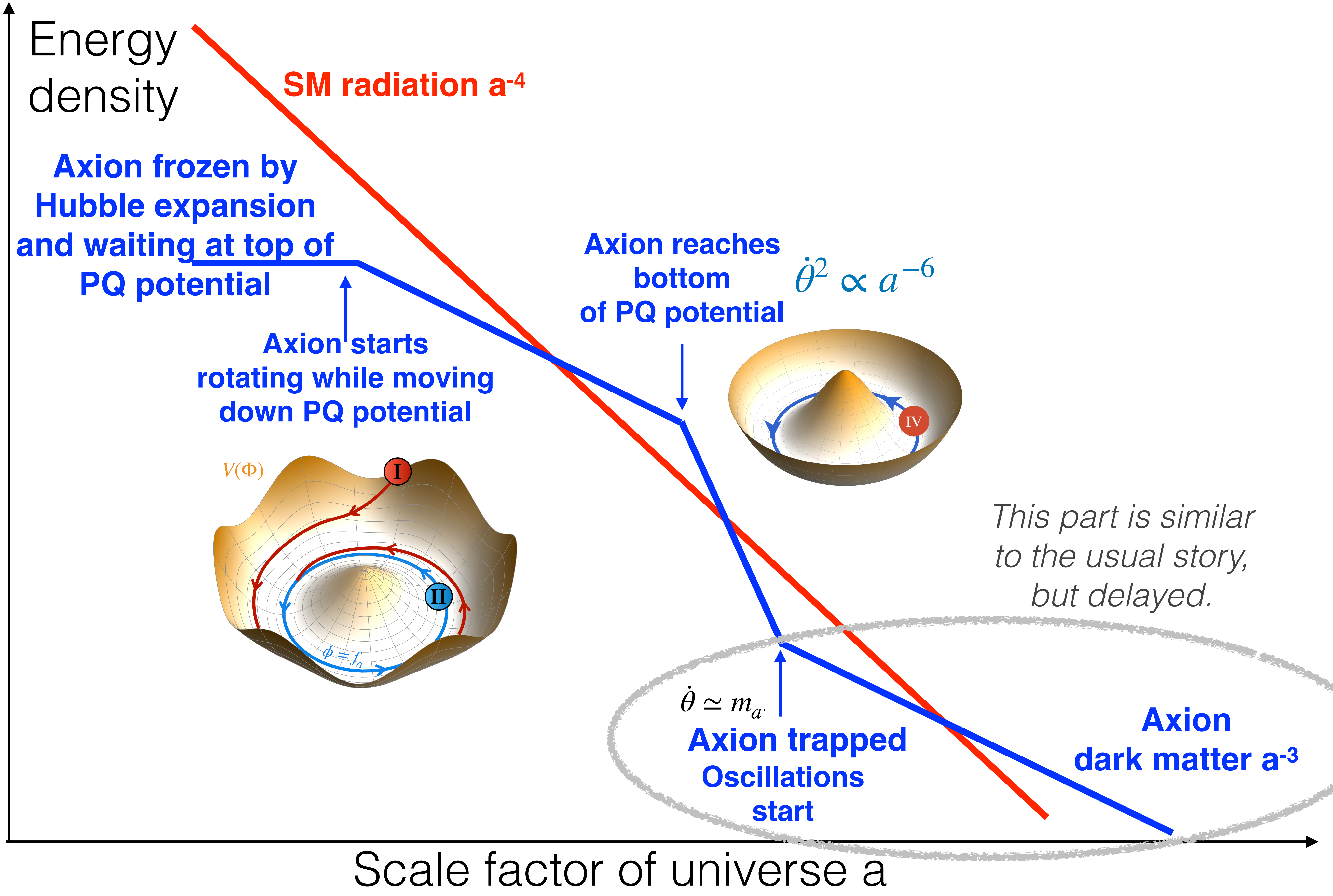
This part is similar to the usual story, but delayed.

Axion trapped Oscillations start

Axion dark matter a^{-3}

Scale factor of universe a

New story.



Energy density

SM radiation a^{-4}

Axion frozen by Hubble expansion and waiting at top of PQ potential

Axion starts rotating while moving down PQ potential

Axion reaches bottom of PQ potential

$\dot{\theta}^2 \propto a^{-6}$

This part is similar to the usual story, but delayed.

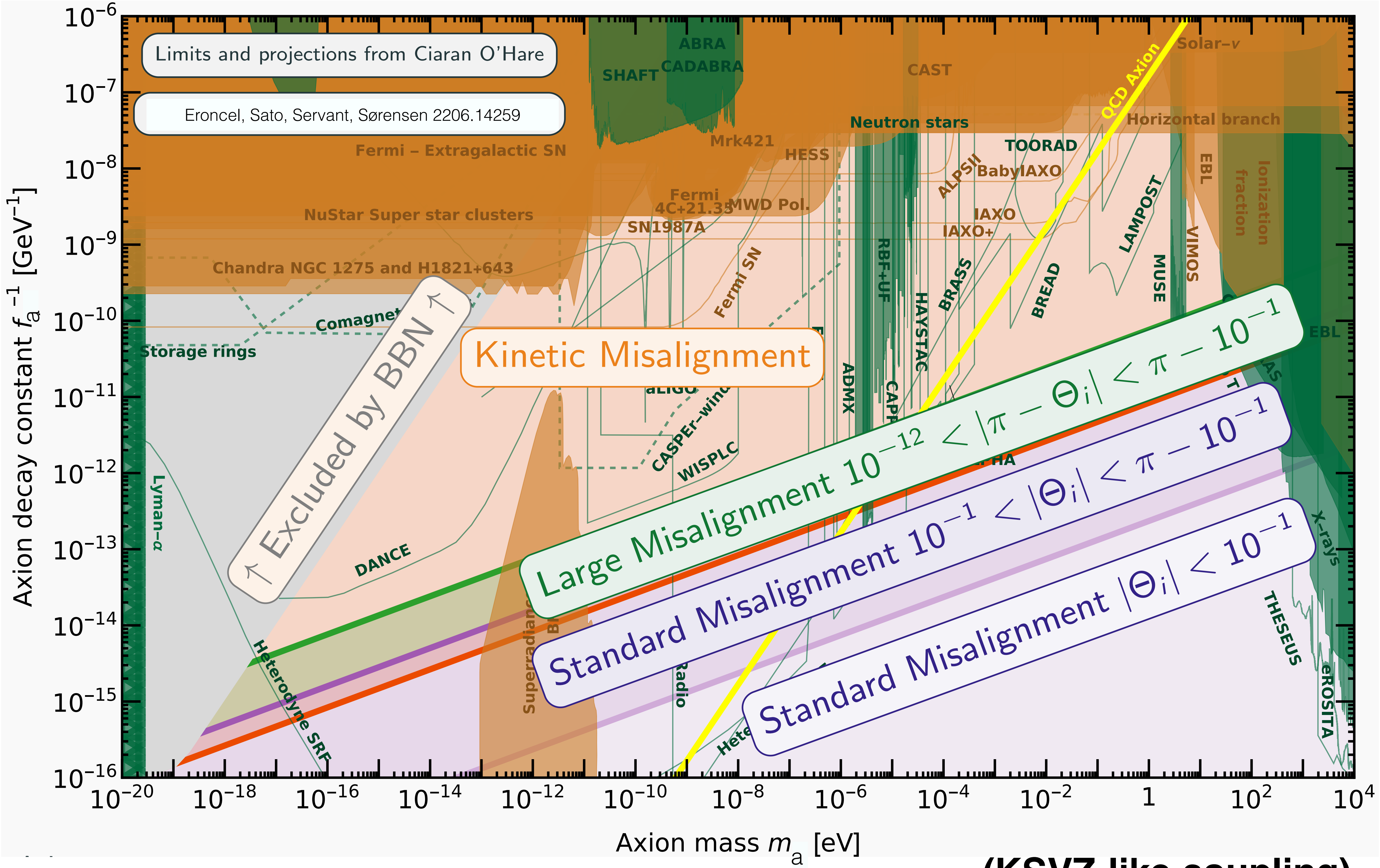
$\dot{\theta} \simeq m_a$

Axion trapped Oscillations start

Axion dark matter a^{-3}

Scale factor of universe a

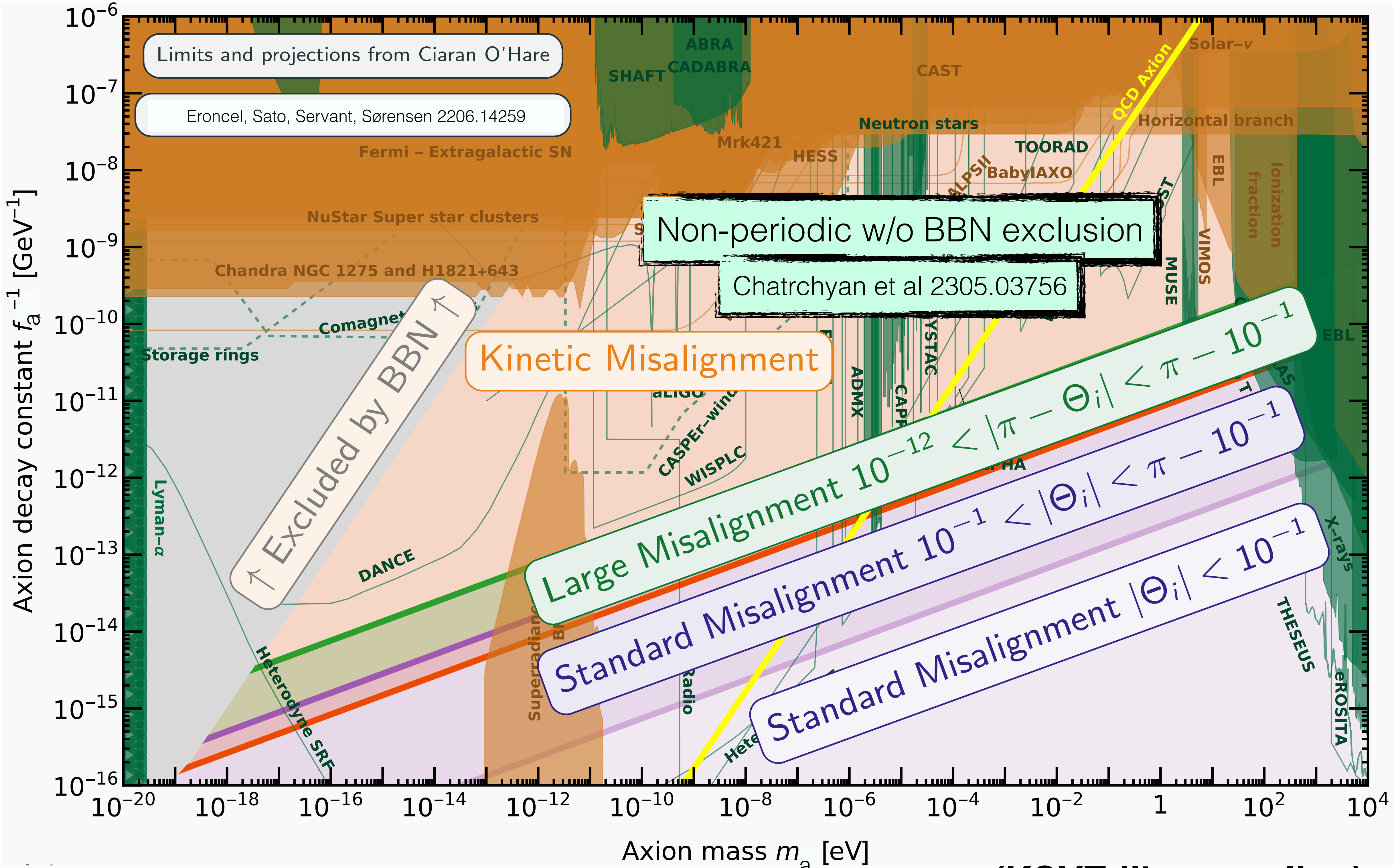
ALP DM parameter space.



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

ALP DM parameter space.



Limits and projections from Ciaran O'Hare

Eroncel, Sato, Servant, Sørensen 2206.14259

Non-periodic w/o BBN exclusion

Chatrchyan et al 2305.03756

Kinetic Misalignment

↑ Excluded by BBN ↑

Large Misalignment $10^{-12} < |\pi - \Theta_i| < \pi - 10^{-1}$

Standard Misalignment $10^{-1} < |\Theta_i| < \pi - 10^{-1}$

Standard Misalignment $|\Theta_i| < 10^{-1}$

(KSVZ-like coupling)

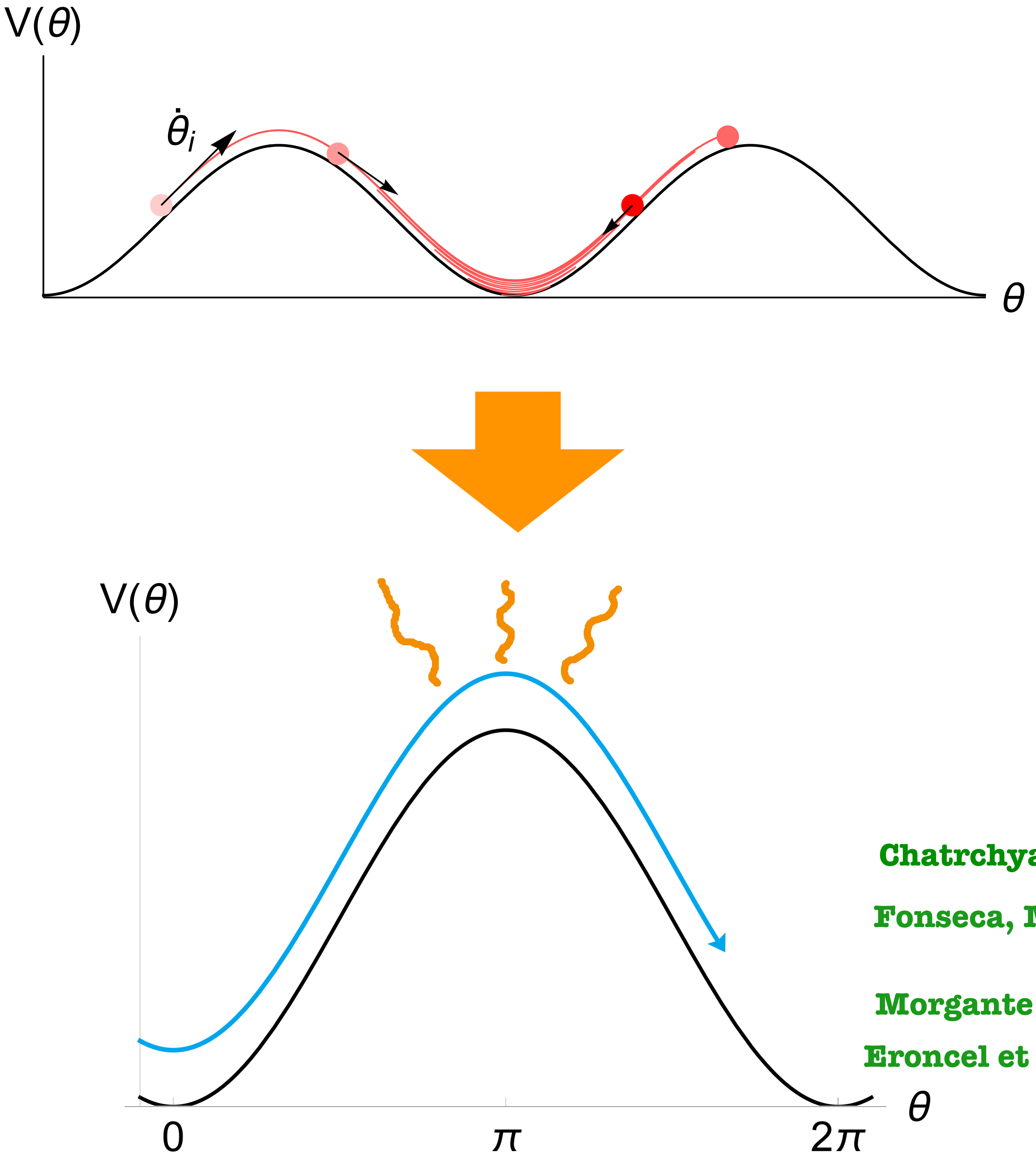
$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

Axion kinetic misalignment:

Axion fragmentation.

Compact axion halos.

Axion fragmentation



Chatrchyan et al, 1903.03116, 2004.07844

Fonseca, Morgante, Sato, Servant, 1911.08472, 1911.08473

Morgante et al, 2109.13823

Eroncel et al'22, 2206.14259

Axion Fragmentation.

Not considered in usual axion phenomenology with oscillations around one minimum: Fragmentation suppressed unless the field starts very close to the top of the potential (“large misalignment mechanism”) or for specific potentials with more than one cosine -> parametric resonance.

Greene, Kofman, Starobinsky, hep-ph/9808477
Chatrchyan et al, 1903.03116, 2004.07844
Arvanitaki et al, 1909.11665

However, becomes very relevant when field crosses many wiggles, with interesting implications, e.g. for the relaxion mechanism, but also as a new axion Dark Matter production mechanism.

Chatrchyan et al, 1903.03116, 2004.07844
Fonseca,Morgante,Sato, Servant’19
Morgante et al, 2109.13823

Generalization Eroncel et al, 2206.14259
(fragmentation before and after trapping + detailed application to DM)

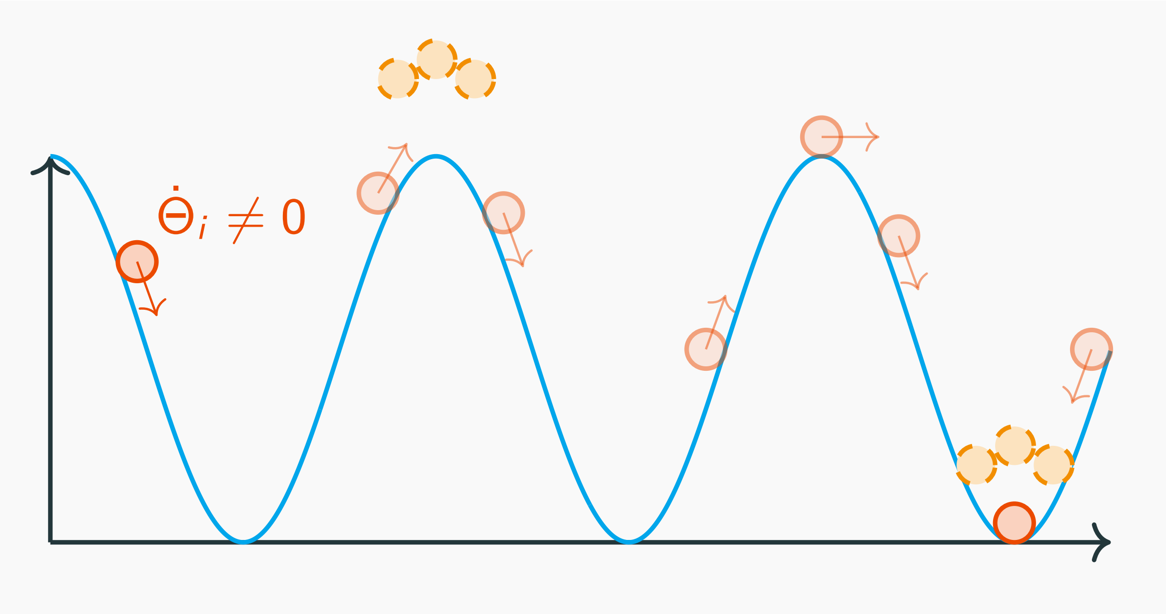
ALP fluctuations.

- Even in pre-inflationary scenario, ALP field has some fluctuations on top of the homogeneous background, which can be described by the mode functions in the Fourier space.

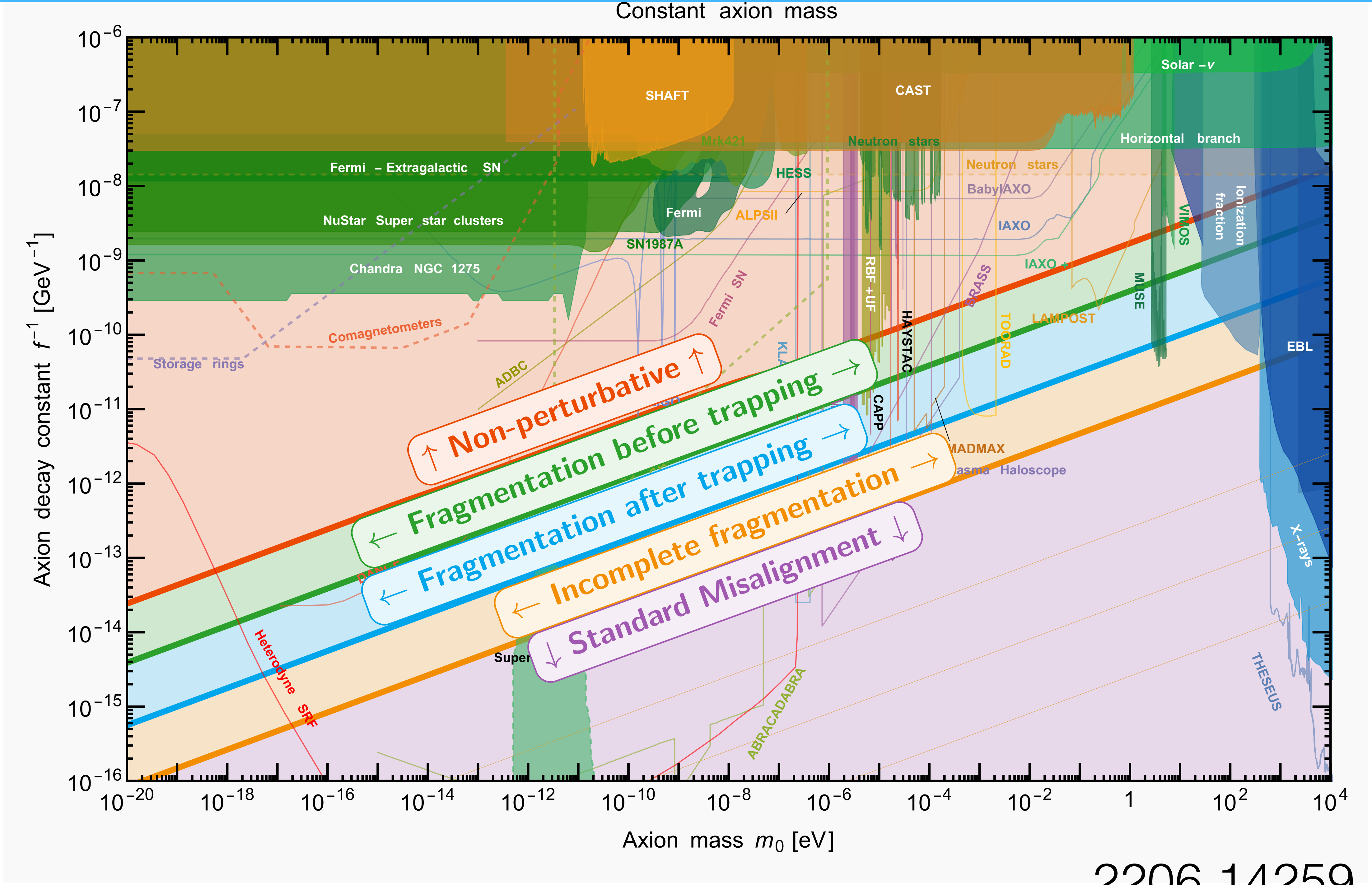
$$\theta(t, \mathrm{x}) = \Theta(t) + \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \theta_k e^{i\vec{\mathrm{k}}\cdot\vec{\mathrm{x}}} + \mathrm{h.c.}$$

- Even though the fluctuations are small initially, they can be enhanced exponentially later via parametric resonance yielding to fragmentation.

- In the case of efficient fragmentation, all the energy of the homogeneous mode can be transferred to the fluctuations. [Fonseca et al. 1911.08472; Morgante et al. 2109.13823]

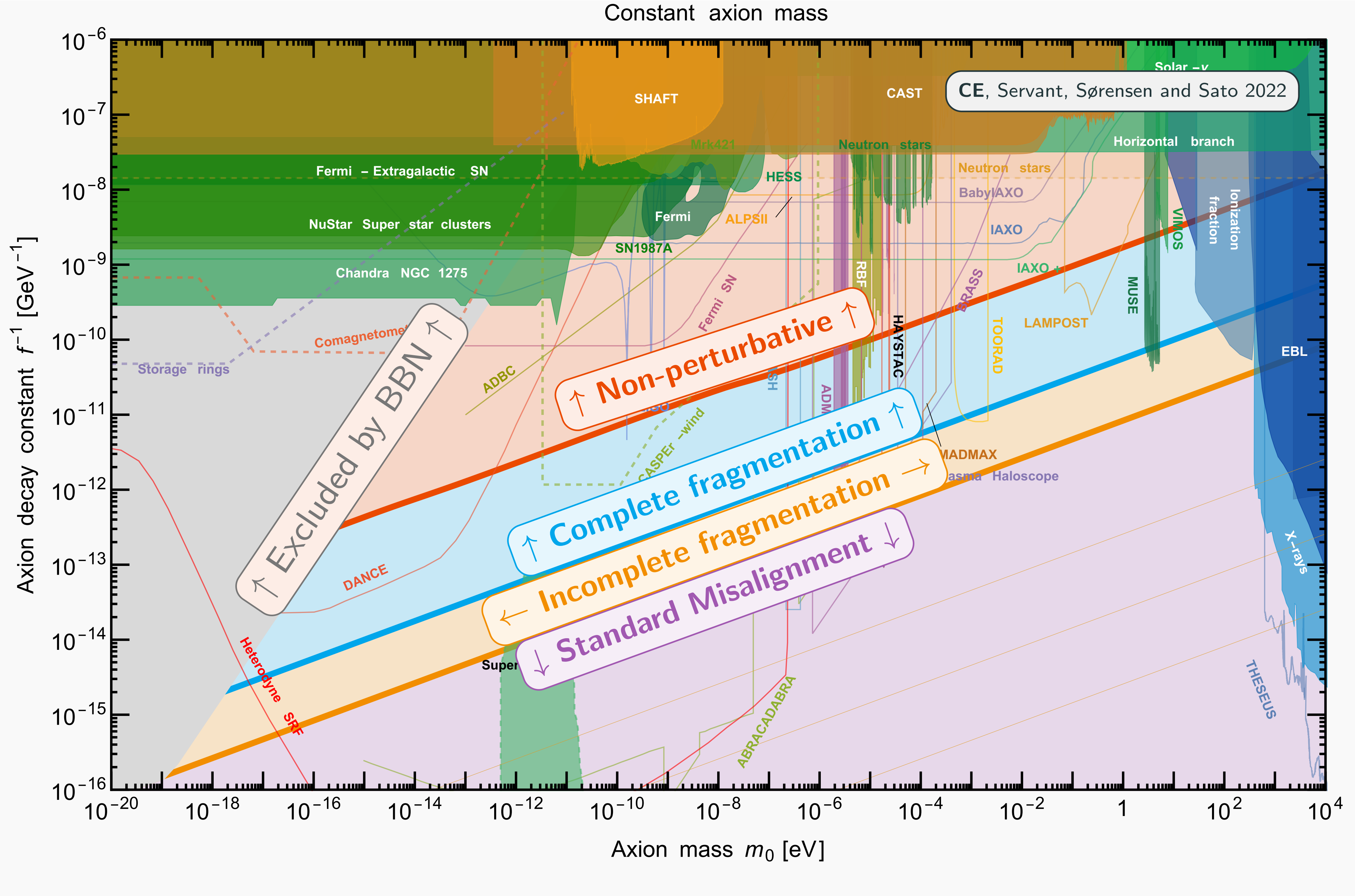


Fragmentation regions in ALP parameter space.


Constant axion mass

2206.14259

Fragmentation regions in ALP parameter space.



ALP fluctuations.

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\phi_k e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}$$

EoM for the unavoidable adiabatic perturbations :

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \underbrace{\left[\frac{k^2}{a^2} + V''(\phi)\Big|_{\bar{\phi}}\right]}_{\text{eff. frequency}}\phi_k = \underbrace{2\,\Phi_k\,V'(\phi)\Big|_{\bar{\phi}} - 4\dot{\Phi}_k\dot{\bar{\phi}}}_{\text{source term}}$$

unstable when the effective frequency

- **becomes negative ⇒ tachyonic instability**
- **is oscillating ⇒ parametric resonance**

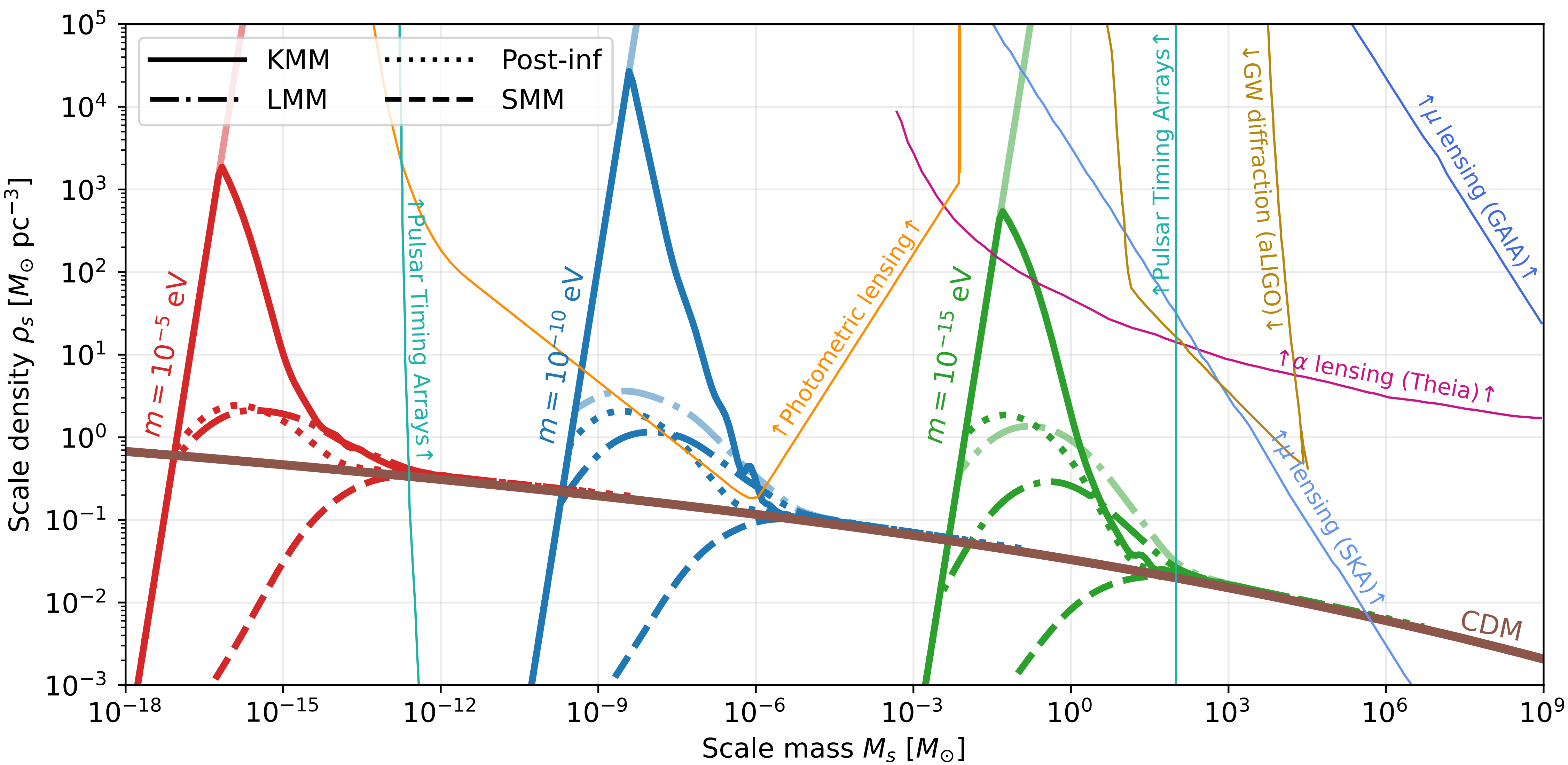
Growth rate of the perturbations depend exponentially on $\left.\frac{m_\phi}{H}\right|_{\text{osc}}$

Dense and compact ALP mini-clusters can also be formed in the pre-inflationary scenario!

Observational tests: compact axion halos.

kinetic misalignment—>axion fragmentation-> structure formation enhancement

Scale density of axion compact structures



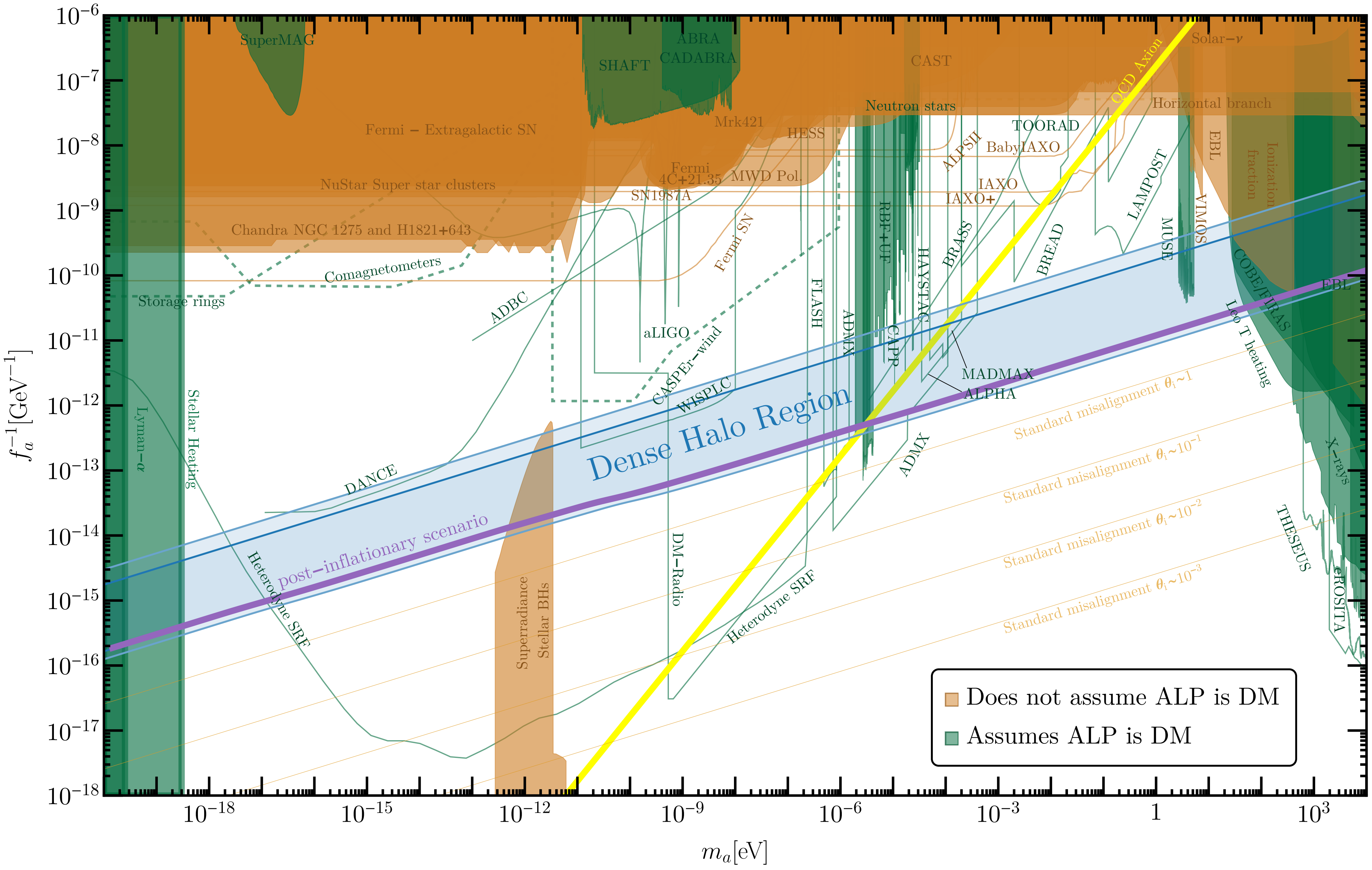
Eroncel et al' 2207.10111

fragmentation
post-inflationary
standard misalignment
large misalignment

was studied in the context of large misalignment scenario in [Arvanitaki et al'19]

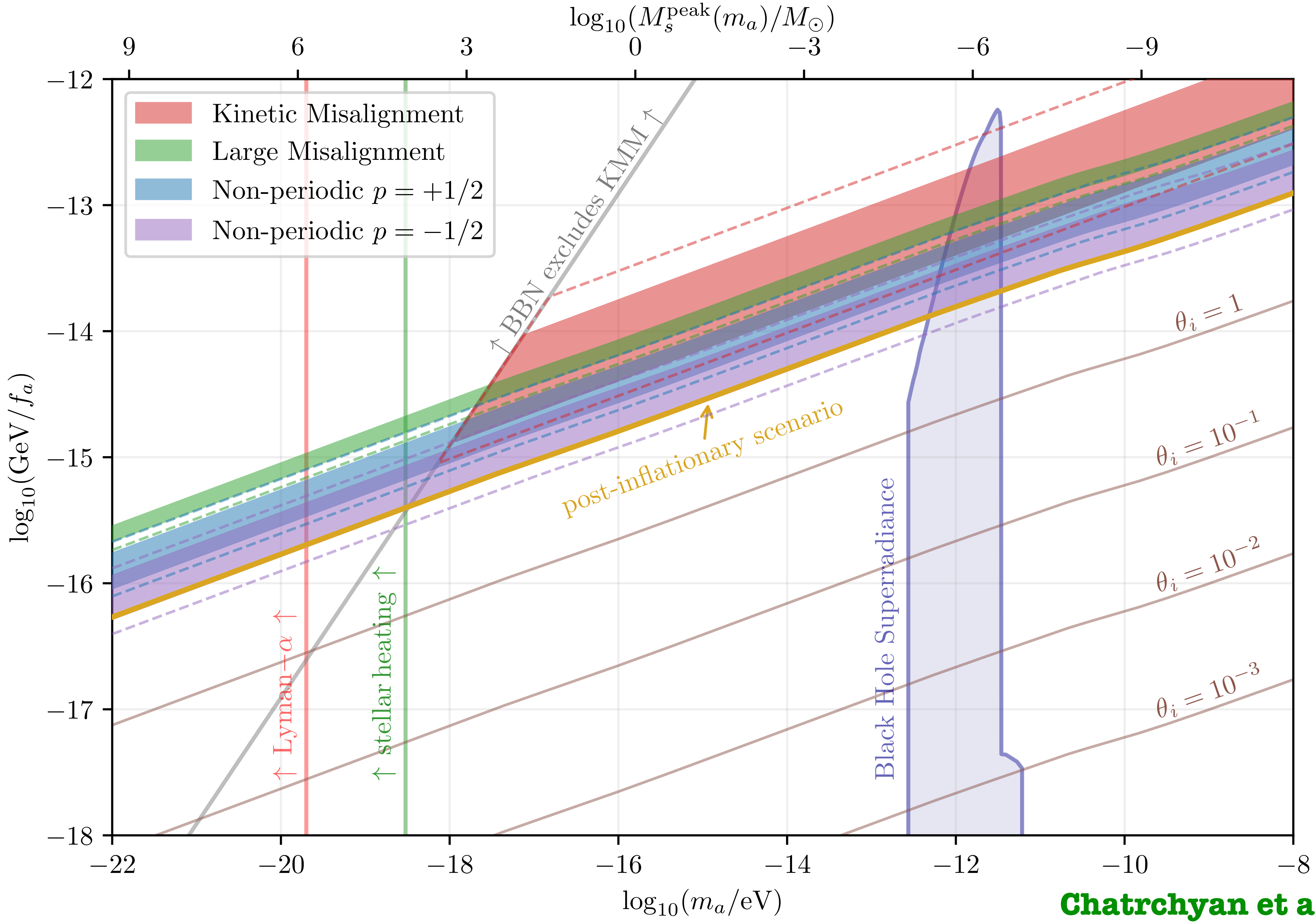
Different in the context of axion kinetic fragmentation: Eroncel et al , 2207.10111

Parameter space where parametric resonance can create compact halos.



Chatrchyan et al, 2305.03756

Parameter space where parametric resonance can create compact halos (with $\rho_s \gtrsim 10\, M_\odot\, \mathrm{pc}^{-3}$).

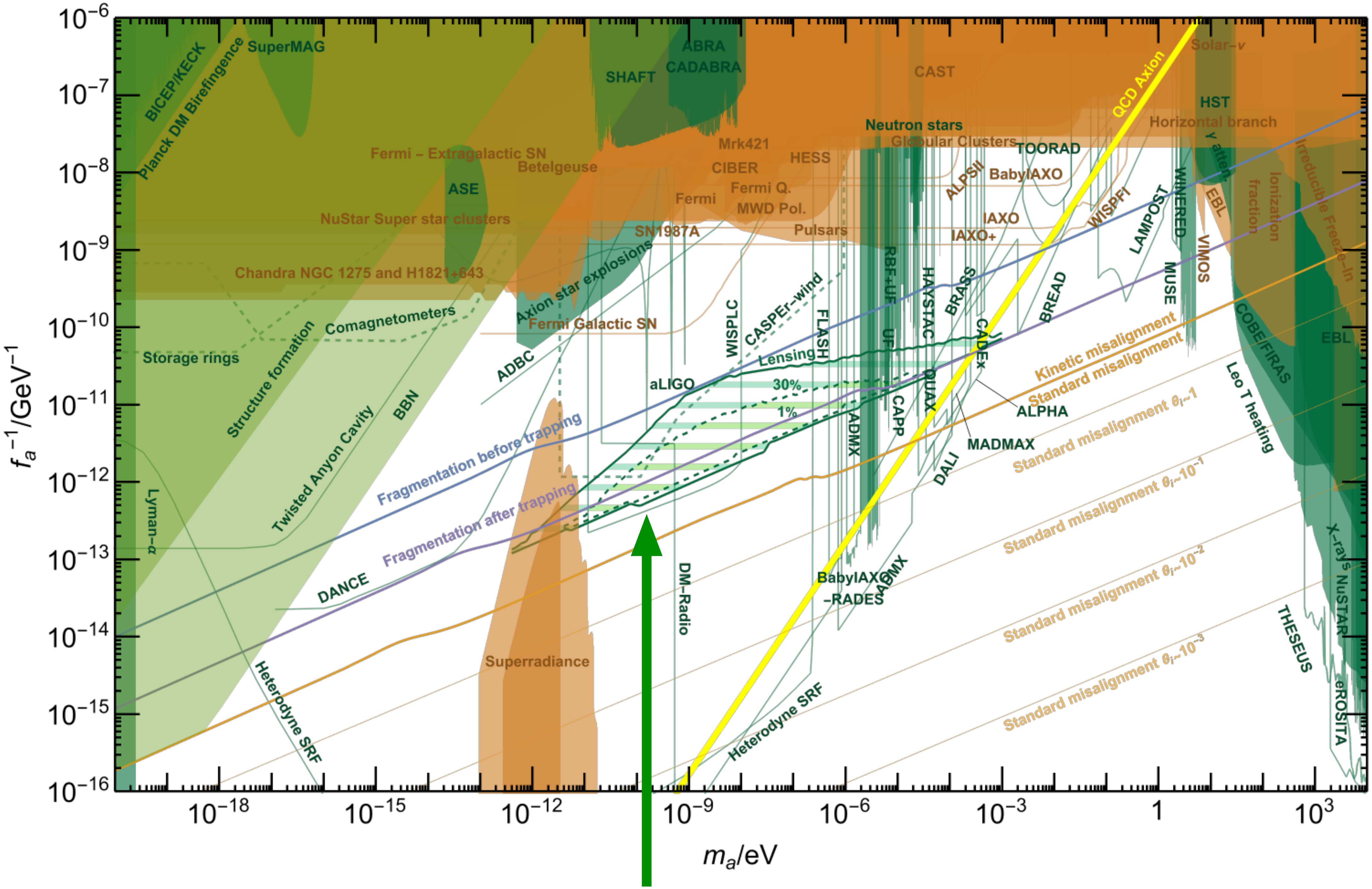


Chatrchyan et al 2305.03756

The dense halo regions from ≠ production mechanisms mostly overlap. Difficult to infer the producion mechanism from observations.

However, observations of dense structure gives information about f_a even when ALP does not couple to the SM!

Observability of compact halos from kinetic misalignment.



Region that can be probed by photometric lensing

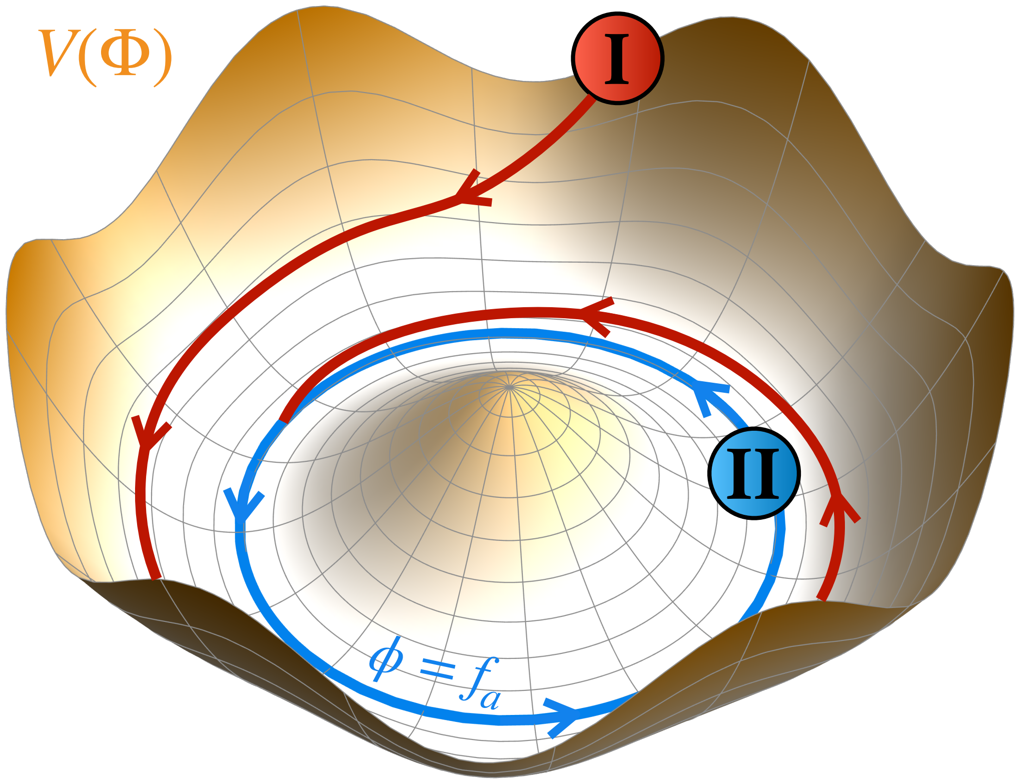
Model implementations of a rotating axion .

Complex scalar field

“Affleck-Dine Baryogenesis” (Affleck, Dine, 1985)

“Axiogenesis” (Co, Hall, Harigaya, et. al., ’19)

“Kination cosmology” (Gouttenoire et al, ’21)



$\Phi \sim \phi e^{i\theta}$ with $U(1)$-symmetry

Radial mode ϕ oscillates in potential with mass $\sqrt{V''(\Phi)}$.

Angular mode θ “axion” spins, with large kinetic energy.

Requirements

1. $U(1)$-symmetric (quadratic) potential with spontaneous symmetry-breaking minimum

2. Large initial scalar VEV

3. Explicit $U(1)$-breaking term (wiggle for angular velocity)

4. Damping of radial motion

Ingredients 1 & 2 : scalar potential

$$V(\Phi) = m_r^2|\Phi|^2\left[\log\left(\frac{|\Phi|^2}{f_a^2}\right) - 1\right] + \left[A\frac{\Phi^n}{M_{Pl}^{n-3}}. + h.c\right] + \frac{|\Phi|^{2n-2}}{M_{Pl}^{2n-6}}$$

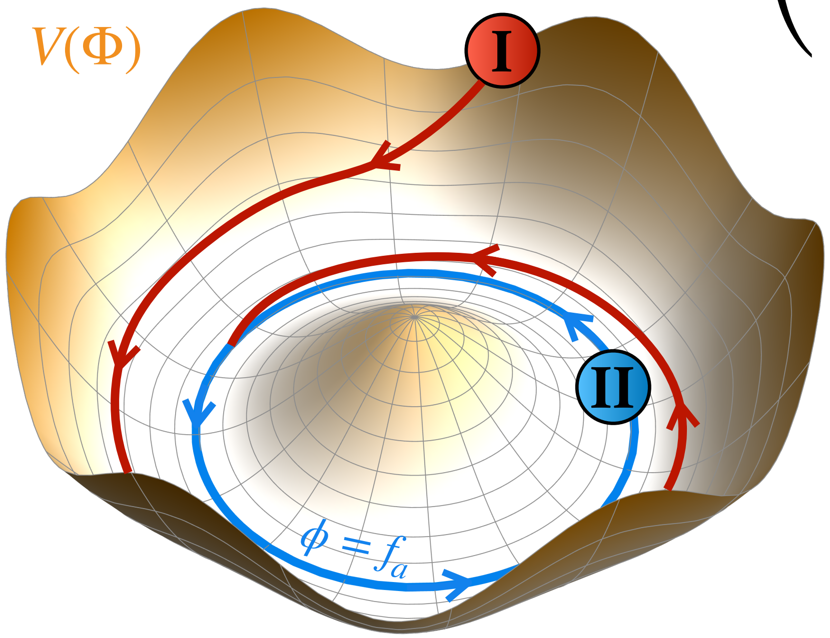
I. $U(1)$-conserving potential
(quadratic)
with a minimum f_a

(motivated by supersymmetric setups)

$\propto \cos(l\theta)$
II. explicit breaking term
(e.g. $U(1)$ is not exact
at high scales.)

stabilization
i.e., at large $|\Phi|$

$$\phi_{\rm ini} \simeq M_{\rm Pl}\left(\sqrt{\frac{c}{\lambda^2}}\cdot\frac{m_r}{M_{\rm Pl}}\right)^{1/(n-2)}$$



Ingredient 3 : large initial VEV $\phi_{\rm ini}$

By adding a negative Hubble mass

$$V_H(\Phi, H) \supset -cH^2|\Phi|^2$$

ϕ is driven away from $\phi = 0$ at early times $(H \gg m_r)$

(e.g. Dine, Randall, Thomas, 1995, Fujita & Harigaya 1607.07058)

Summary of Part 2 .

ALPs can be the DM everywhere in the [m_a ,f_a] plane.

Kinetic Misalignment Mechanism:

A well-motivated alternative production mechanism for ALP Dark Matter

Moves the ALP Dark Matter window into testable territory.
->All axion experiments are in principle sensitive to axion dark matter
(even helioscopes and light-shining-through-the-wall experiments)

QCD axion Dark Matter inside MADMAX and Iaxo sensitivities

Kinetic fragmentation :

A promising probe: Much denser compact axion dark matter halos

Axion cosmology: Rich spectrum of possibilities, role of radial mode!

In next lecture:
Other observational tests: Gravitational waves from a rotating axion

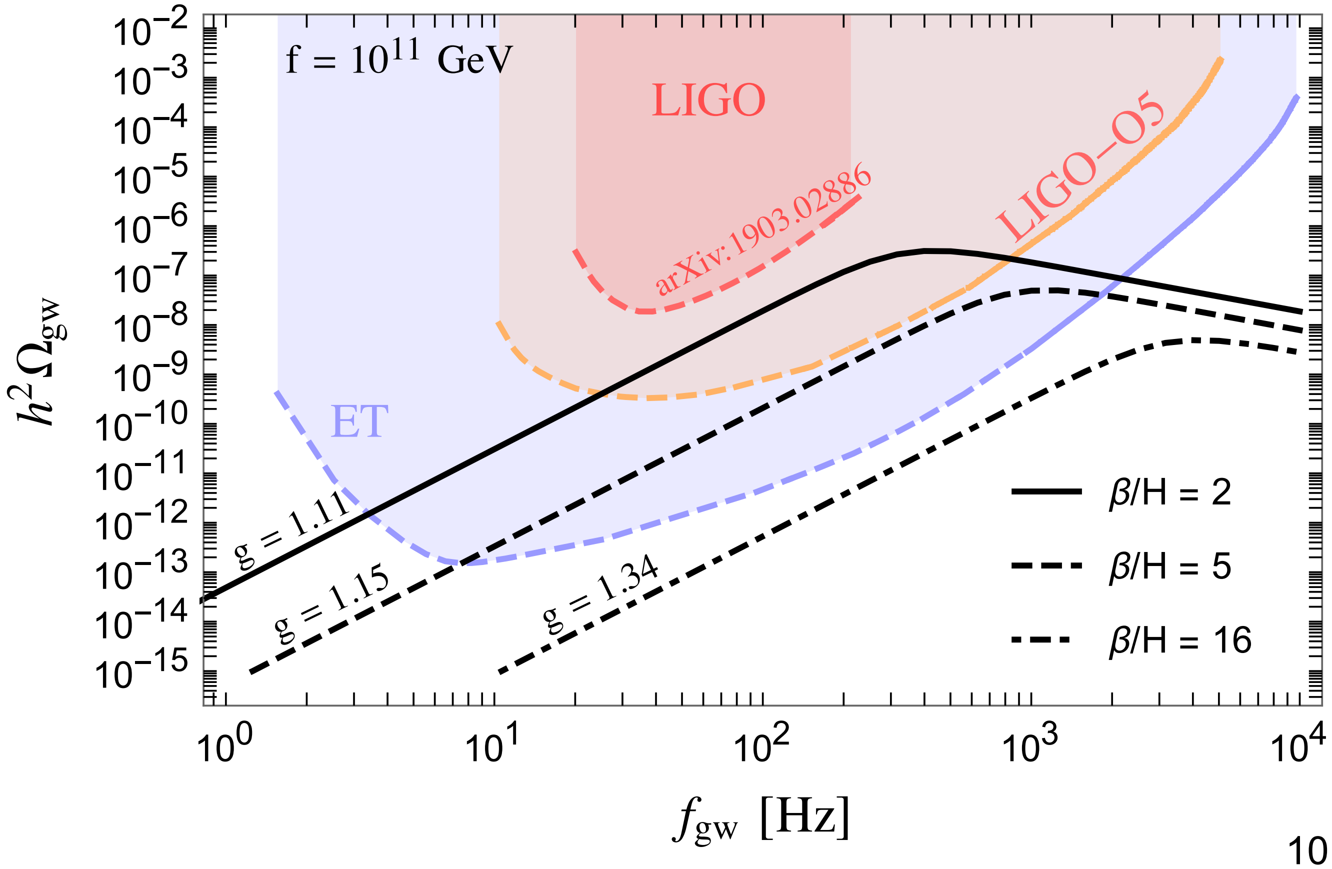
Lecture 2:

GW backgrounds from axion early-universe dynamics .

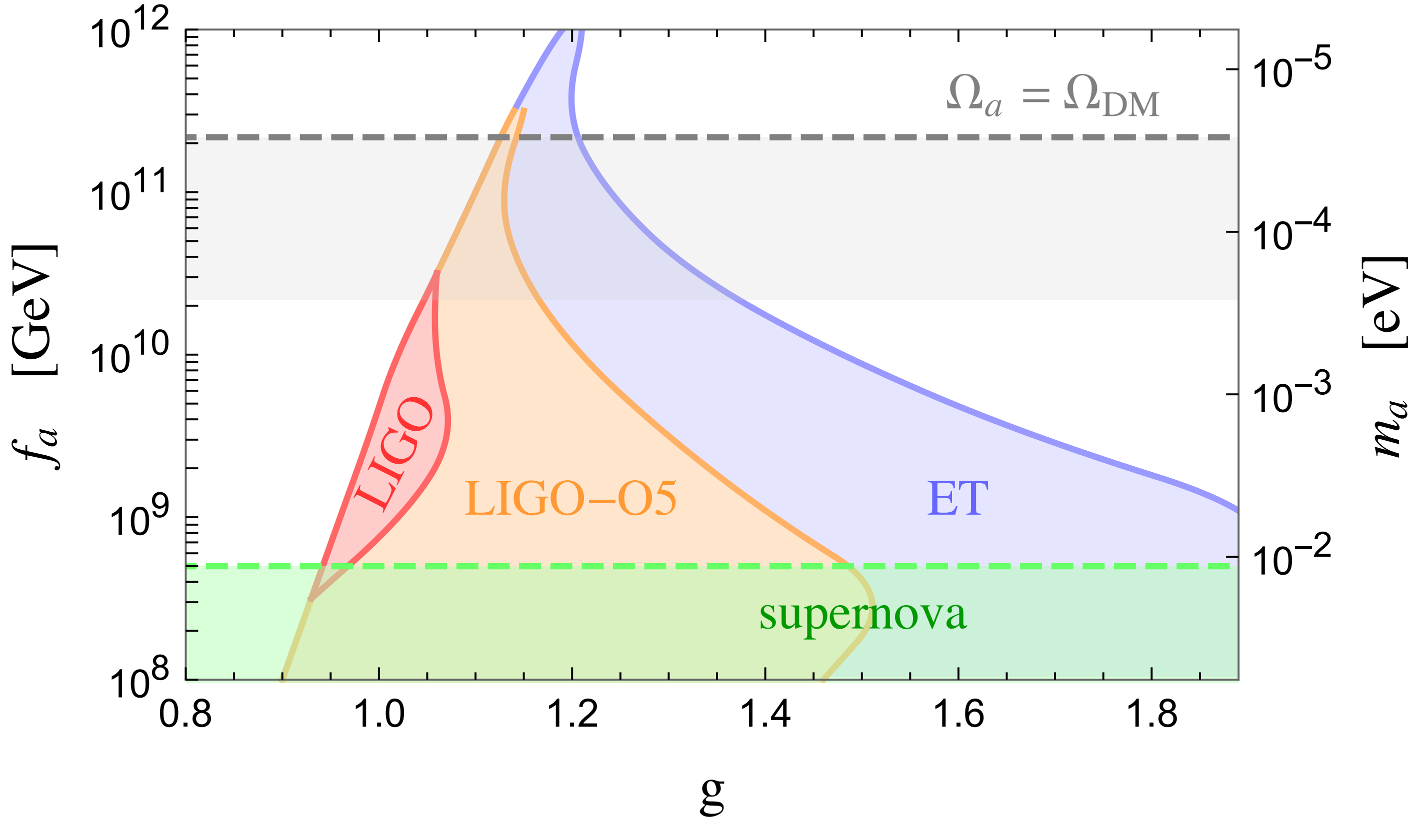
3 distinct sources:

- GWs from the Peccei-Quinn phase transition (if first-order)
- GWs from axionic (global) cosmic strings
- GW signatures from kination induced by rotating axions
- GWs from axion fragmentation

GW from a first-order Peccei-Quinn phase transition.



Delle Rose et al,1912.06139
Von Harling et al, 1912.07587



GW from axion-related defects.

Some references

GW from GLOBAL Cosmic Strings:

Chang & Cui, [1910.04781], [2106.09746].

Gouttenoire et al,[1912.02569].

Gorghetto, Hardy & Nicolaescu, [2101.11007].

Ramberg & Visinelli, [1904.05707], [2012.06882].

GW from Domain Walls:

T. Hiramatsu, M. Kawasaki and K. Saikawa, On the estimation of gravitational wave spectrum from cosmic domain walls, JCAP 02 (2014) 031 [1309.5001].

R. Zambujal Ferreira, A. Notari, O. Pujolas & F. Rompineve, High Quality QCD Axion at Gravitational Wave Observatories, Phys. Rev. Lett. 128 (2022) 141101 [2107.07542].

K. Saikawa, Gravitational waves from cosmic domain walls: a mini-review, J. Phys. Conf. Ser. 1586 (2020) 012039.

. **R. Z. Ferreira, A. Notari, O. Pujolas and F. Rompineve, Gravitational waves from domain walls in Pulsar Timing Array datasets, JCAP 02 (2023) 001 [2204.04228].**

. **E. Madge, E. Morgante, C. P. Iba´n˜ez, N. Ramberg and S. Schenk, Primordial gravitational waves in the nano-Hertz regime and PTA data – towards solving the GW inverse problem, 2306.14856.**

GW from cosmic strings.

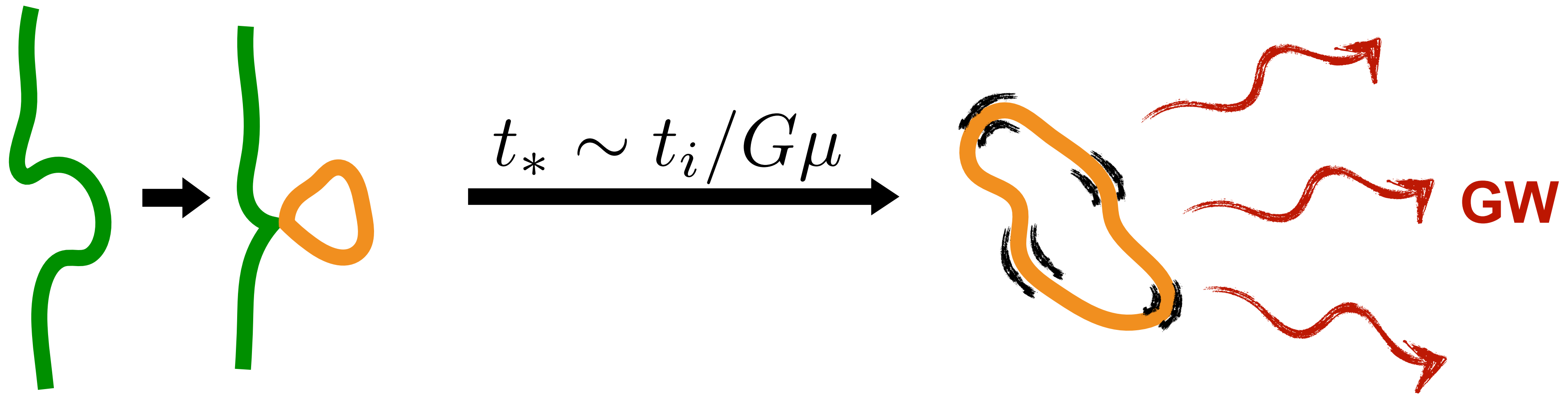
GW spectrum ⇨ GW emission from a loop × loop-number density

$$\Omega^{(k)}_{\rm GW}(f) = \frac{1}{\rho_c}\cdot\frac{2k}{f}\cdot\underbrace{\frac{(0.1)\Gamma^{(k)}G\mu^2}{\alpha(\alpha+\Gamma G\mu)}}_{\text{string's nature}}\int_{t_F}^{t_0} d\tilde{t}\,\underbrace{\frac{C_{\rm eff}(t_i)}{t_i^4}}_{\text{loop number}}\underbrace{\left[\frac{a(\tilde t)}{a(t_0)}\right]^5\overbrace{\left[\frac{a(t_i)}{a(\tilde t)}\right]^3}^{a^{-3}}}_{\text{red-shift}}\Theta(t_i-t_F)$$

$t_i \equiv$ loop production, $\tilde t \equiv$ loop emission

GW: a^{-4} , loop size: a^{-1}

cosmic string (local)



loop production @ t_i

$t_* \sim t_i/G\mu$

loop emission @ t_*

Relation between observed frequency & Hubble radius at emission

$$f \approx H_* \left(\frac{a_i}{a_0}\right) \left(\frac{1}{G\mu}\right)^{1/2}$$

LOCAL STRINGS vs GLOBAL STRINGS.

With respect to local strings, the GW spectrum from global strings in standard radiation cosmology is:

- suppressed by the shorter Hubble time $\tilde{t}_M$ at the time of GW emission: factor $\tilde{t}_{\rm M}^{\rm global}/\tilde{t}_{\rm M}^{\rm local} \propto G\mu_{\rm local} \propto \left(\eta/M_{\rm pl}\right)^2$,
- suppressed by the larger GW redshift factor since emission occurs earlier: factor $\left[\frac{a\left(\tilde{t}_{\rm M}^{\rm global}\right)}{a\left(\tilde{t}_{\rm M}^{\rm local}\right)}\right]^4 \propto \left(\eta/M_{\rm pl}\right)^4$,
- enhanced by the lower loop redshift factor since GW emission occurs right after loop production: factor $\left(a\left(\tilde{t}_{\rm M}^{\rm local}\right)/a\left(\tilde{t}_{\rm M}^{\rm global}\right)\right)^3 \propto \left(\eta/M_{\rm pl}\right)^{-3}$,
- increased by the logarithmically-enhanced GW power emission rate: factor $\log^2\left(\eta t_i\right)$,
- increased by the logarithmically-enhanced loop lifetime: factor $\log\left(\eta t_i\right)$.

LOCAL STRINGS
VS
GLOBAL STRINGS.

See comparison in Appendix F of [1912.02569] .

Loops from global strings : short-lived

Loops from local strings : long-lived.

—> different GW spectra in both frequency and amplitude.

LOCAL STRINGS vs GLOBAL STRINGS.

Global strings: no gauge field, instead massless Goldstone mode, with logarithmically-divergent gradient energy.

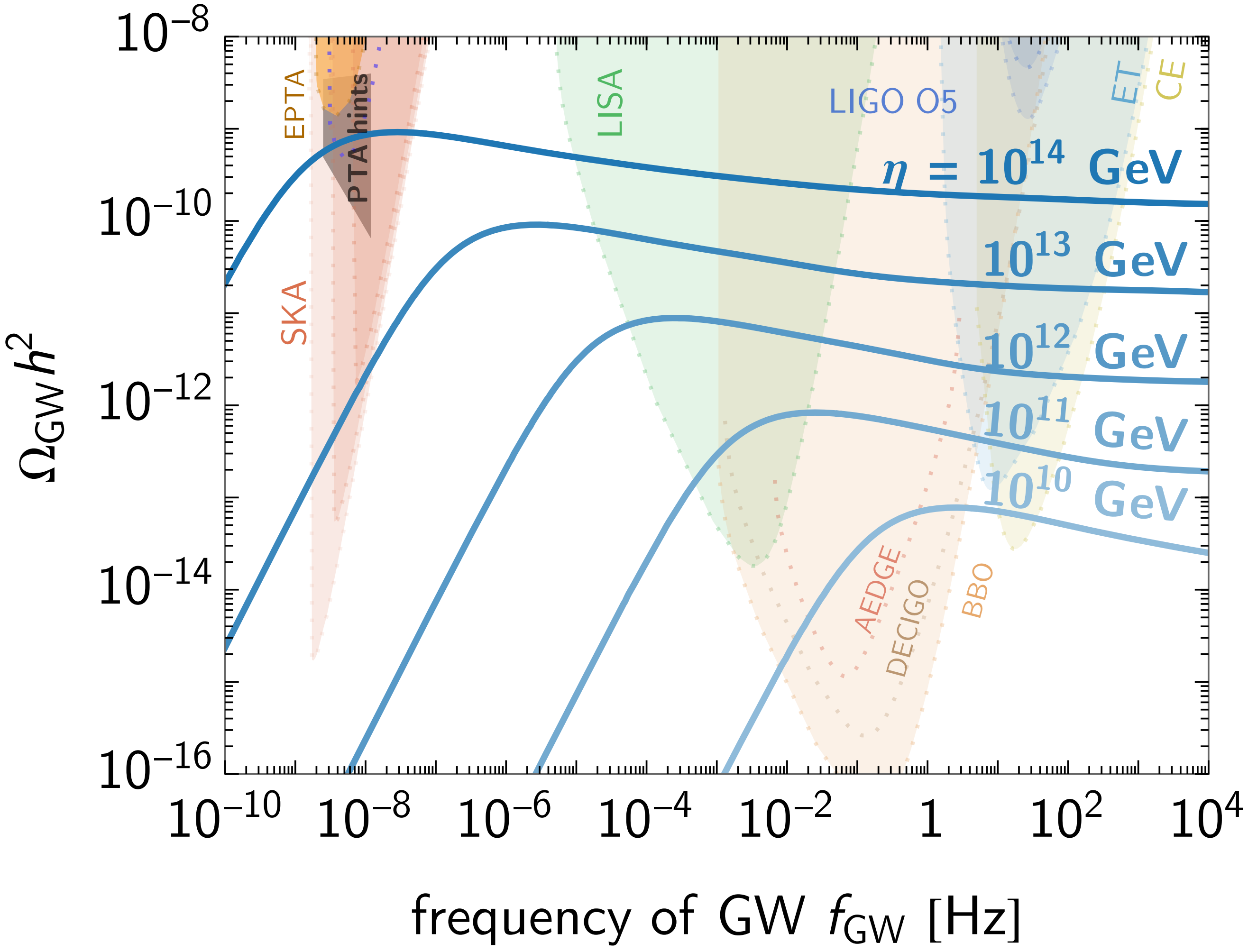
Loops quickly decay into axion particles.

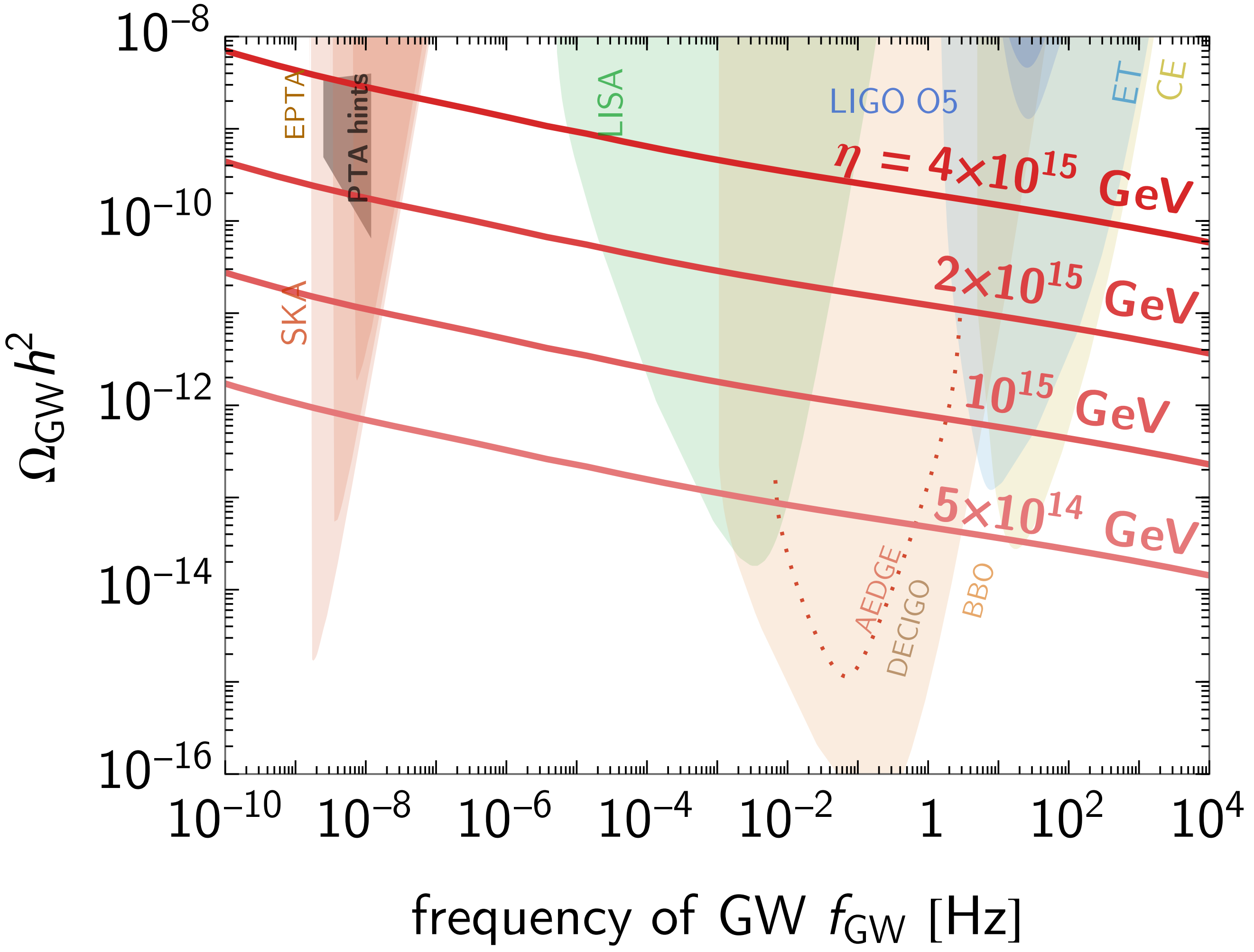
GW are mainly produced at the time of the loop production.

$$\Omega_{\mathrm{GW}}^{\mathrm{local}} \simeq \Omega_r \frac{\eta}{M_{\mathrm{pl}}}, \qquad \Omega_{\mathrm{GW}}^{\mathrm{global}} \simeq \Omega_r \left(\frac{\eta}{M_{\mathrm{pl}}}\right)^4 \log^3\left(\eta t_i\right).$$

$\eta = f_a$

LOCAL STRINGS versus GLOBAL STRINGS (m_a=0)





spectral shape changes with η

local loops live longer before decaying (& lifetime depends on η)

global loops decay fast.

To reach the same amplitude as the local strings, global strings need a larger η since GW production is not the leading energy loss.

Temperature-frequency relation.

A loop population produced at temperature T quickly decays into GW of frequency

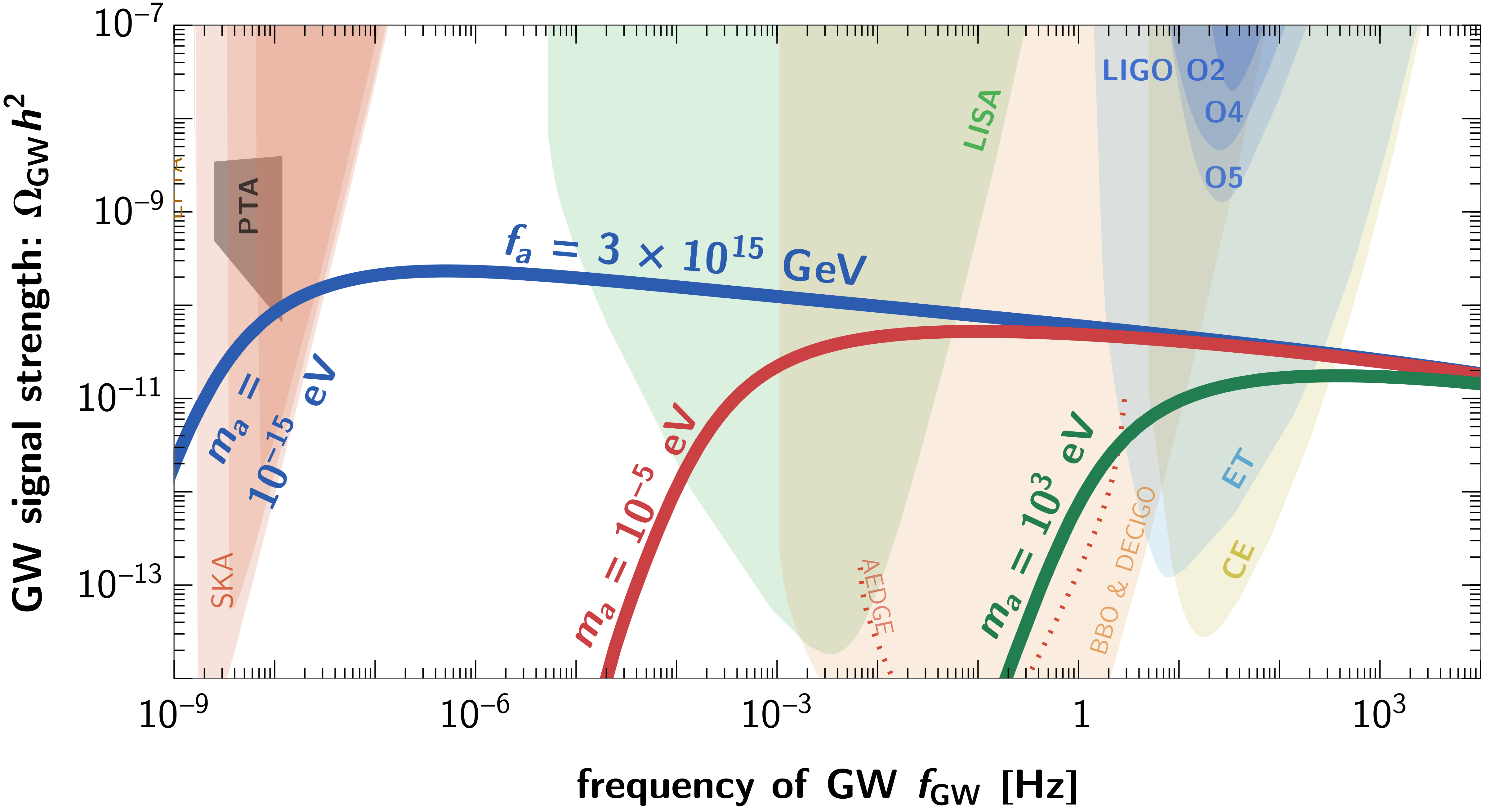
$$f_{\rm GW}^{\rm cs}(T) \simeq 63~{\rm nHz} \left(\frac{\alpha}{0.1}\right) \left(\frac{T}{10{\rm MeV}}\right) \left[\frac{g_*(T)}{10.75}\right]^{\frac{1}{4}},$$

α: typical loop size in units of Hubble horizon

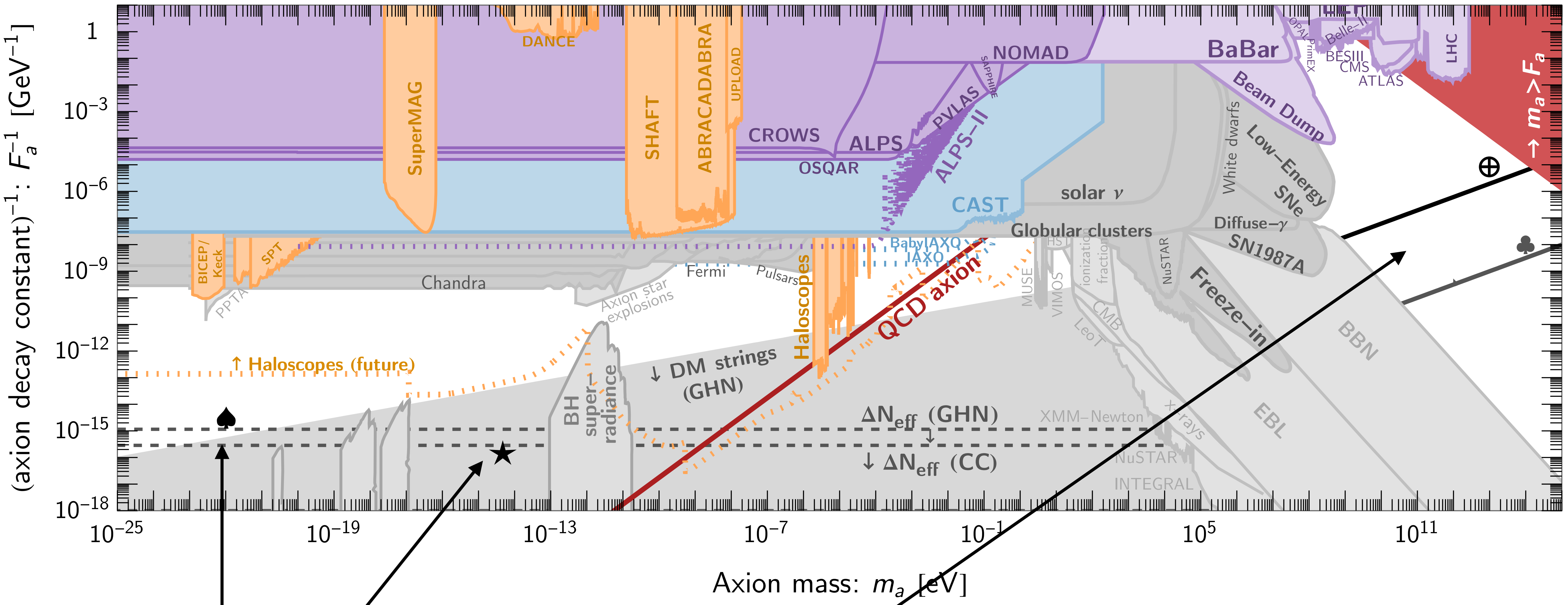
IR cutoff of GW spectrum fixed by axion mass .

Network decays when H ~ m_a

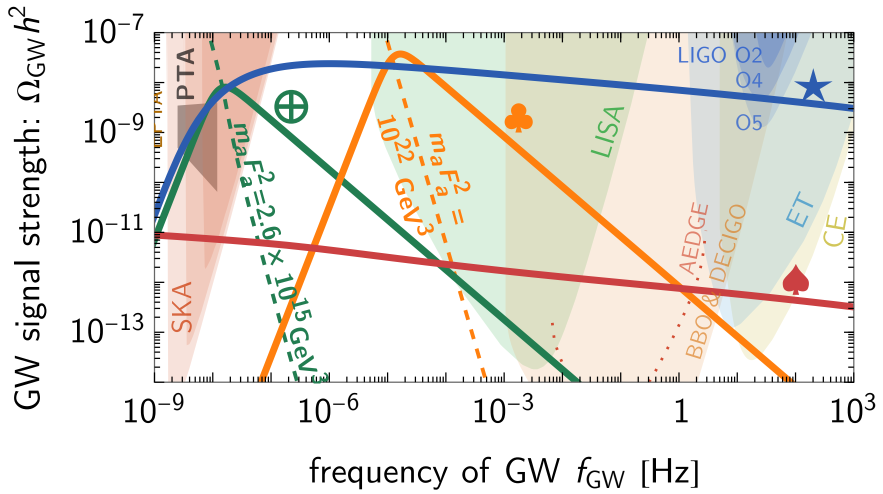
$$f_{\mathrm{GW}}^{\mathrm{cs}}(m_a) \simeq 9.4\ \mathrm{nHz}\left(\frac{\alpha}{0.1}\right)\left(\frac{m_a}{10^{-15}\mathrm{eV}}\right)^{\frac{1}{2}}.$$



Gravitational-wave constraints on axion parameter space from axionic strings.

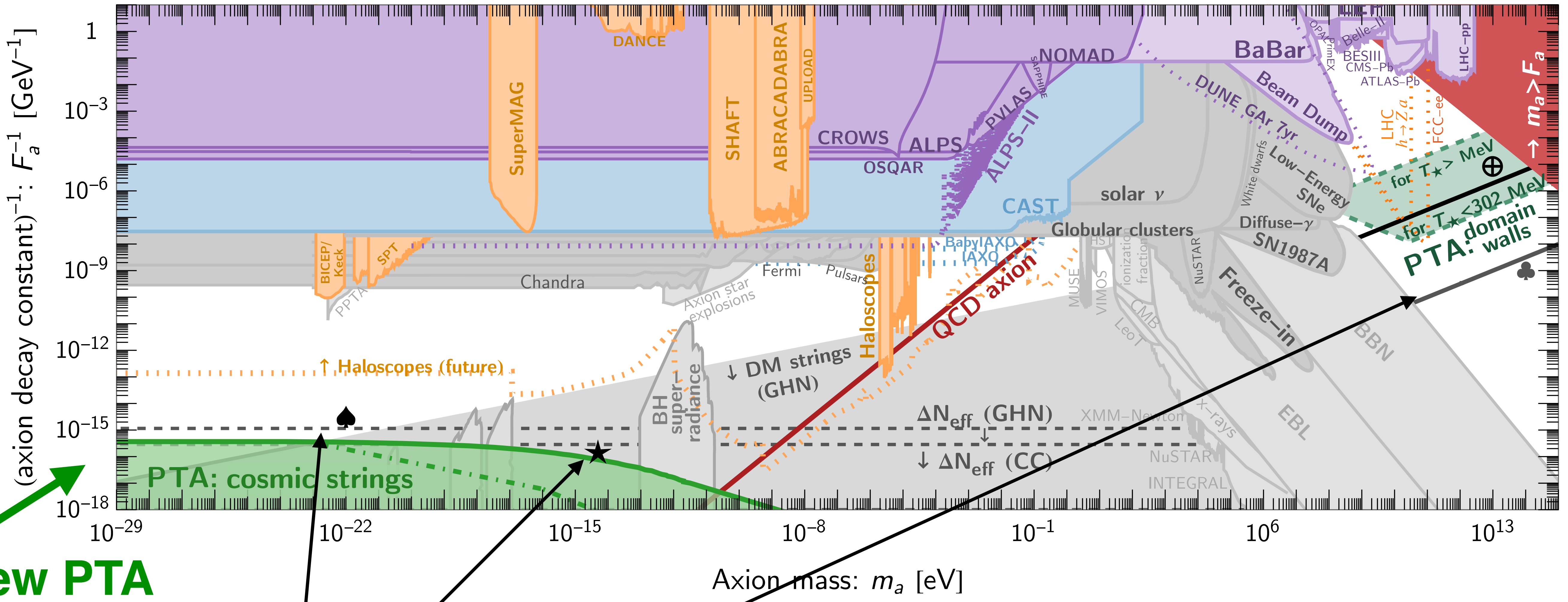


Benchmark spectra



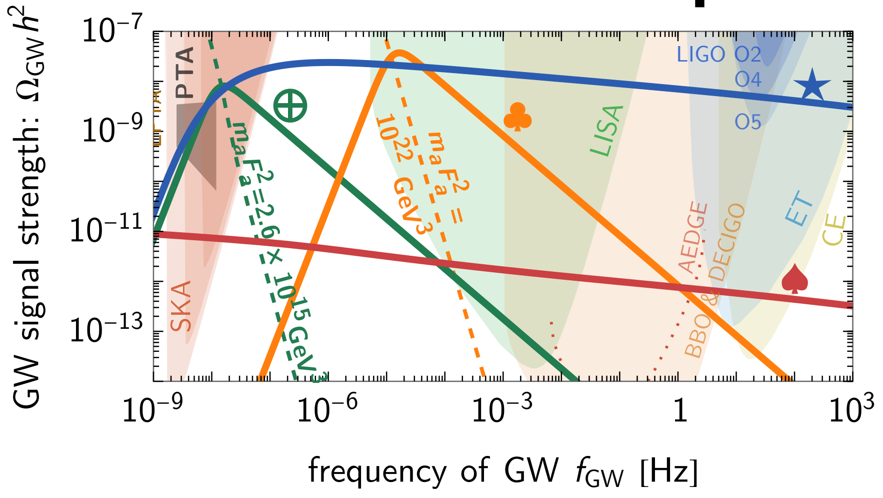
Constraining post-inflationary axions with Pulsar Timing Arrays .

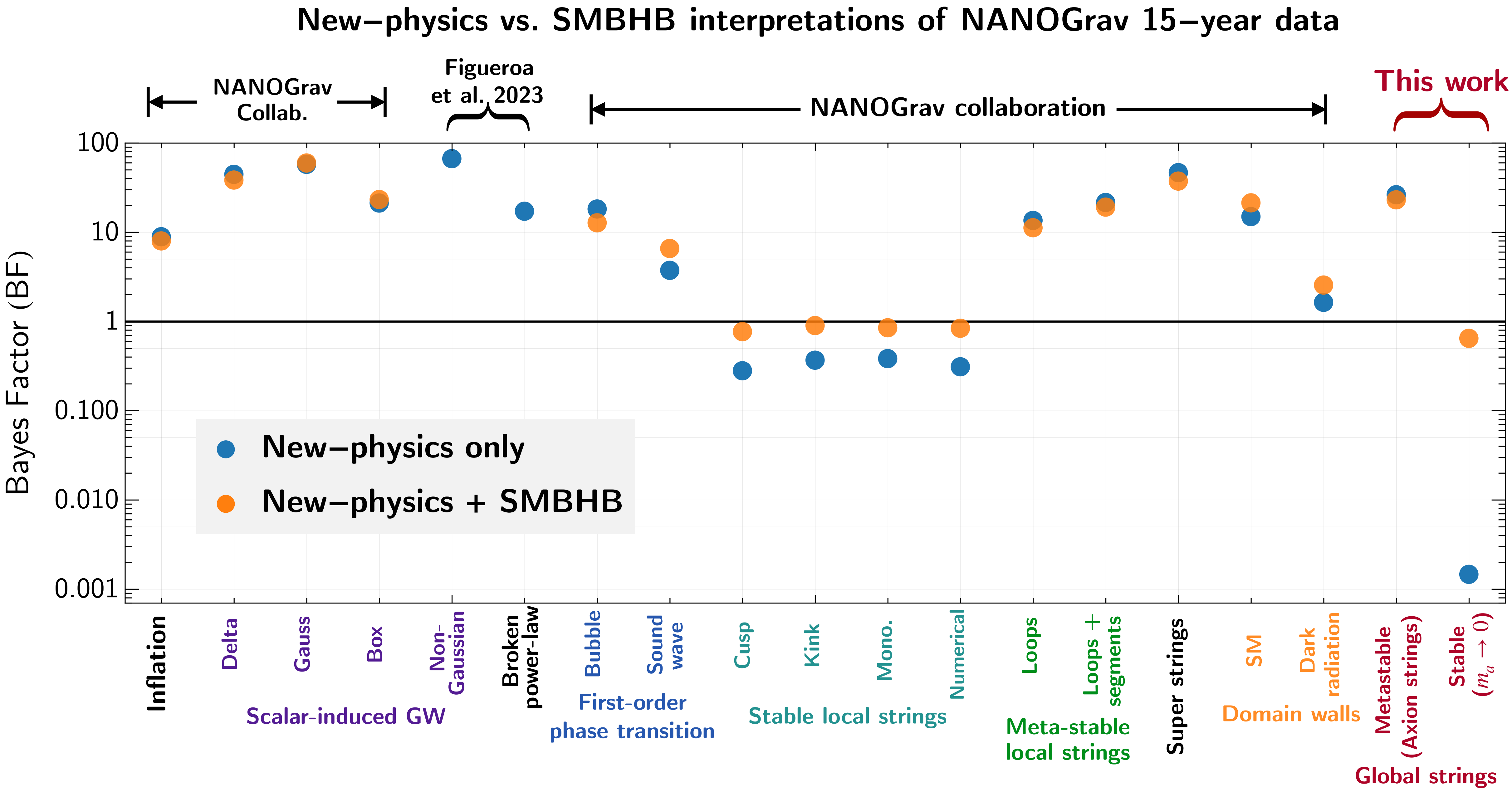
[Servant, Simakachorn, 2307.03121]



New PTA constraint

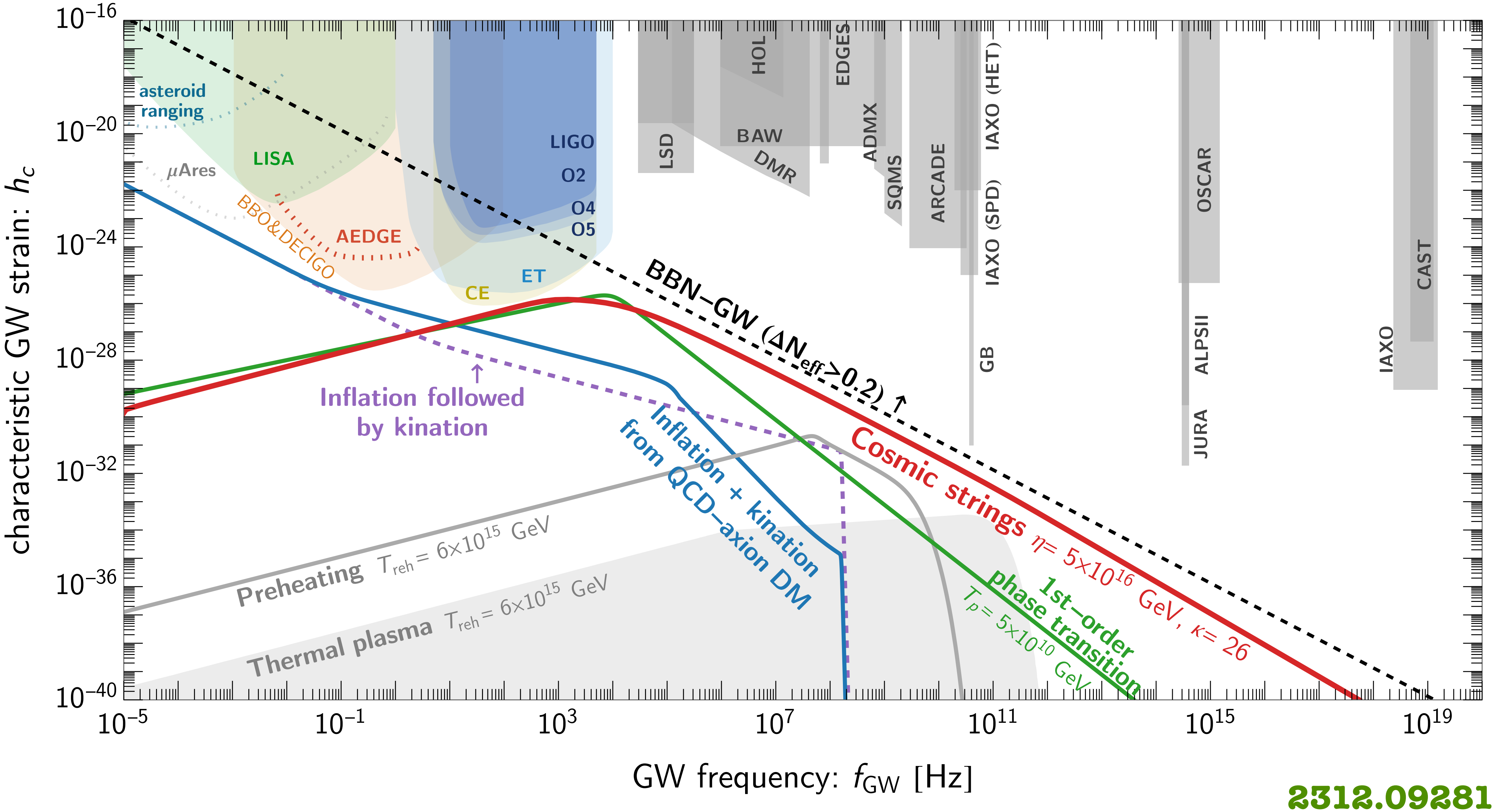
Benchmark spectra





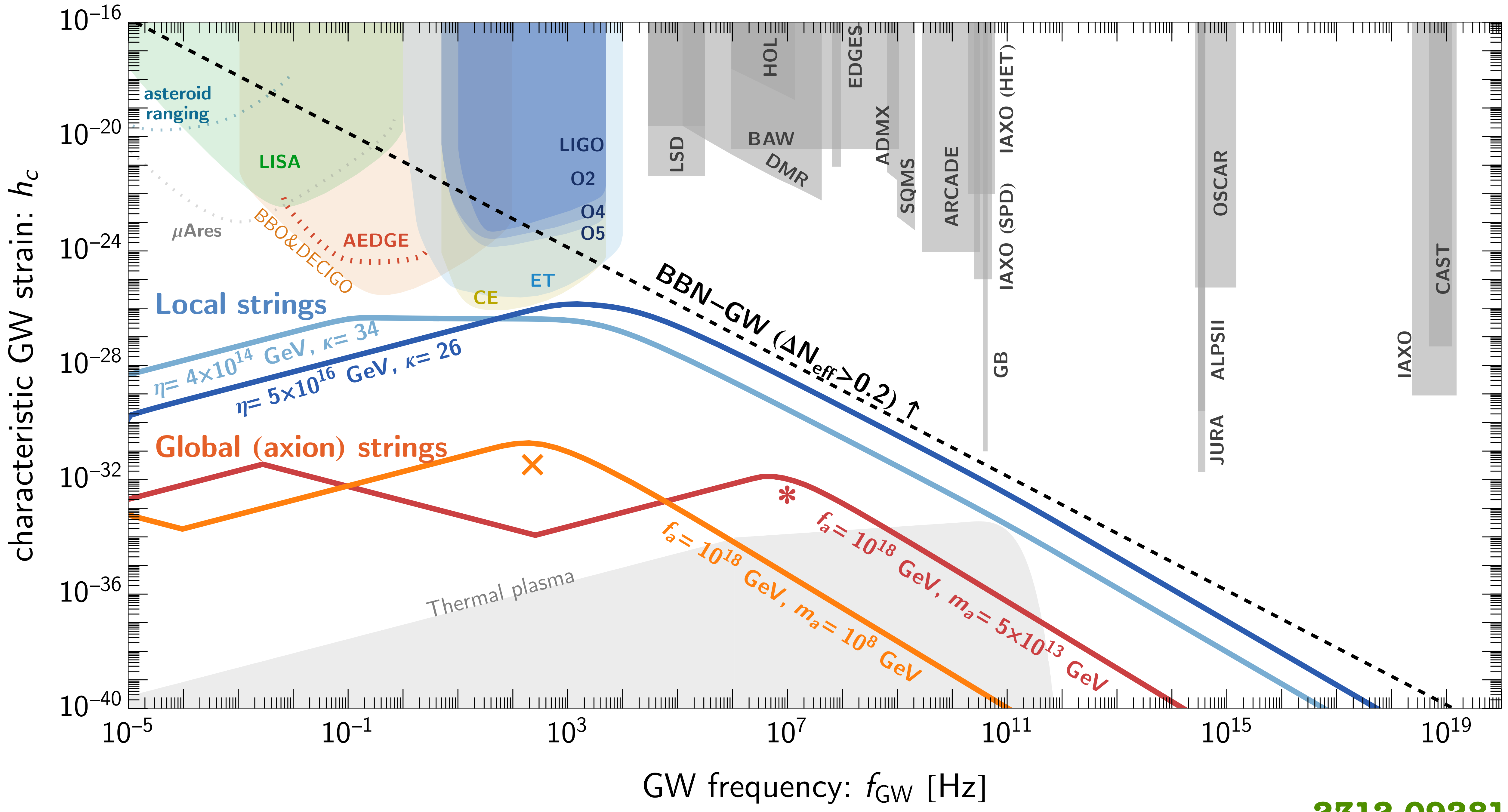
[Servant, Simakachorn, 2307.03121]

Ultra-high frequency primordial GWs .



2312.09281

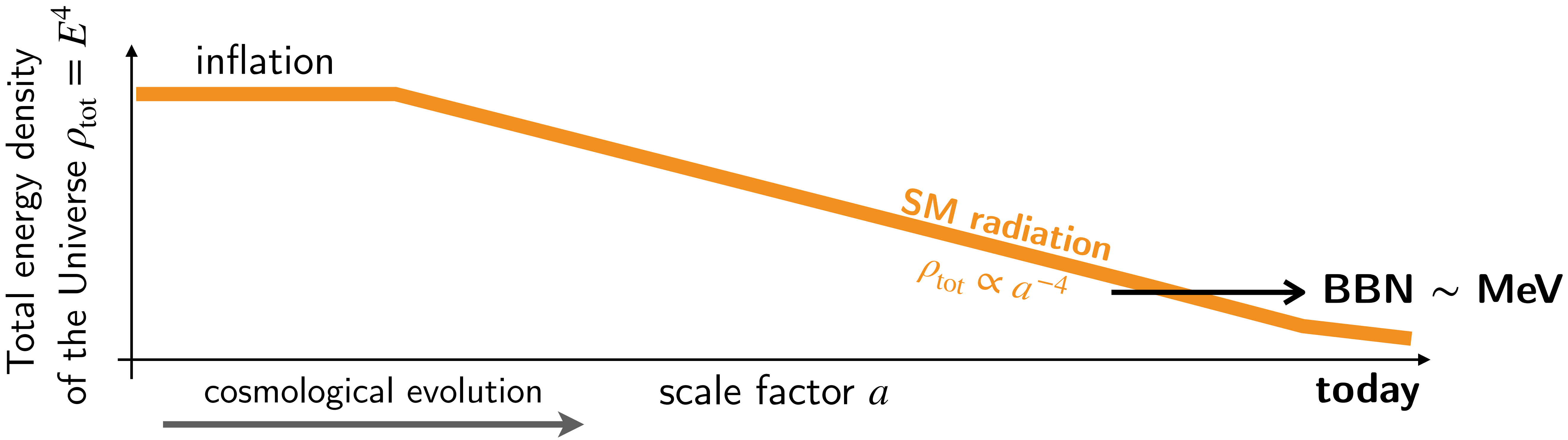
Ultra-high frequency GWs from local versus global cosmic strings .



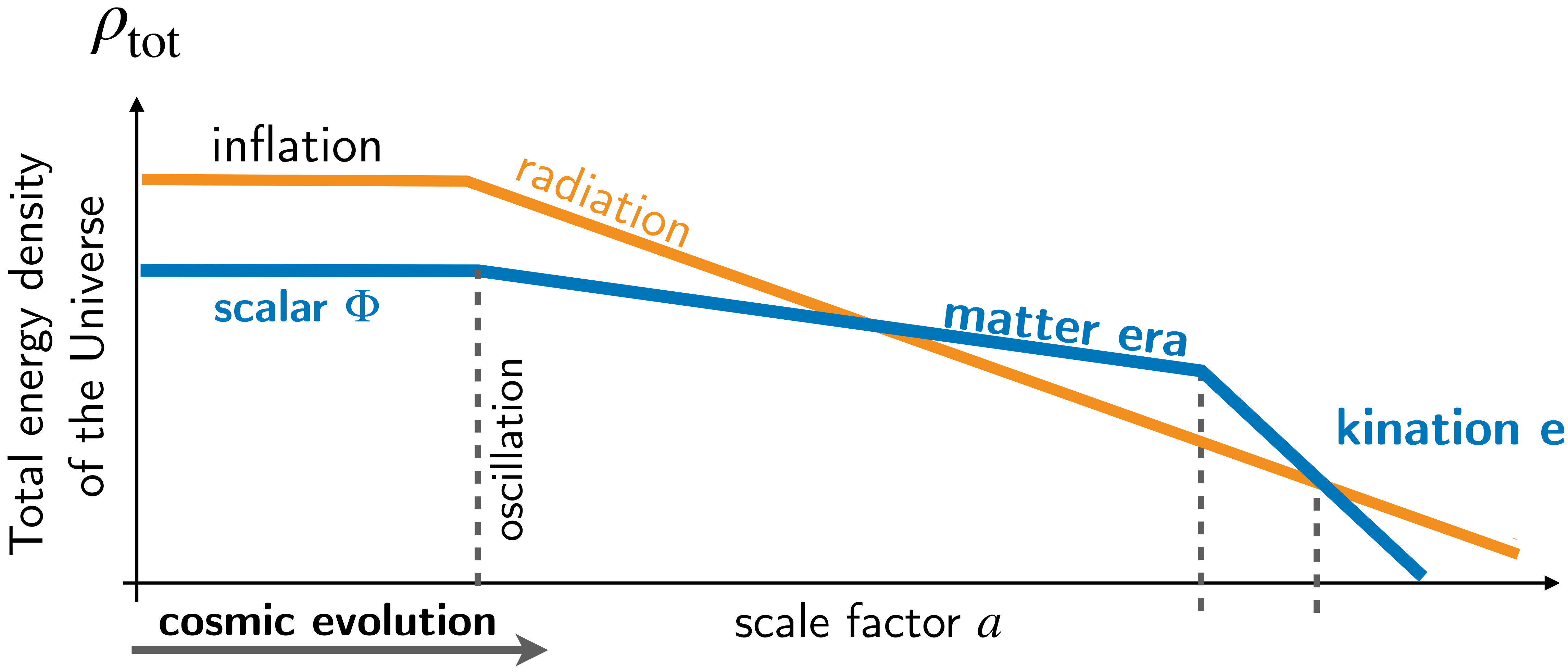
2312.09281

Effect of non-standard cosmology on primordial GW spectra.

Standard cosmological history



Early matter+kination era

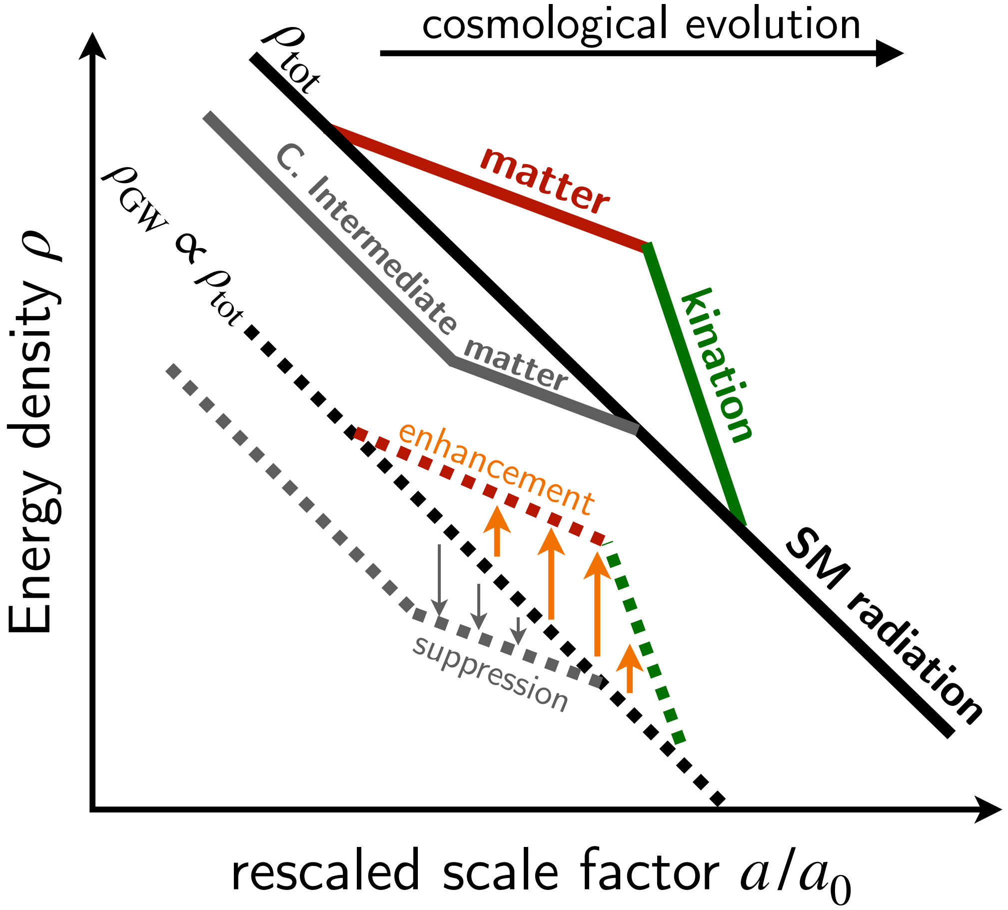


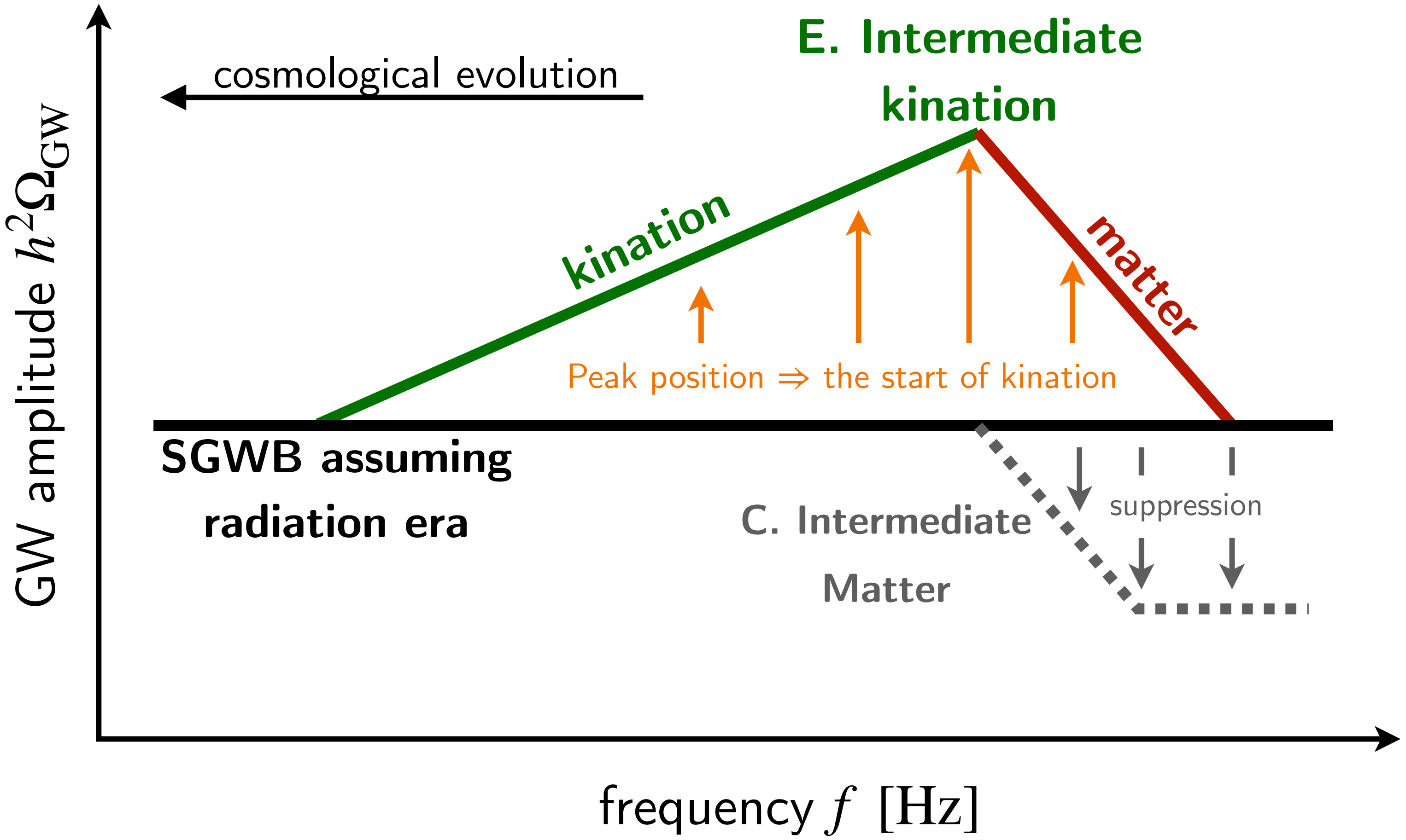
GW from cosmic strings and from inflation track the total energy density of the universe.

—> Significantly enhanced by a matter + kination era

Impact of the cosmological history on Gravitational Waves:

[1912.02569] [2111.01150]

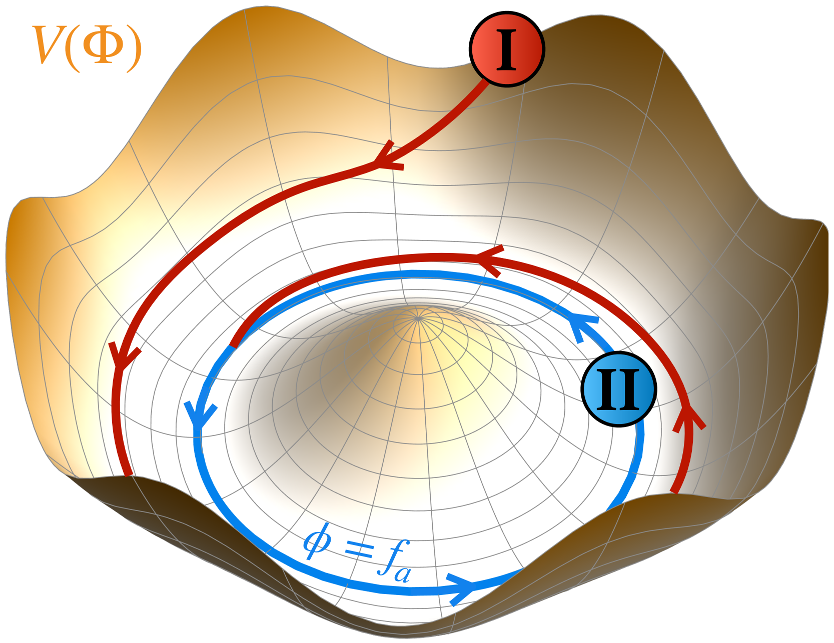


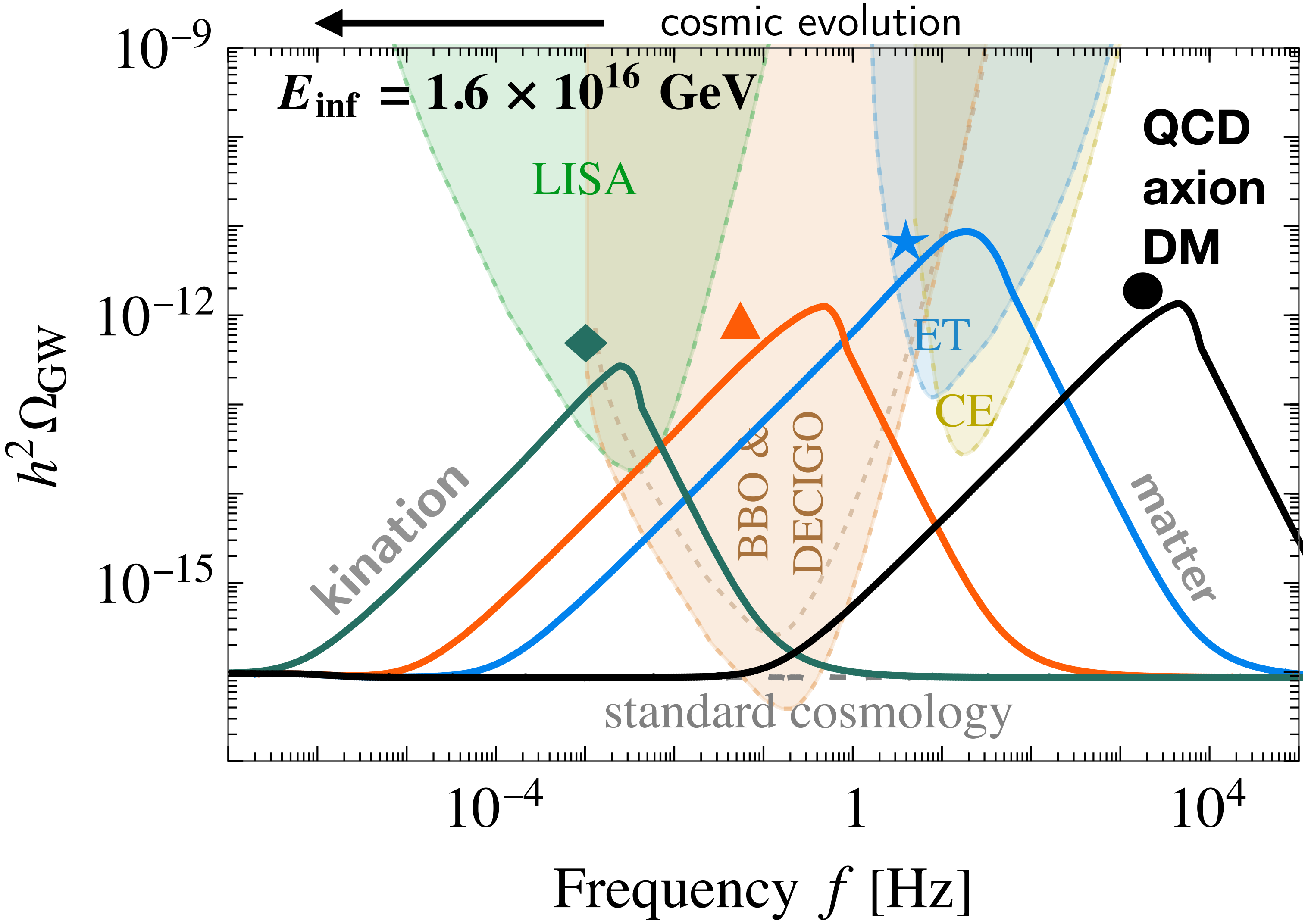


Fraction of energy density in GW today

$$\Omega_{\rm GW,0} = \left(\frac{\rho_{\rm GW,prod}}{\rho_{\rm tot,0}}\right)\left(\frac{a_{\rm prod}}{a_0}\right)^4 = \left(\frac{\rho_{\rm GW,prod}}{\rho_{\rm tot,prod}}\right)\left(\frac{\rho_{\rm tot,prod}}{\rho_{\rm tot,0}}\right)\left(\frac{a_{\rm prod}}{a_0}\right)^4$$

Amplification of inflationary GW from axion-induced kination era.

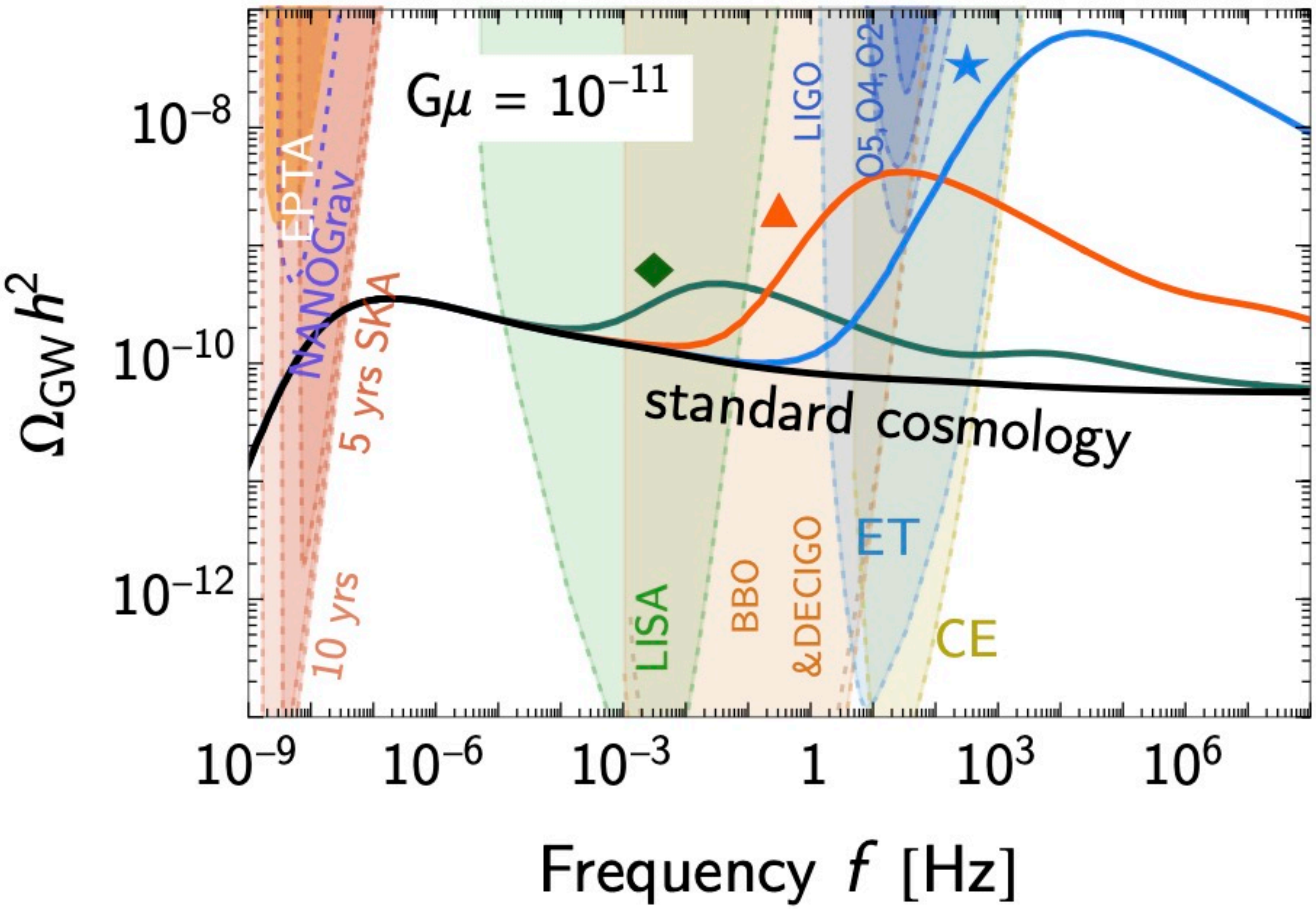




f_a~10^{10}GeV
N_{KD} =5

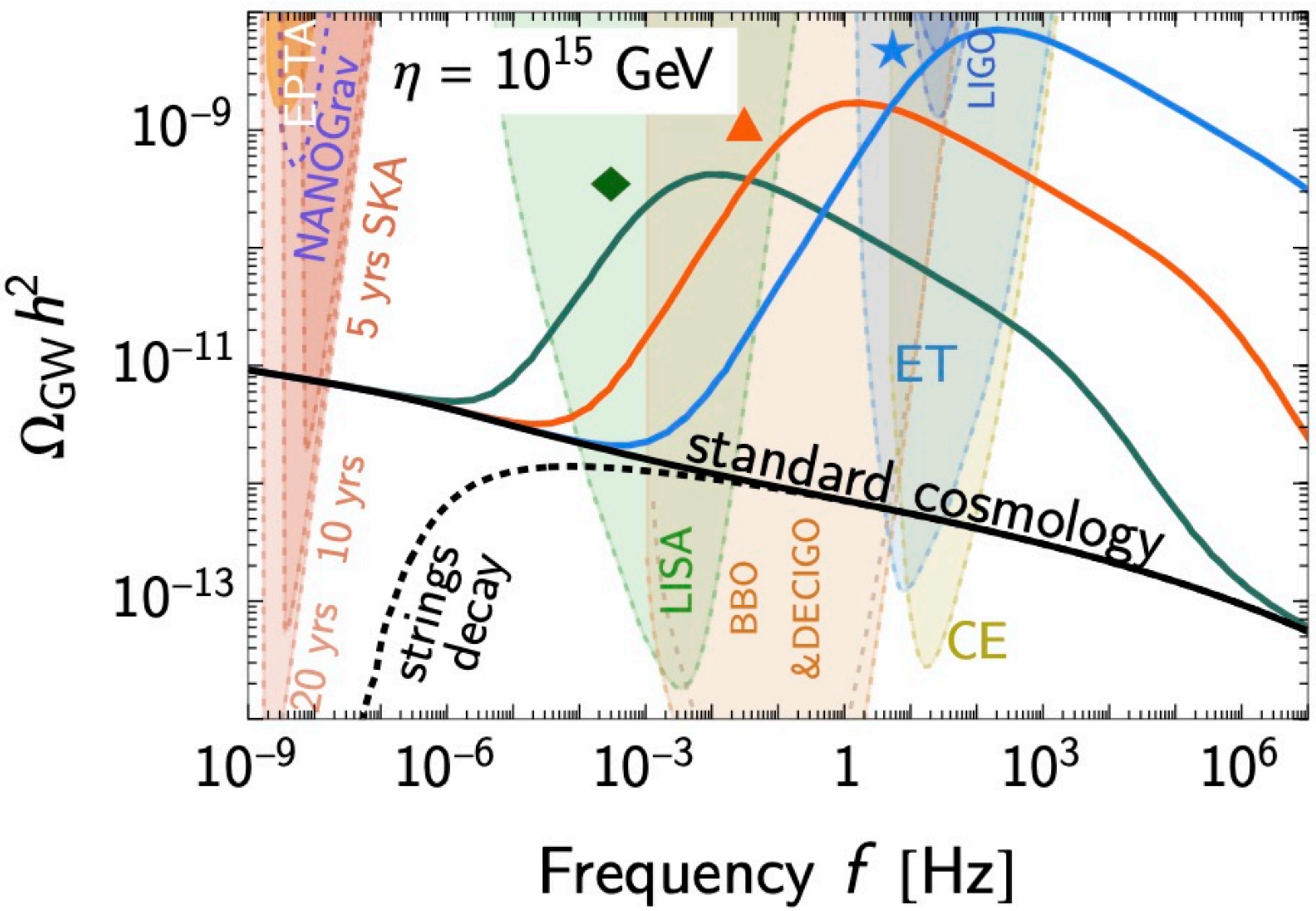
[Gouttenoire et al 2108.10328 & 2111.01150]

Amplification of GW from local cosmic strings due to an axion-induced kination era.



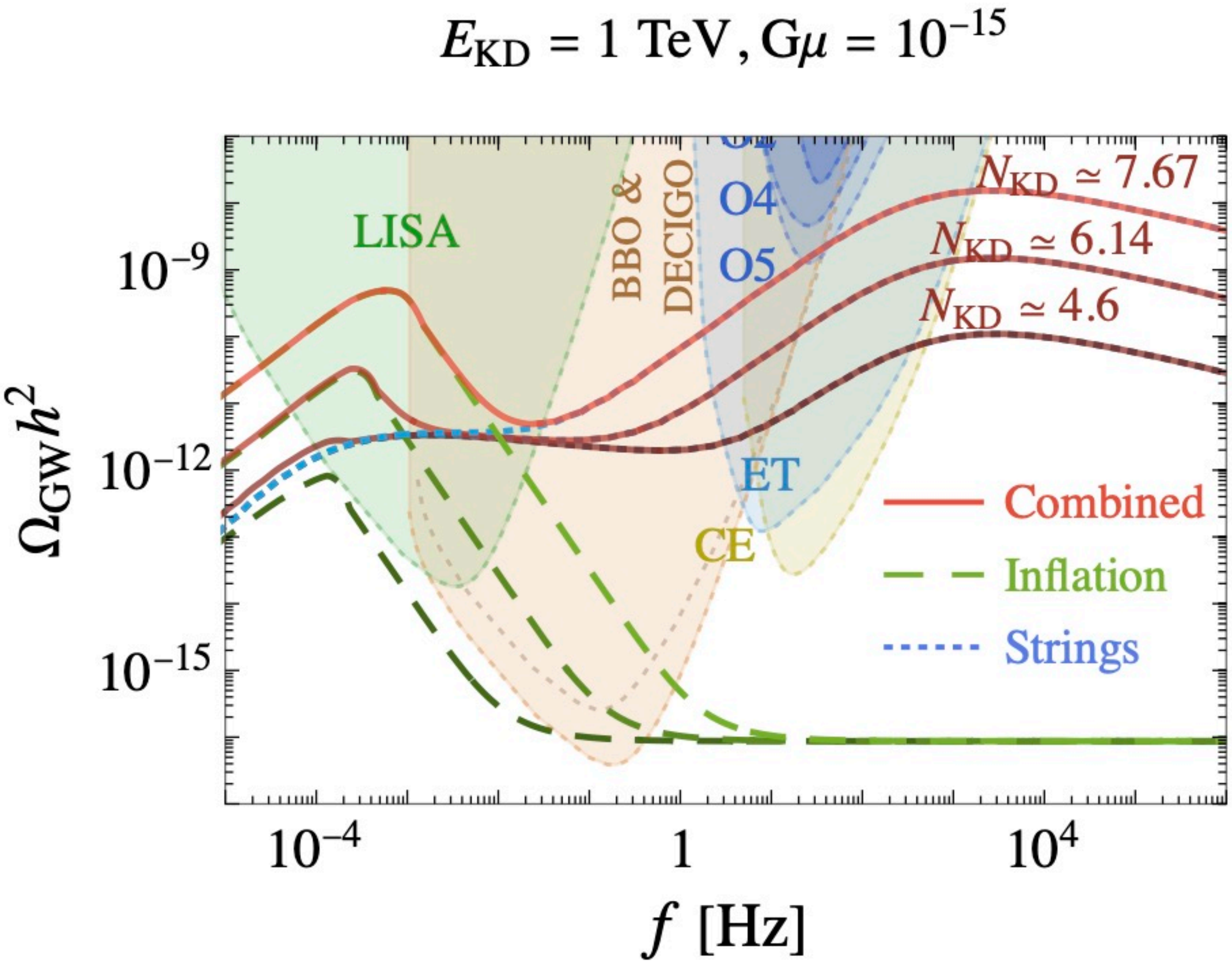
[2111.01150]

Amplification of GW from global cosmic strings due to an axion-induced kination era.



[2111.01150]

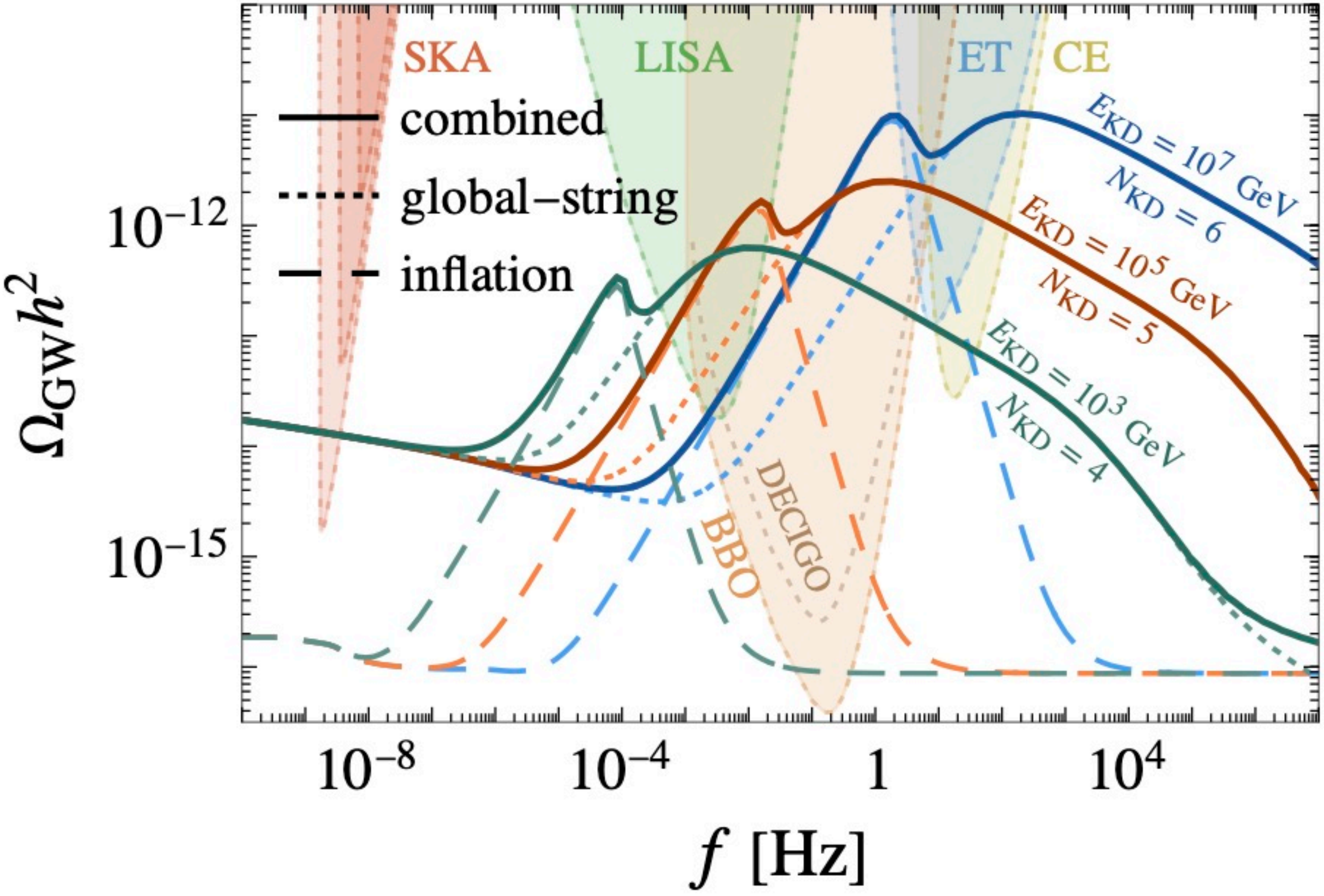
Gravitational Waves from inflation & local cosmic strings in non-standard cosmology induced by rotating axions.



$E_{\mathrm{KD}} = 1$ TeV, $G\mu = 10^{-15}$

[2111.01150]

Gravitational Waves from inflation & global cosmic strings in non-standard cosmology induced by rotating axions.



[2111.01150]

GWs
from axion fragmentation.

GWs from axion fragmentation.

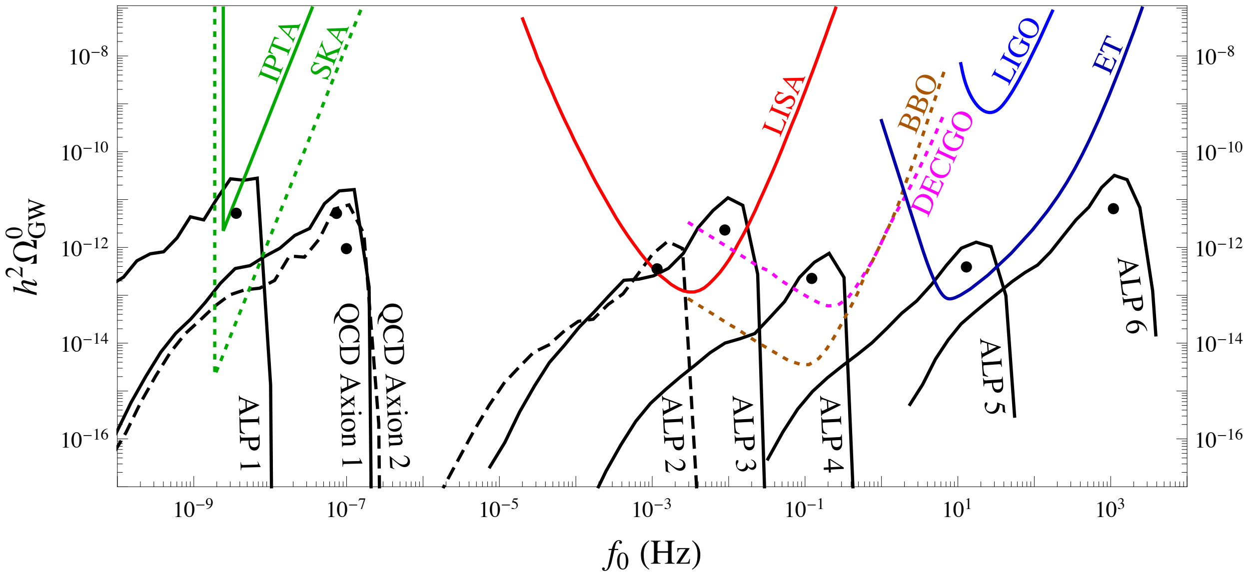
The transfer of energy in the early universe from the homogeneous axion field into axion quantum fluctuations, inevitably produces a stochastic background of gravitational waves of primordial origin with a peak frequency controlled by the axion mass.

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta h_{ij}}{a^2} = \frac{16\pi}{M_{\rm pl}^2}\Pi_{ij}^{\rm TT},$$

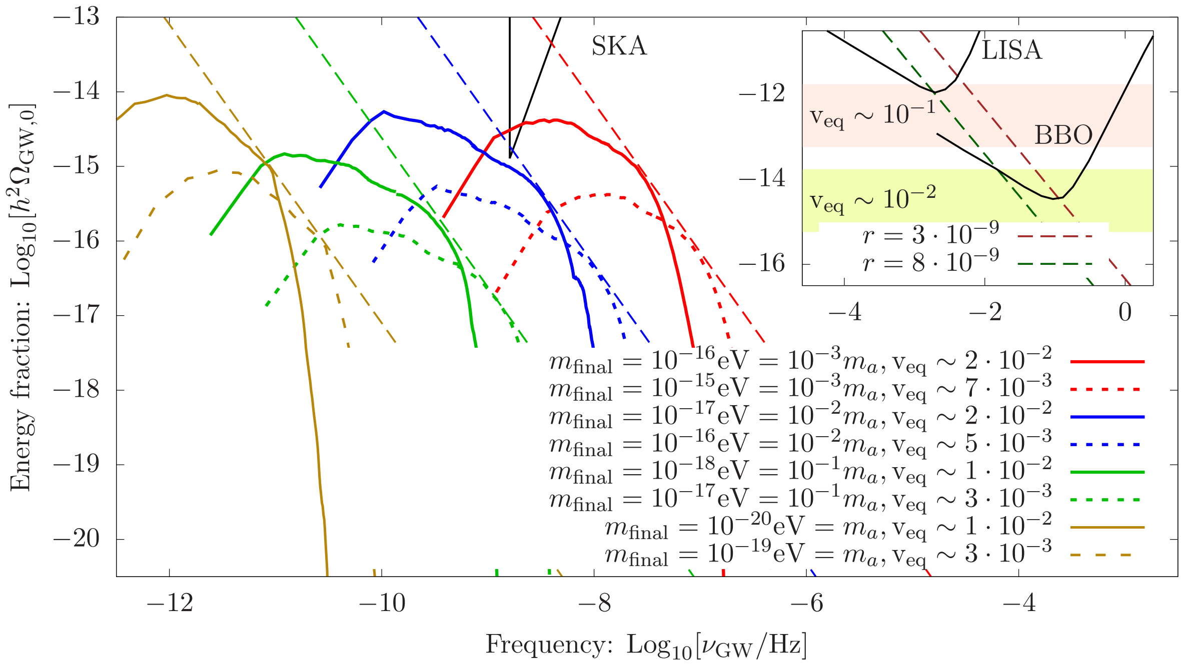
$$\Pi_{ij}^{\rm TT}(t,\vec{x}) = \frac{1}{a^2}\left[\partial_i\phi(t,\vec{x})\partial_j\phi(t,\vec{x}) - \frac{1}{3}\delta_{ij}(\partial_k\phi(t,\vec{x})\partial_k\phi(t,\vec{x}))\right]$$

Examples.

Machado et al,
1811.01950
‘Audible axions’
(Excite dark photon)



Chatrchyan, Jaeckel
2004.07844



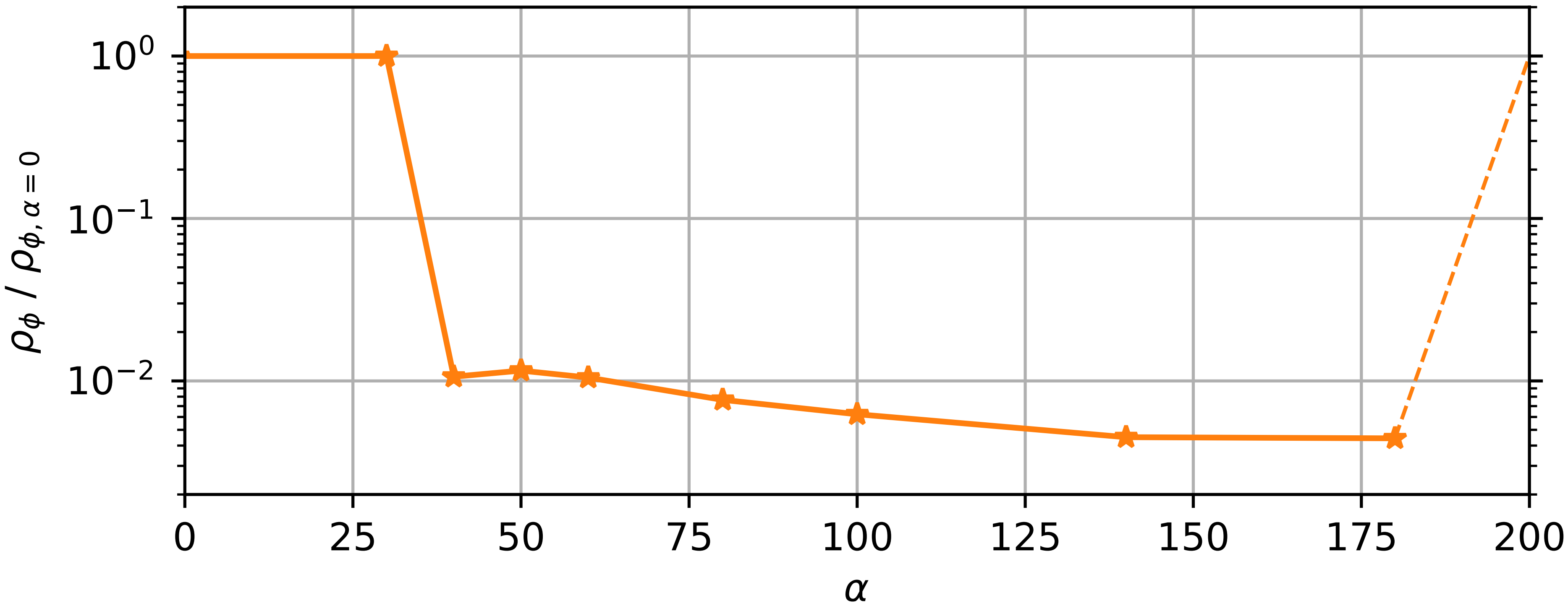
However.

The signal is generally suppressed when imposing the upper bounds from either the axion dark matter abundance or the axion dark radiation.

Schwaller et al, 2012.11584 (from coupling to dark photon)
Eroncel et al, 2206.14259
Geller et al, 2307.03724

—>Dilution of ALP energy density needed, not easy

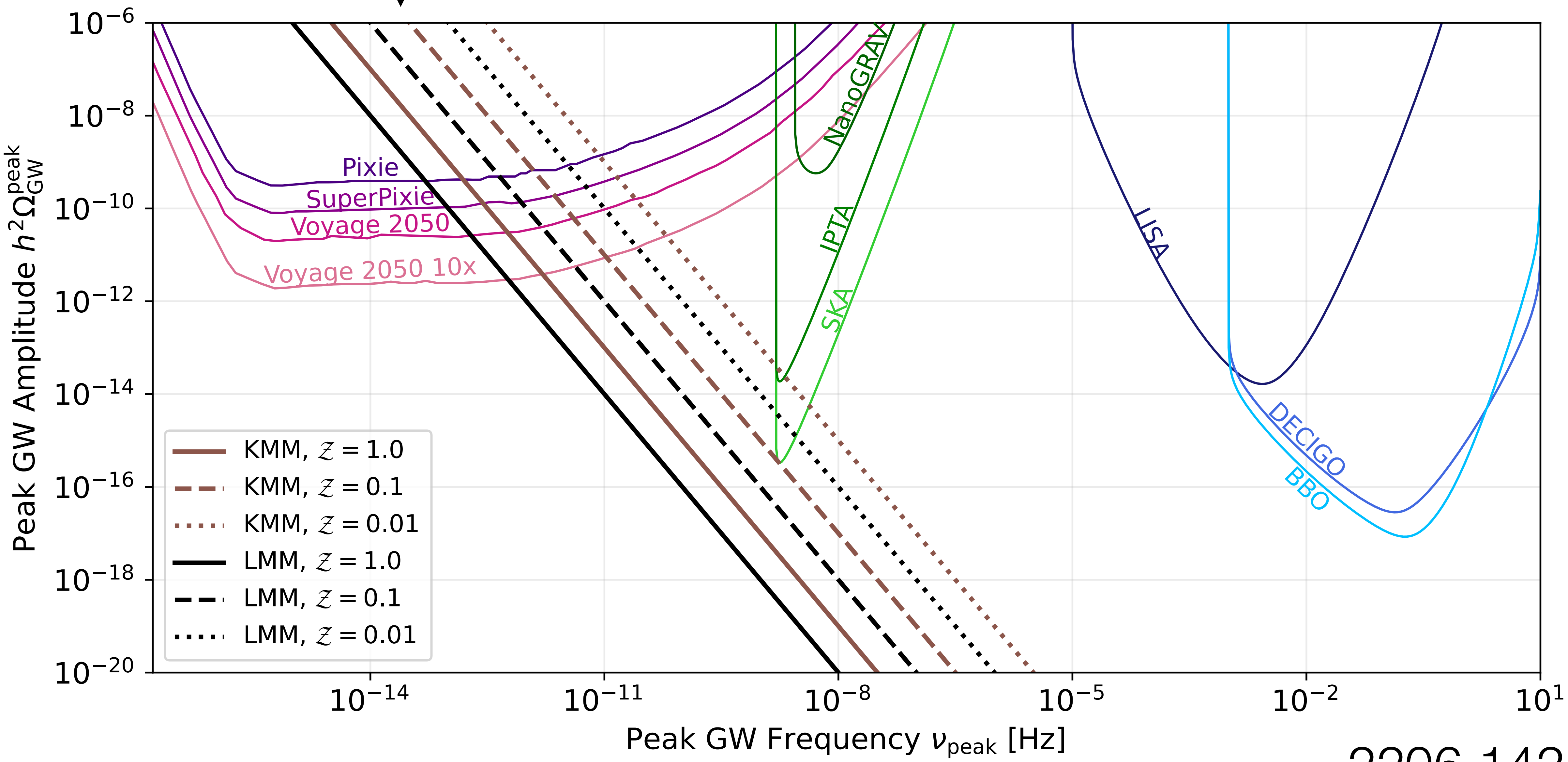
Achieved dilution factor of ALP energy density



Ratzinger, Schwaller, Stefanek, 2012.11584

Gravitational waves from ALP DM fragmentation.

position of the peak of the GW spectrum



2206.14259

$\mathcal{Z}$ = needed dilution factor of ALP energy density

(Can arise from non-linear effects after the fragmentation.)

Conclusion.

Gravitational waves: complementary probes of

- Cosmological phase transitions
- Early equation of state of the universe
- Scalar field dynamics. (before/during/after inflation)
- Scalar Dark Matter production mechanism (from misalignment or from decay of defects)

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

DESY THEORY WORKSHOP

WHISPERS FROM THE DARK UNIVERSE – PARTICLES & FIELDS IN THE GRAVITATIONAL WAVE ERA

HELMHOLTZ

24 - 27 September 2024 DESY Hamburg, Germany

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

DESY
DESY Theory Group

Programme on INDICO

Andreas Ringwald Fest

The annual DESY Theory Workshop is organized by the elementary particle physics community in Germany. The focus is on a topical subject in theoretical particle physics and related fields. The workshop features:

- **Plenary sessions** of specialized talks by invited speakers.
- **Parallel sessions**, allowing young researchers to present their work (Wednesday and Thursday afternoon).
- The **DESY Heinrich-Hertz-Lecture on Physics** for public outreach.

Plenary Talks

P. Agrawal (Oxford U.)
O. Buchmueller (ICL London)
M. Buschmann (GRAPPA/UvA)
A. Chou (Fermilab)
S. Ellis (Geneva U.)
R. Flauger (UC, San Diego)
G. Franciolini (CERN)
J. Harz (Mainz U.)
L. Heisenberg (Heidelberg U.)
A. Hook (Maryland U.)
M. Kamionkowski (J. Hopkins)
E. Lim (King's College)
M. Peloso (Padua U.)
N. Porayko (MPI Bonn)
R. Porto (DESY)
C. Prescod-Weinstein (N. Hampsh.)
E.-M. Rossi (Leiden U.)
K. Schutz (McGill U.)
X. Siemens (Oregon State U.)
J. van de Vis (Leiden U.)

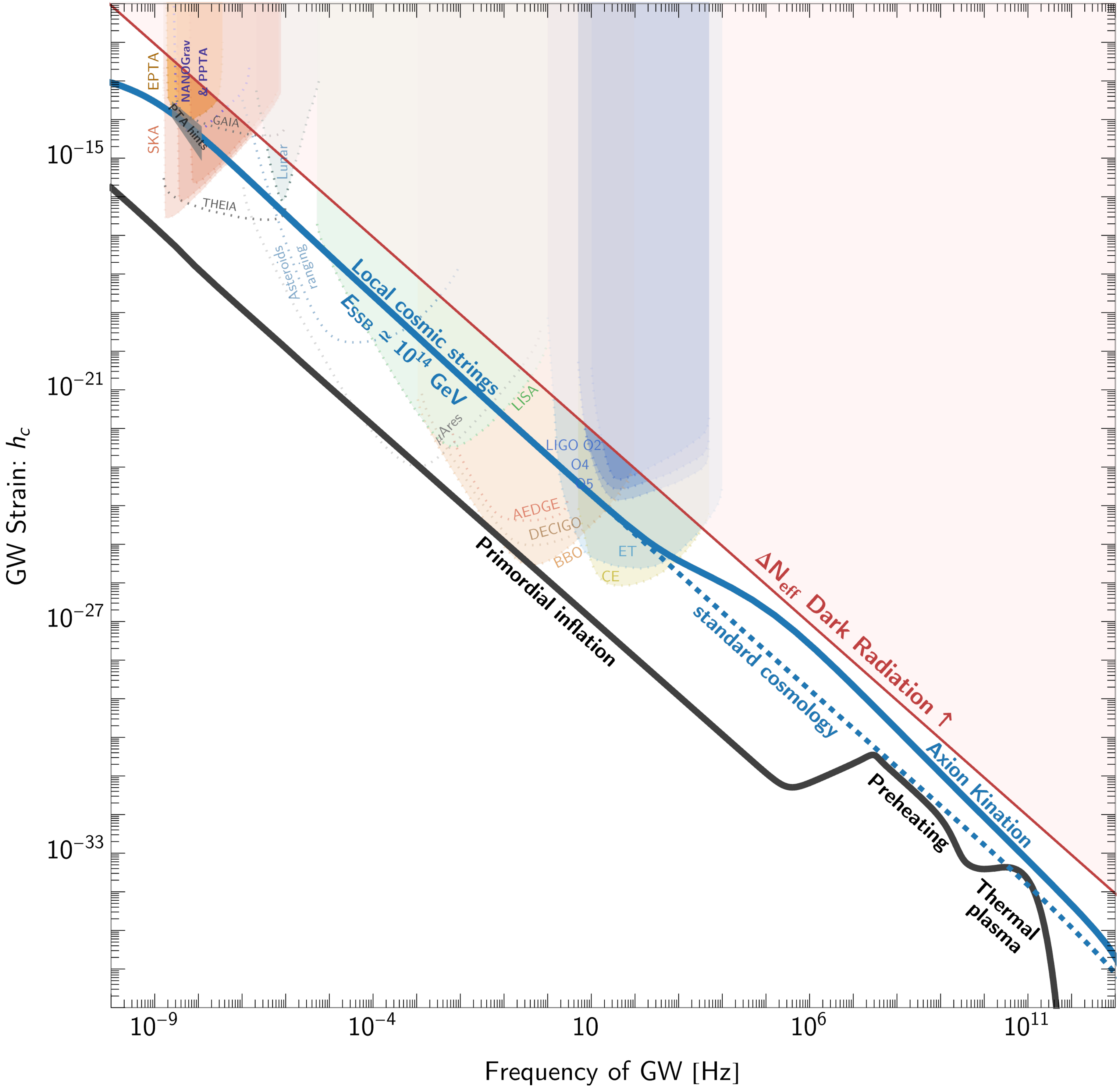
DESY Heinrich Hertz Lecture on Physics

Marc Kamionkowski (Johns Hopkins University)
Thursday, September 26, 2024, DESY Auditorium

Parallel Sessions

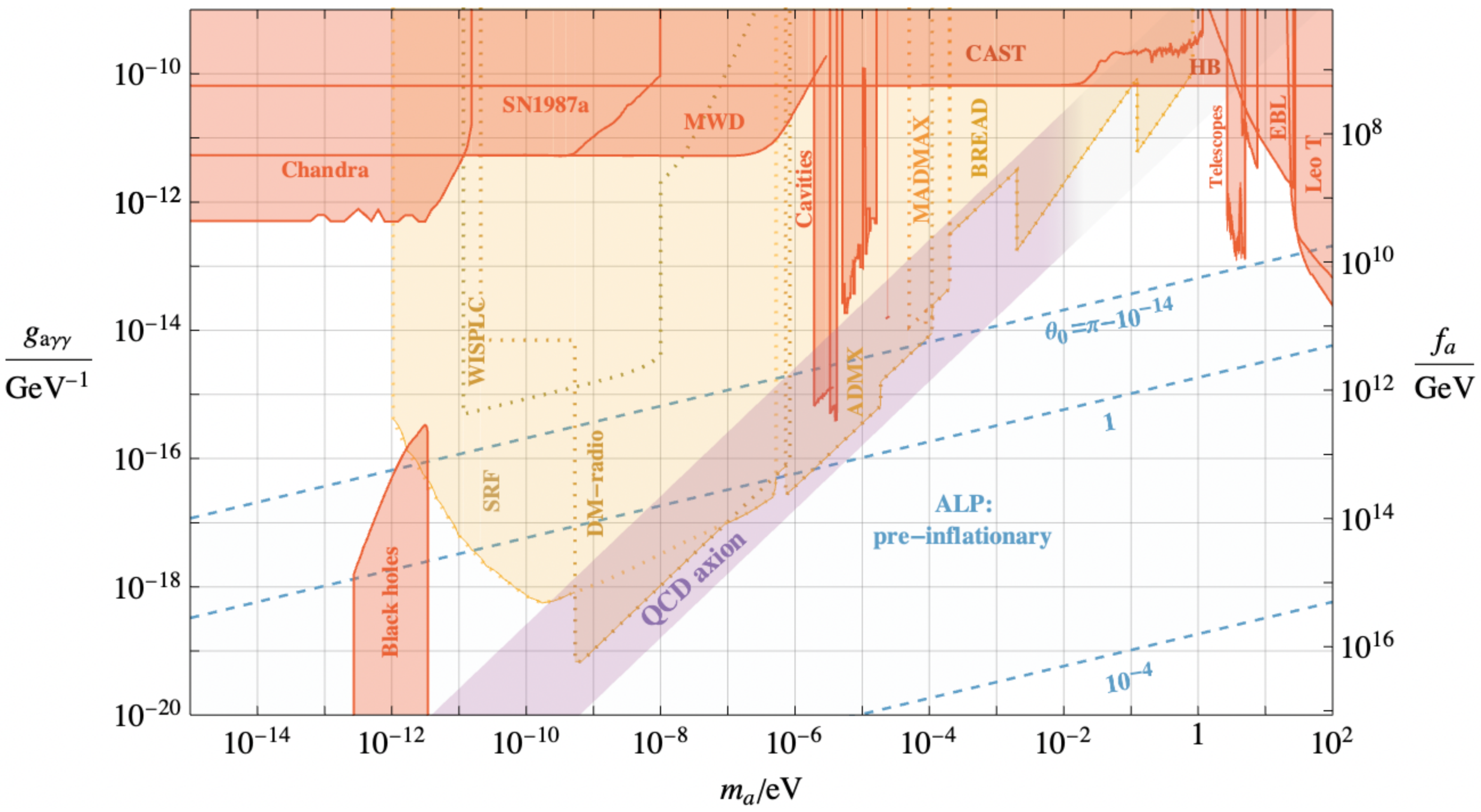
Extra material.

$$h_c \simeq 1.26 \times 10^{-18} (\mathrm{Hz}/f_{\mathrm{GW}}) \sqrt{\Omega_{\mathrm{GW}} h^2}.$$



Slide by Marco Gorghetto

ALPs: Targets for haloscopes



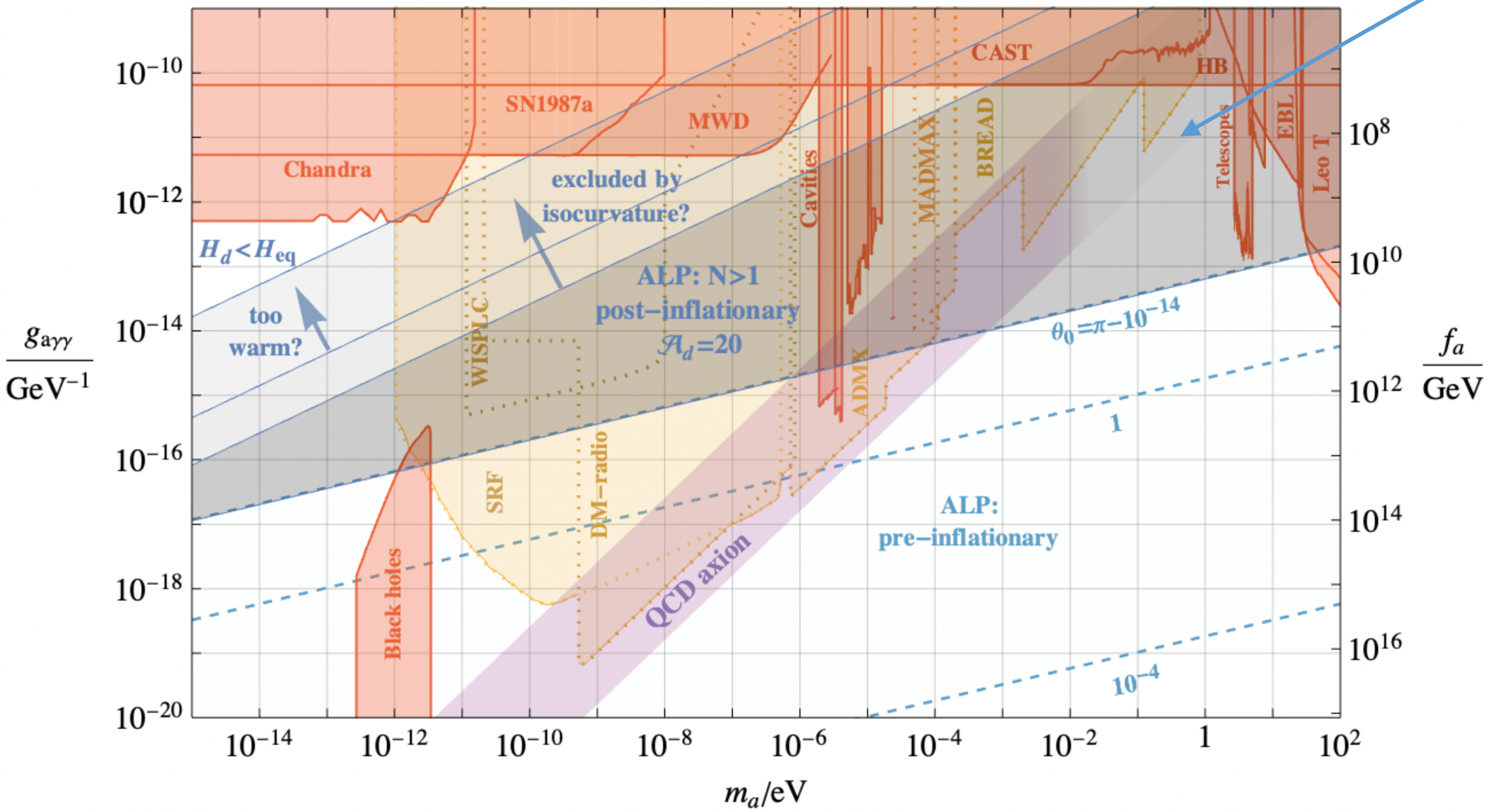
$$\frac{\Omega_a^{\rm mis}}{\Omega_{\rm DM}} \simeq 2.2\cdot 10^{-3}\left(\frac{f_a}{10^{12}\ \rm GeV}\right)^2\left(\frac{m_a}{10^{-6}\ \rm eV}\right)^{1/2} h(\theta_0)\,\theta_0^2$$

$$g_{a\gamma\gamma}\simeq\frac{\alpha_{em}}{2\pi f_a}$$

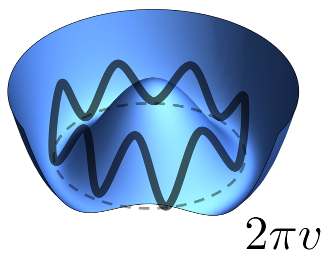
The case $N_{DW} > 1$.

Slide by Marco Gorghetto

ALPs: Targets for haloscopes



Domain wall number



$$v = N f_a$$

$$\boxed{\frac{\Omega_a}{\Omega_{DM}} \simeq 2 \left(\frac{\mathcal{A}_d}{20}\right) \left(\frac{m_a}{H_d}\right)^{1/2} \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{2} \left(\frac{m_a}{10^{-6}\ \text{eV}}\right)^{1/2}}$$

[2212.13263]

M. Gorghetto, E.Hardy

Equation of motion of complex scalar field in expanding universe .

$$\ddot{\Phi} - a^{-2}\nabla^2\Phi + 3H\dot{\Phi} + \frac{\partial V}{\partial \Phi^\dagger} = 0$$

with $\Phi = \phi e^{i\theta}$

$$\begin{aligned}
&\ddot{\phi} - a^{-2}\nabla^2\phi + 3H\dot{\phi} + V'(\phi) = \phi\dot{\theta}^2 - a^{-2}\phi(\nabla\theta)^2, \\
&\phi\ddot{\theta} - a^{-2}\phi\nabla^2\theta + 3H\phi\dot{\theta} = -2\dot{\phi}\dot{\theta} + 2a^{-2}\nabla\phi\nabla\theta.
\end{aligned}$$

For homogeneous field, these are Kepler problem:

centrifugal force

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \phi\dot{\theta}^2$$

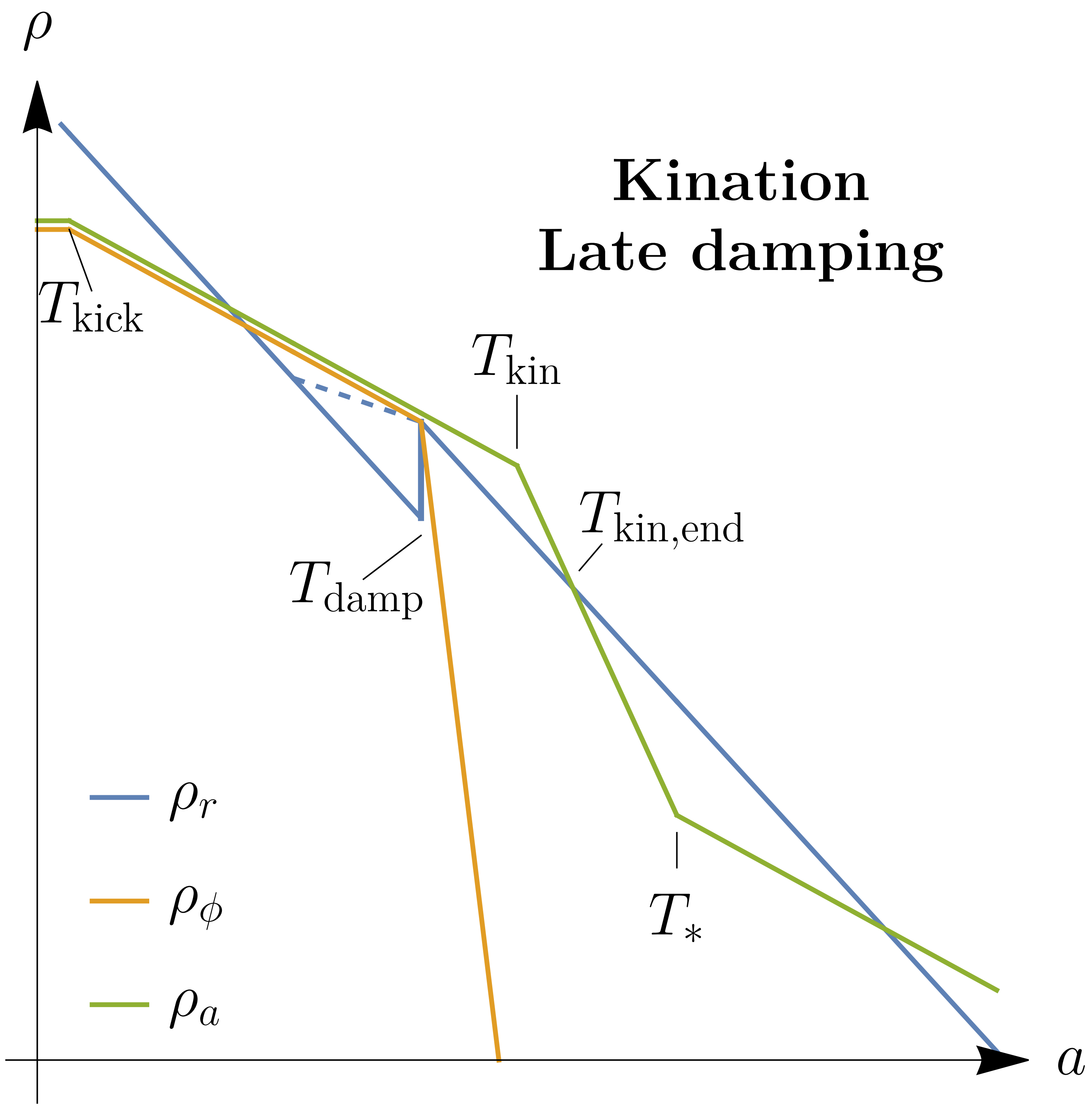
coriolis force

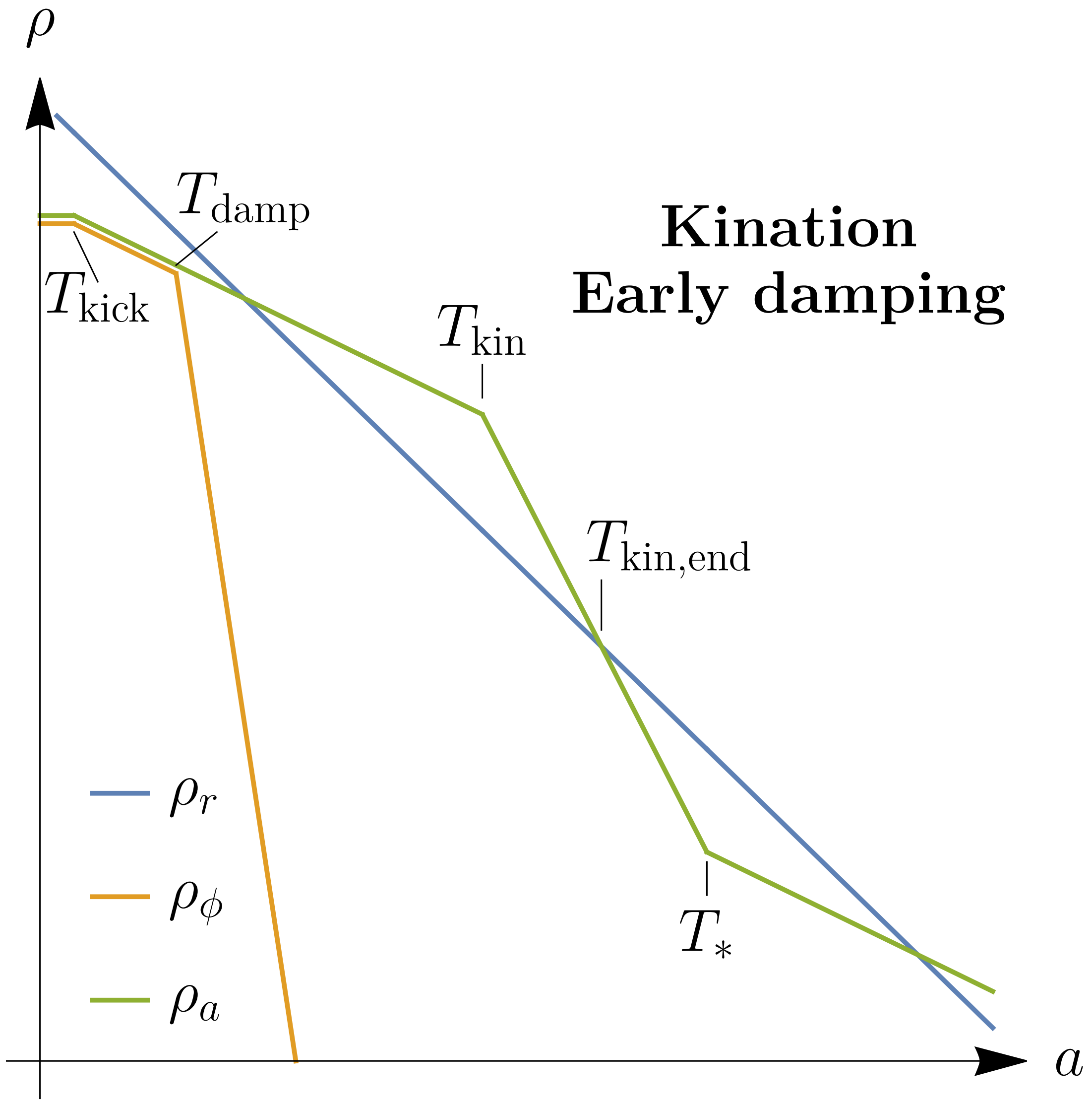
$$\ddot{\theta} + 3H\dot{\theta} = -2\frac{\dot{\phi}}{\phi}\dot{\theta}$$

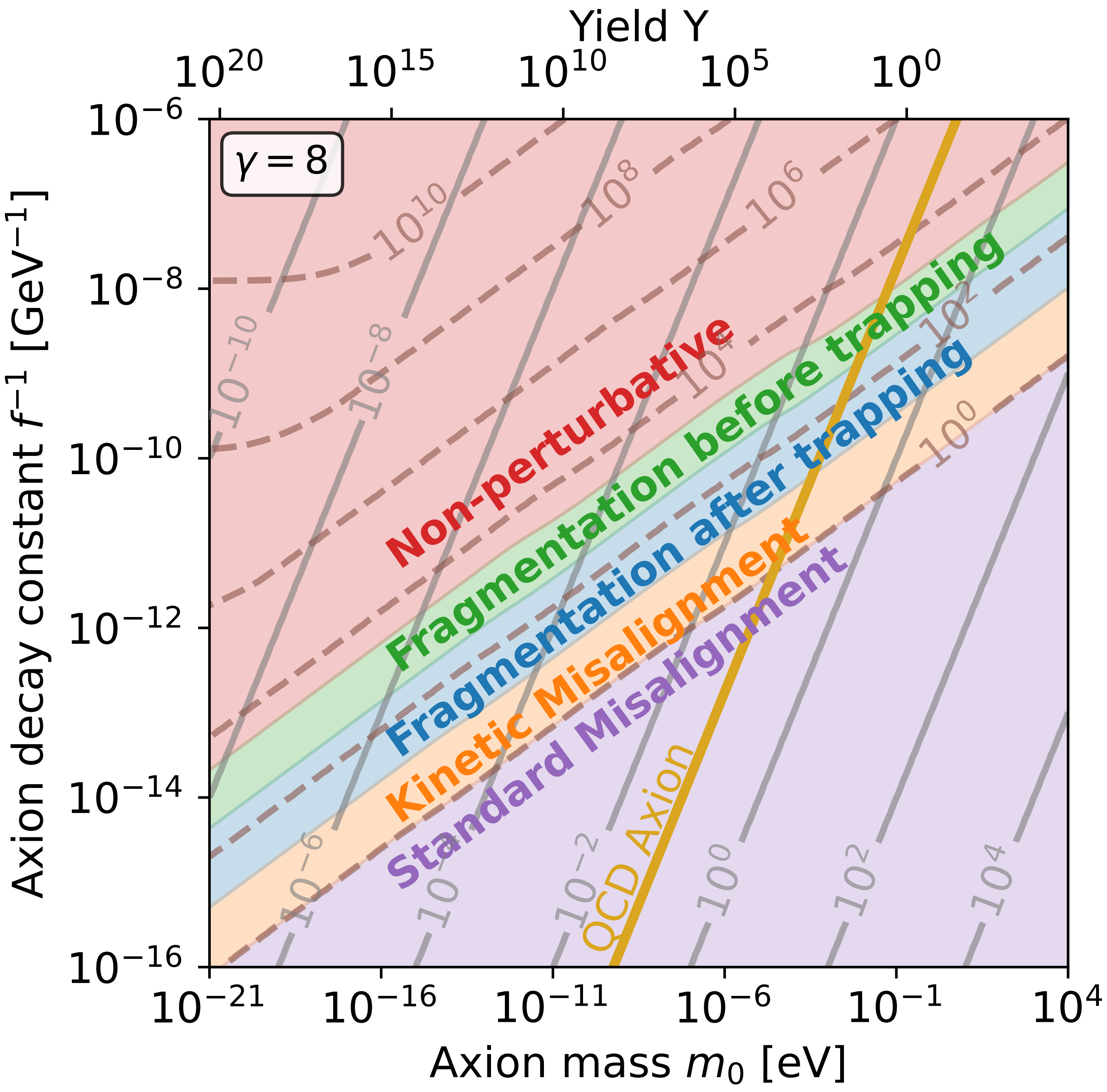
conservation of charge (angular momentum):

$$\frac{d}{dt}(a^3\phi^2\dot{\theta}) = 0$$

Ingredient 4 for kination: Damping of radial mode energy





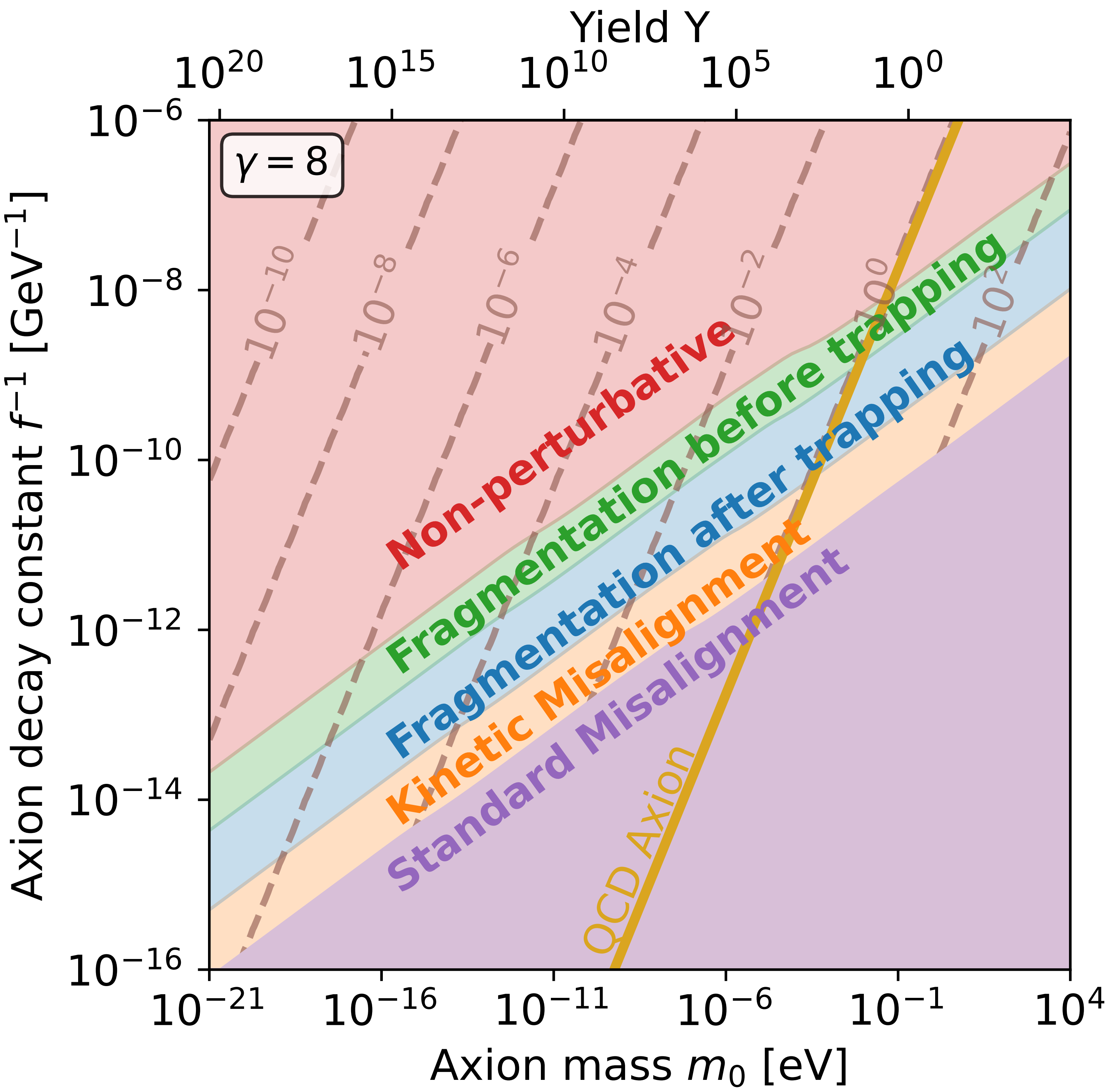


Solid lines: contours of zero-temperature barrier heights,

Dashed lines: (m/3H)_* contours

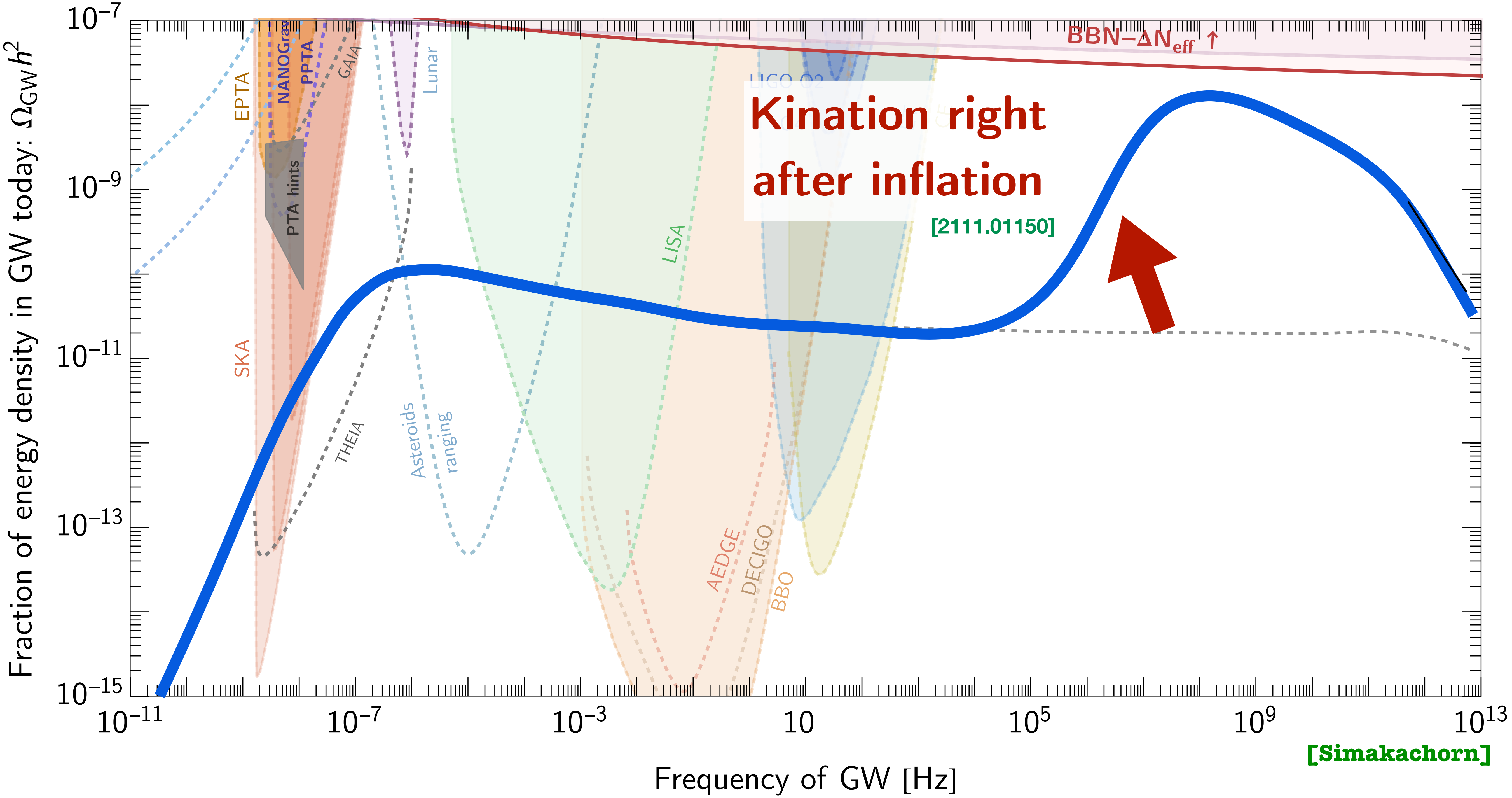
2206.14259

Contours of trapping temperature in GeV.



2206.14259

Gravitational Waves from cosmic strings in non-standard cosmology (kination after inflation).

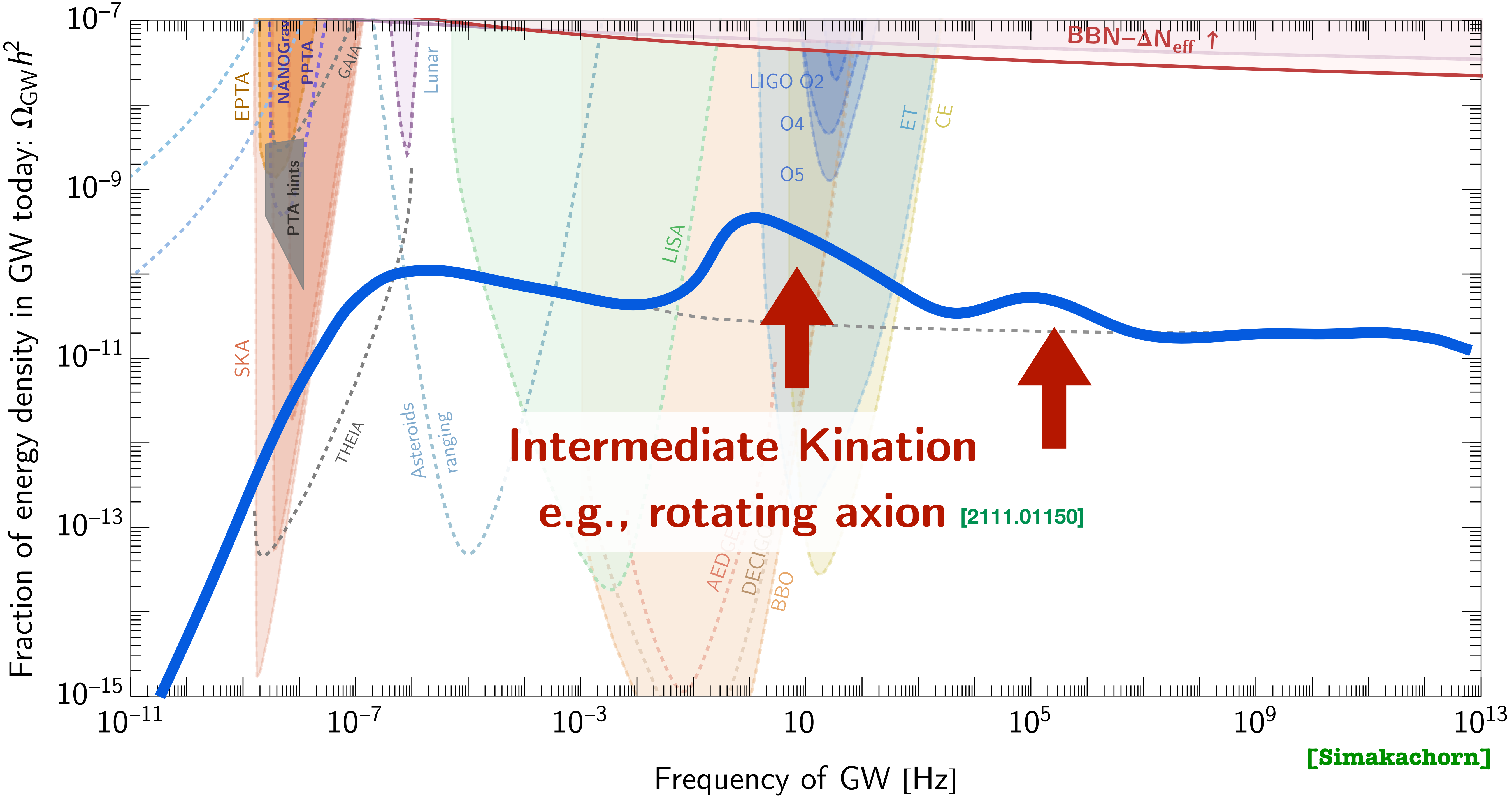


Kination right after inflation

[2111.01150]

[Simakachorn]

Gravitational Waves from cosmic strings in non-standard cosmology induced by rotating axions.

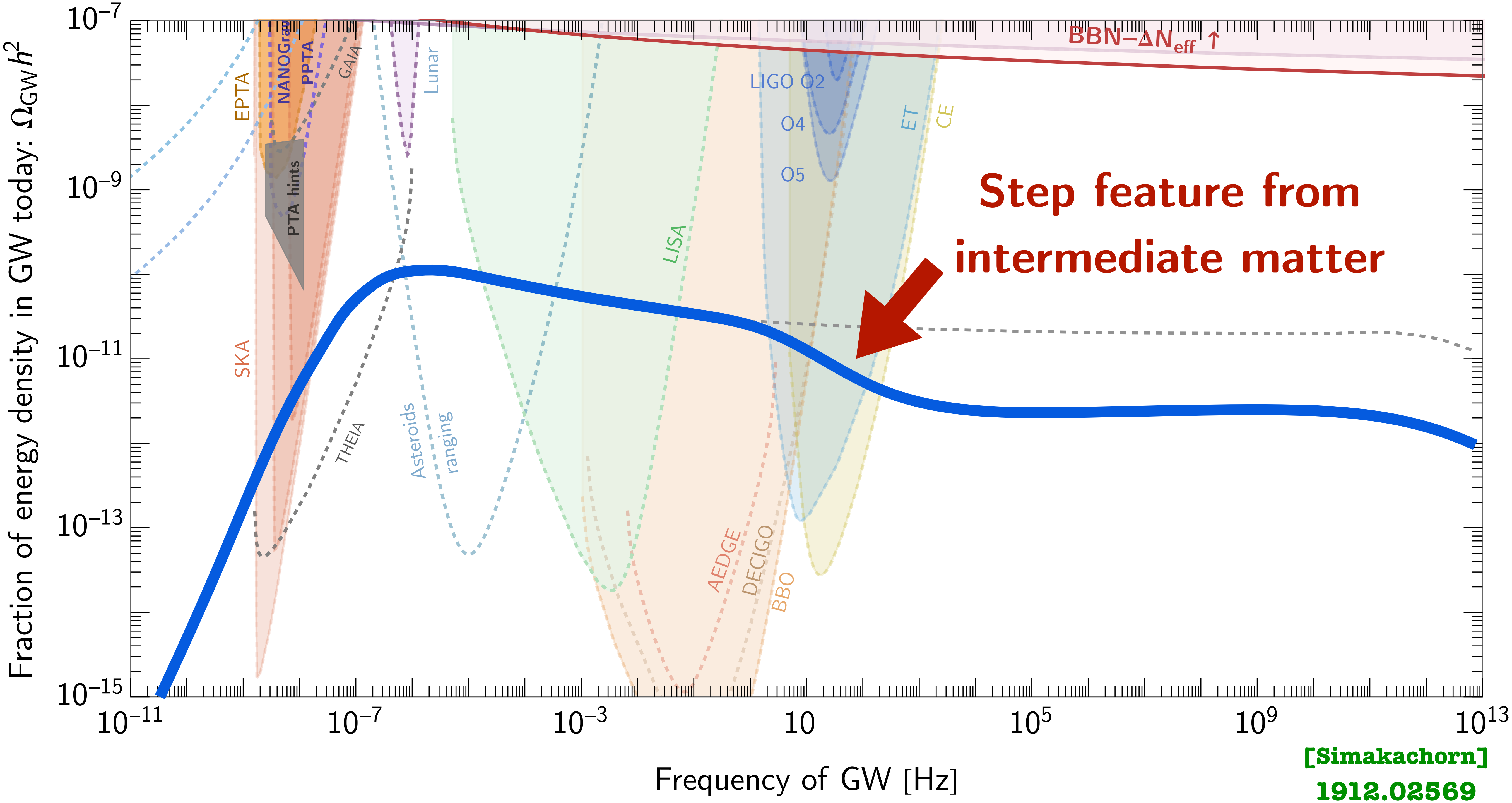


Intermediate Kination

e.g., rotating axion [2111.01150]

[Simakachorn]

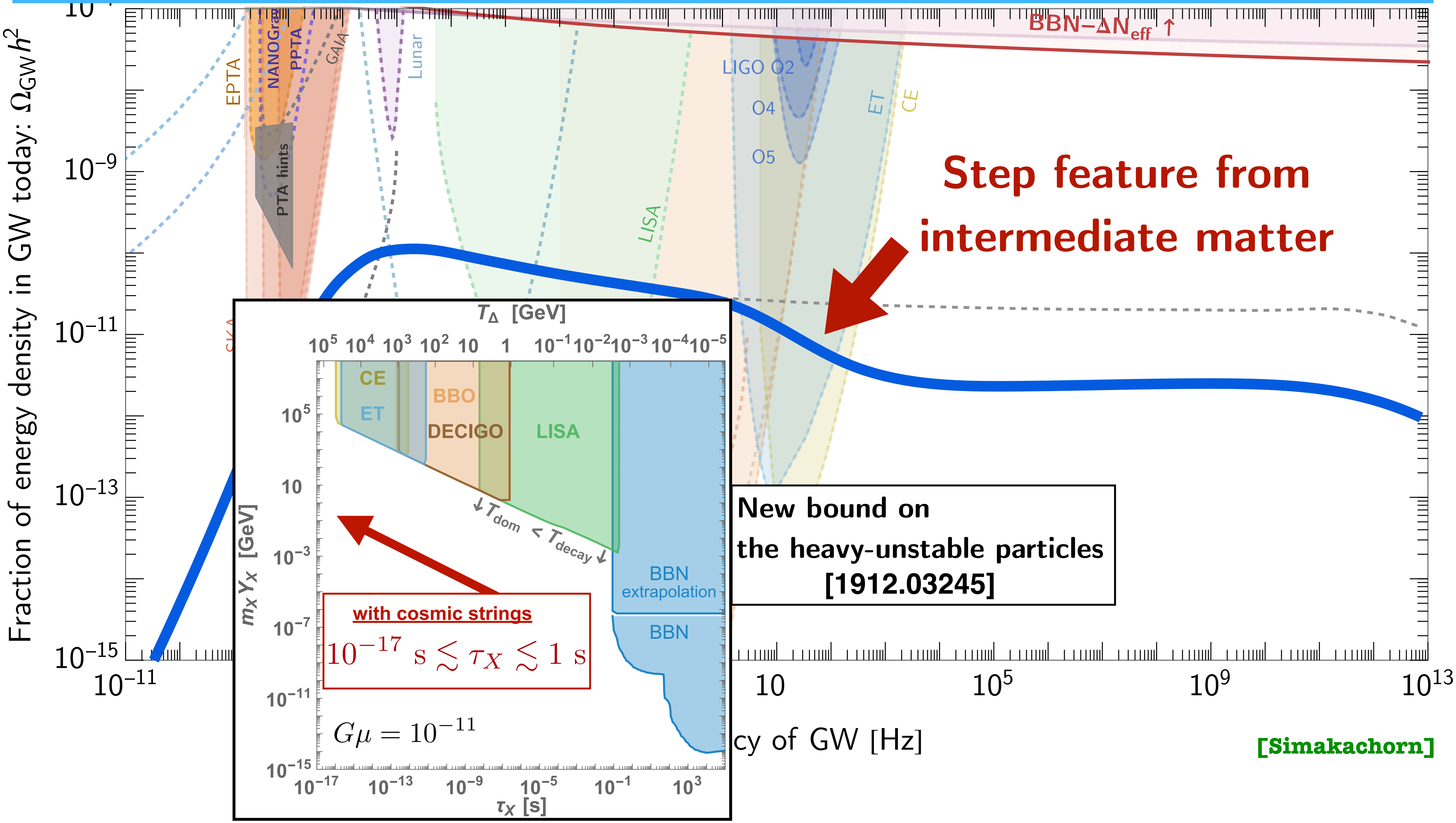
Gravitational Waves from cosmic strings in non-standard cosmology.



Step feature from intermediate matter

[Simakachorn]
1912.02569

Gravitational Waves from cosmic strings in non-standard cosmology.



Step feature from intermediate matter

New bound on the heavy-unstable particles [1912.03245]

with cosmic strings

$$10^{-17}\ {\rm s} \lesssim \tau_X \lesssim 1\ {\rm s}$$

[Simakachorn]