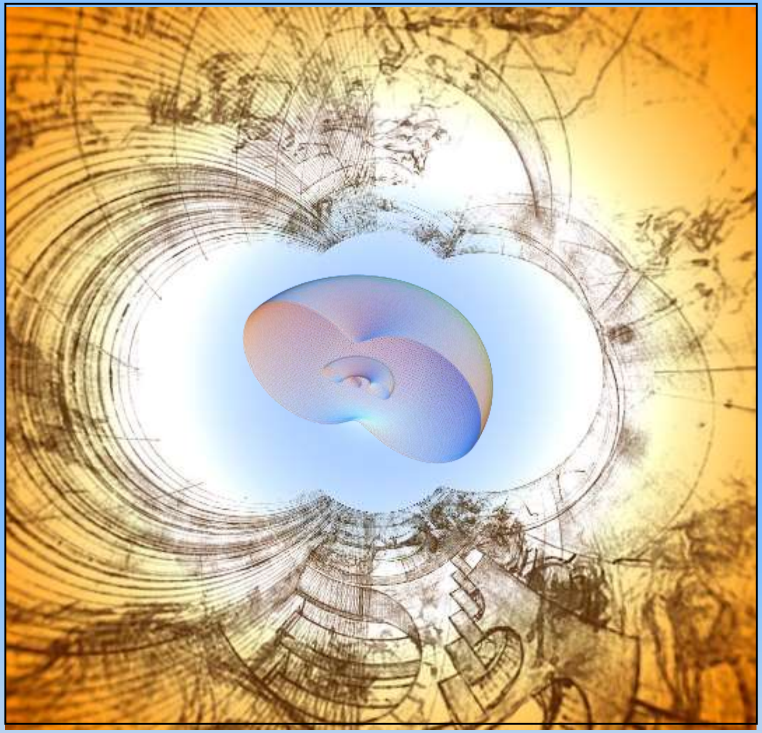
New Horizons for Psi -- Lisbon 2/07/24

Black Hole uniqueness and dirty black holes



Eugen Radu

Universidade de Aveiro, Portugal

some remarks:

- very large literature (thousands of papers)

- ***Black Hole uniqueness***: classic subject
  (review: e-Print: 1205.6112)

- ***dirty Black Holes:*** subjective choice of models
  (minimize the deviation from GR&SM)

- *here:* dirty ⇔ hairy

the no hair conjecture can be violated!

(validity is rather an exception)

- *here:* no dynamics or contact with observations..

- *exercises/questions*

Outline of the lectures

L1

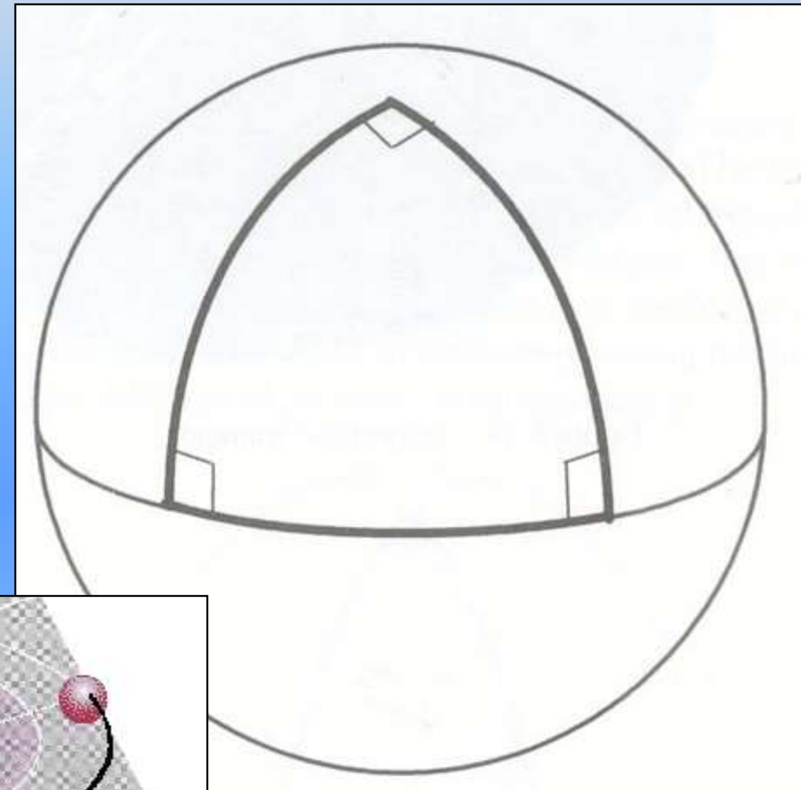
- review of (electro-)vacuum black hole solutions
- black hole uniqueness
- the "no hair" concept
- Beckenstein theorem
- scalar hair: *various mechanisms*

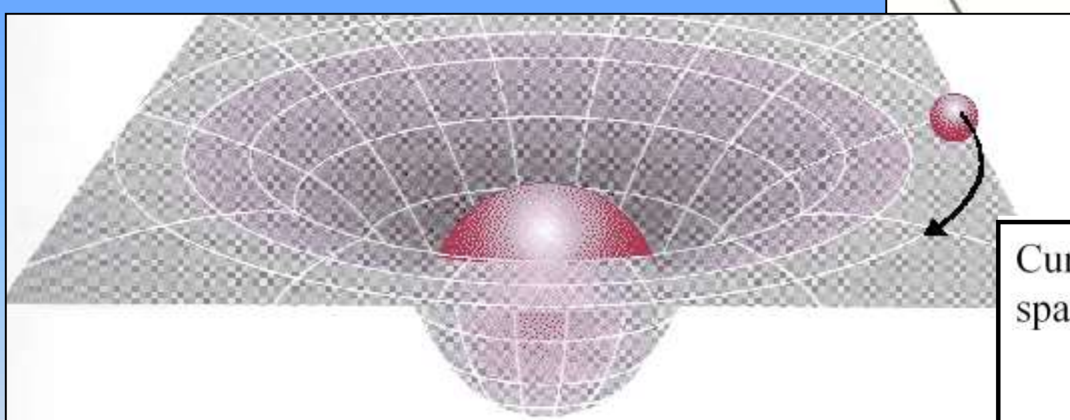
L2

- black holes with synchronized hair: *scalar and Proca fields*
- beyond GR: *higher order curvature terms*
- black hole scalarization
- vector hair
- other models of hairy black holes (*e.g. gauged scalars*)

Modern description of black holes

- ***geometric theory of gravitation***: (non-Euclidean geometry)
- non-linear partial differential equations





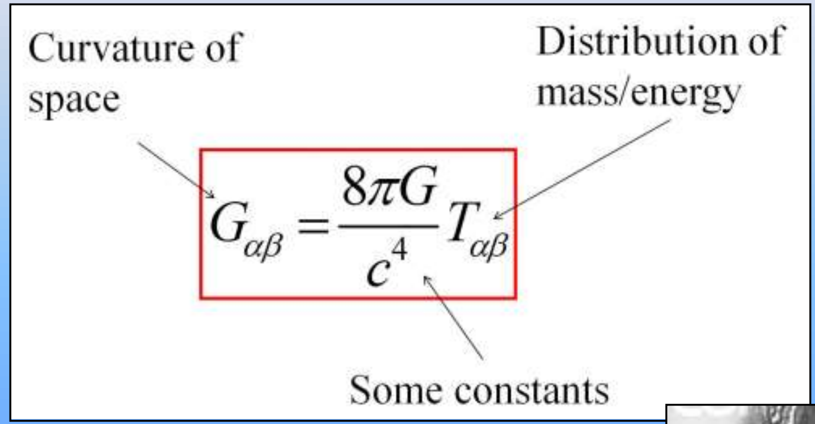
Curvature of space

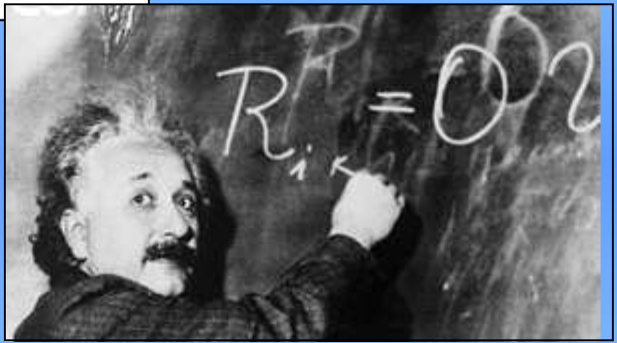
Distribution of mass/energy

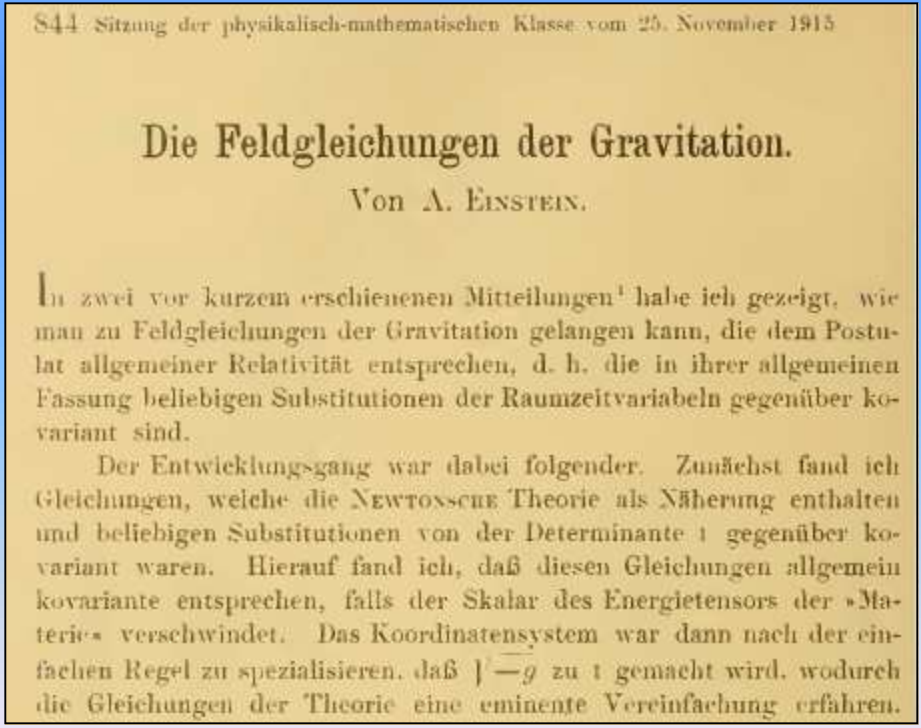
$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Some constants

1915: **Einstein's General Relativity**







Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren.

1916: **Schwarzschild's solution**

Über das Gravitationsfeld eines Massenpunktes nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. EINSTEIN hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\left.\begin{aligned} &\delta\int ds = 0, \\ \text{wobei} \quad & \\ &ds = \sqrt{\sum g_{\mu\nu}\, dx_\mu dx_\nu} \quad \mu,\nu = 1,2,3,4 \end{aligned}\right\} \quad (1)$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltig

$$ds^2 = \frac{dr^2}{1-\frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - (1 - \frac{2M}{r})dt^2$$

1963: *Kerr solution*

a rotating black hole

GRAVITATIONAL FIELD OF A SPINNING MASS AS AN EXAMPLE OF ALGEBRAICALLY SPECIAL METRICS

Roy P. Kerr*
University of Texas, Austin, Texas and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio
(Received 26 July 1963)

Goldberg and Sachs[1] have proved that the algebraically special solutions of Einstein's empty-space field equations are characterized by the existence of a geodesic and shear-free ray congruence, k_μ. Among these spaces are the plane-fronted waves and the Robinson-Trautman metrics[2] for which the congruence has nonvanishing divergence, but is hypersurface orthogonal.

where ζ is a complex coordinate, a dot denotes differentiation with respect to u, and the operator D is defined by

$$D = \partial/\partial\zeta - \Omega\partial/\partial u.$$

P is real, whereas Ω and m (which is defined to be $m_1 + im_2$) are complex. They are all independent of the coordinate r. Δ is defined by

geometry:

$$G_{\alpha\beta} = 0$$

$$T_{\alpha\beta} = 0$$

no matter

the vacuum, axially symmetric, stationary Black Hole

Roy Kerr

$$ds^2 = -\frac{(\Delta - a^2\sin^2\theta)}{\Sigma}dt^2 - 2a\sin^2\theta\frac{(r^2+a^2-\Delta)}{\Sigma}dtd\phi + \left(\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

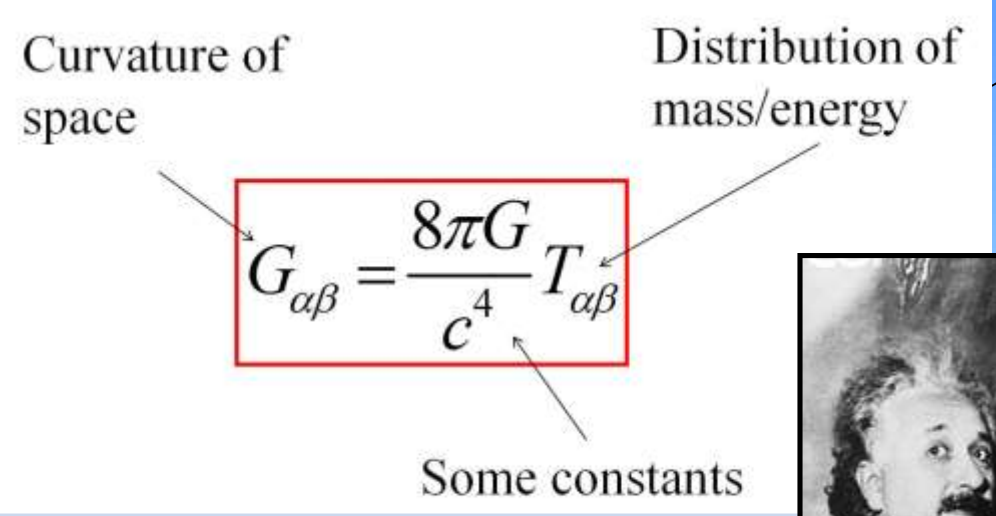
$$\Delta = r^2 - 2GMr + a^2$$

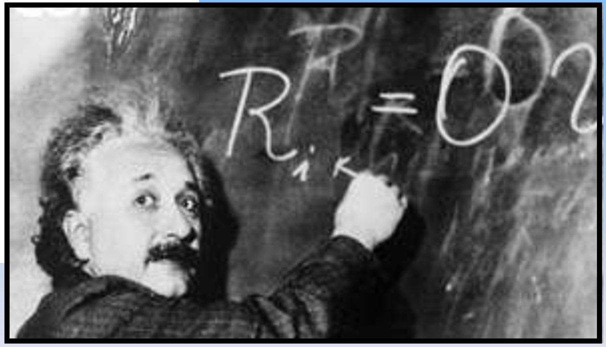
a=0: spherical symmetry **(Schwarzschild solution)**

so far vacuum case only

- how general are these results?

- what about matter fields?





Maxwell field

1916,1918: **Reissner-Nordström solution**

106

4. Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie; von H. Reissner.

Nachdem Hr. Einstein durch die Erklärung der Perihelbewegung des Merkur die Fruchtbarkeit seiner neuen kovarianten Feldgleichungen der Gravitation und damit des Postulats der allgemeinsten Relativität gezeigt und an anderer Stelle die allgemein kovariante Fassung der Maxwell-Lorentzschen Gleichungen des elektromagnetischen Feldes gegeben hat, erschien es mir als nächste Aufgabe, den Einfluß der Eigengravitation des elektrischen Feldes von Kugelsymmetrie an einem einfachen Beispiel zu untersuchen. Ich ging dabei allerdings von der Hoffnung aus, einen statischen Zusammenhalt von Elementarladungen durch deren Eigengravitation zu finden, ohne den Boden der Maxwellschen Theorie verlassen zu brauchen, konnte aber im Verlauf der Arbeit zunächst nur feststellen, daß die Einsteinsche Gravitation zwar das Feld der elektrischen Elementarladung in bestimmter, übrigens ungeheuer geringer Weise verzerrt, aber ihrem Wesen nach die gegenseitige elektrostatische Abstoßung der Ladungselemente nicht aufheben kann.

H. Reissner,
Annalen der Physik,
355 (1916) 106-120

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} + r^2 d\Omega^2$$

$$A = \frac{Q}{r} dt \quad \text{(Maxwell 1-form potential } F = dA\text{)}$$

Maxwell field

1965: **Kerr-Newman solution**

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 6, NUMBER 6 JUNE 1965

Metric of a Rotating, Charged Mass*

E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence
Physics Department, University of Pittsburgh, Pittsburgh, Pennsylvania
(19 June 1964)

A new solution of the Einstein–Maxwell equations is presented. This solution has certain characteristics that correspond to a rotating ring of mass and charge.

$$ds^2 = -\frac{(\Delta - a^2\sin^2\theta)}{\Sigma}dt^2 - 2a\sin^2\theta\frac{(r^2+a^2-\Delta)}{\Sigma}dtd\phi + \left(\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2GMr + a^2 + Q^2$$

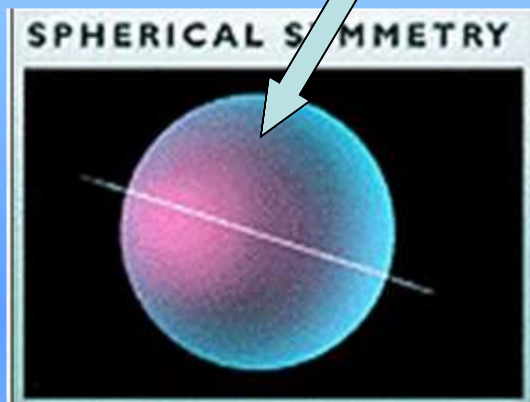
also R. H. Boyer and R. W. Lindquist
J. Math. Phys. 8 (1967) 265

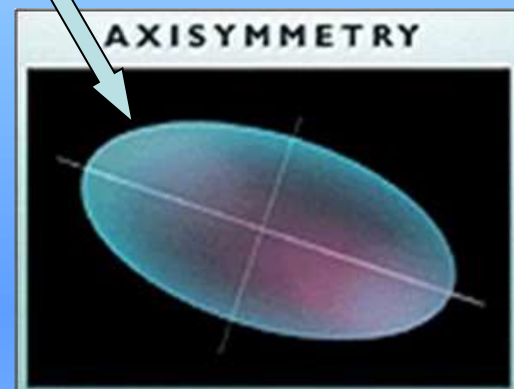
gravity + electromagnetism

Schwarzschild/Kerr Black Hole

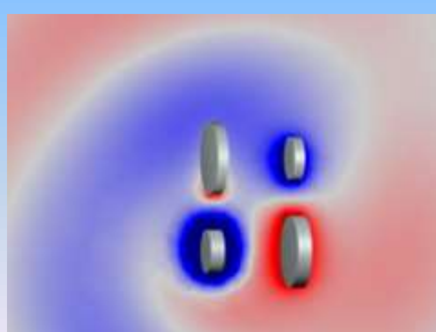
add electric charge **Q**

Reissner-Nordström/Kerr-Newmann Black Hole





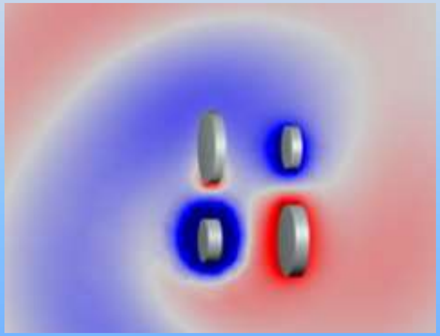
Q: are there more general (electrovacuum) BHs?



what abour the multipolar field structure?

Q: are there more general (electrovacuum) BHs?

e.g.



Maxwell field: *multipolar structure*

⇓

BH with e.g. electric dipole hair?

(Exercise 1)

U(1) potential $A = V(r,\theta,\varphi)dt$

$$V(r,\theta,\phi) = \sum_{\ell\geq 1}\sum_{m=-\ell}^{m=\ell} c_{\ell m} R_\ell(r) Y_{\ell m}(\theta,\varphi)$$

Minkowski spacetime:

$$R_\ell(r) = c_1 r^\ell + \frac{c_2}{r^{\ell+1}}$$

(probe limit)

Schwarzschild Black Hole::

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-2M/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$R_1(r) = c_1(r-2M) + c_2\left[\frac{M}{r} - 1 + \left(1-\frac{r}{2M}\right)\ln\left(1-\frac{2M}{r}\right)\right]$$

singularities

uniqueness theorems for Black Hole

1967: **Israel's theorem**

PHYSICAL REVIEW VOLUME 164, NUMBER 5 25 DECEMBER 1967

Event Horizons in Static Vacuum Space-Times

WERNER ISRAEL
Mathematics Department, University of Alberta, Alberta, Canada
and
Dublin Institute for Advanced Studies, Dublin, Ireland
(Received 27 April 1967)

The following theorem is established. Among all static, asymptotically flat vacuum space-times with closed simply connected equipotential surfaces g_{00}=constant, the Schwarzschild solution is the only one which has a nonsingular infinite-red-shift surface $g_{00}=0$. Thus there exists no static asymmetric perturbation of the Schwarzschild manifold due to internal sources (e.g., a quadrupole moment) which will preserve a regular event horizon. Possible implications of this result for asymmetric gravitational collapse are briefly discussed.

Israel's theorem:
An asymptotically flat static vacuum spacetime that is non-singular on and outside an event horizon, must be isometric to the Schwarzschild spacetime.

uniqueness theorems for Black Hole

1967-...: **The electro-vacuum uniqueness theorems**

Axisymmetric Black Hole Has Only Two Degrees of Freedom

B. Carter
Institute of Theoretical Astronomy, University of Cambridge, Cambridge CB3 0EZ, England
(Received 18 December 1970)

A theorem is described which establishes the claim that in a certain canonical sense the Kerr metrics represent "the" (rather than merely "some possible") exterior fields of black holes with the corresponding mass and angular-momentum values.

Phys. Rev. Lett. 26 (1971) 331-333

Vacuum:

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}R$$

Kerr Kerr 1963

Uniqueness Israel 1967; Carter 1971; D.C. Robinson, Phys. Rev. Lett. 34, 905 (1975).

Electro-vacuum:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Kerr-Newman Newman et al. 1965

Uniqueness

W. Israel, Commun. Math. Phys. 8 (1968) 245;
D.C. Robinson, Phys. Rev. 10, 458 (1974)
(...)

uniqueness theorems for Black Hole

Carter-Robinson theorem:
An asymptotically-flat stationary and axi-symmetric vacuum spacetime that is non-singular on and outside an event horizon, is a member of the two-parameter Kerr family.

The assumption of axi-symmetry was subsequently shown to be unnecessary, i.e. for black holes, stationarity $\Rightarrow$ axisymmetry (via the "rigidity theorem", relating the teleologically defined "event horizon" to the local "Killing Horizon" Hawking 1972; I. Rácz and R.Wald, Class. Quant. Grav. 13 (1996) 539).

review: e-Print: 1205.6112

limitations: *(e.g.) analyticity, assumes connected event horizon, causality*
- Einstein gravity

remark: **-** *D=4 is very special (e.g.* ***Black Rings*** *in D>4)*

- *also* <u>*asymptotic flatness*</u>

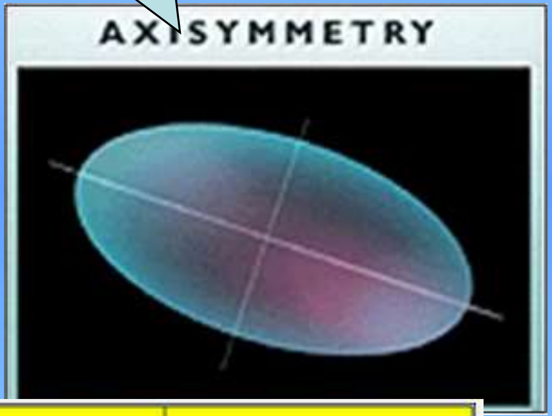
(e.g. a very different picture in AdS)

gravity + electromagnetism

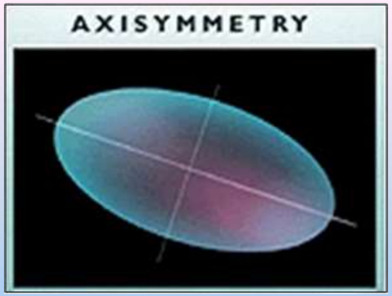
Schwarzschild/Kerr Black Hole

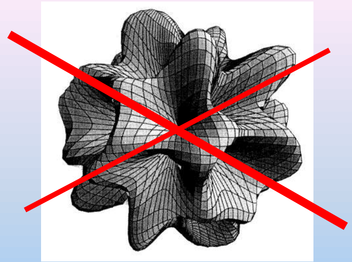
electric charge **Q**

Reissner-Nordström/Kerr-Newmann Black Hole



BH Metric	Mass	Charge	Momentum
Schwarzschild	Yes	No	No
Reissner–Nordström	Yes	Yes	No
Kerr	Yes	No	Yes
Kerr–Newman	Yes	Yes	Yes





uniqueness theorems for Black Hole

*"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's field equations of general relativity, discovered by the New Zealand mathematician, Roy **Kerr**, provides the **absolutely exact representation** of untold numbers of black holes that populate the Universe."*

S. Chandrasekhar, in *Truth and Beauty* (1987)

Nobel Prize (1983)

"Kerr paradigm"

“Kerr paradigm”

UNIQUENESS OF KERR’S SOLUTION

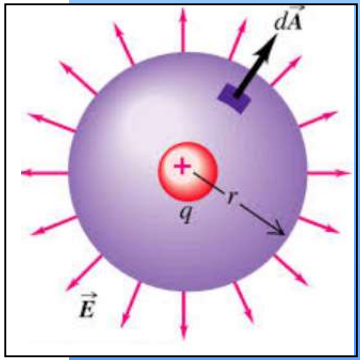
- Kerr’s solution describes *all* black holes without electric charge
- More generally,

  “BLACK HOLES HAVE NO HAIR”
- No-hair theorem: All traces of the matter that formed a BH disappear except for:

MASS

ANGULAR MOMENTUM

CHARGE ← *not important In astrophysics*

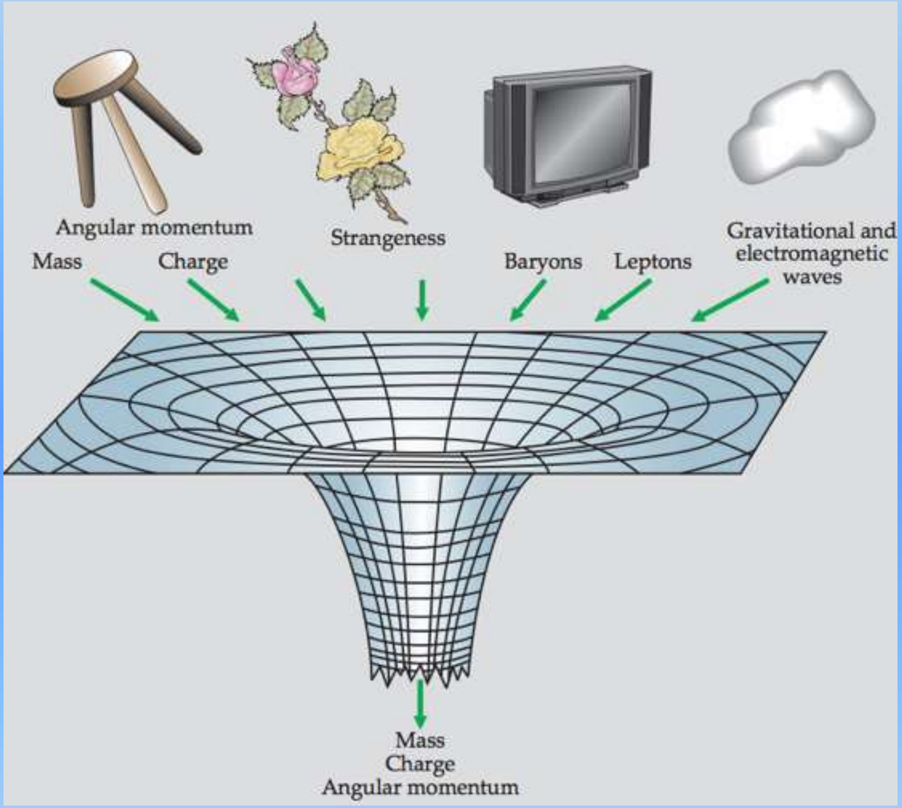


*“**hair”** is a metaphor for any messy/ complicated details (other fields, multipoles etc)*

1971: **Ruffini and Wheeler coin the expression "*a black hole has no hair*"**

The collapse leads to a black hole endowed with mass and charge and angular momentum but, so far as we can now judge, no other adjustable parameters: "A black hole has no hair." Make one black hole out of matter; another, of the same mass, angular momentum, and charge, out of antimatter. No one has ever been able to propose a workable way to tell which is which. Nor is any way known to distinguish either from a third black hole, formed by collapse of a much smaller amount of matter and then built up to the specified mass and angular momentum by firing in enough photons, or neutrinos, or gravitons. And on an equal footing is a fourth black hole, developed by collapse of a cloud of radiation altogether free from any "matter."

Electric charge is a distinguishable quantity because it carries a long-range force (conservation of flux; Gauss's law). Baryon number and strangeness carry no such long-range force. They have no Gauss's law. It is true that no attempt to observe a change in baryon number has ever succeeded. Nor has anyone ever been able to give a convincing reason to expect a direct and spontaneous violation of the principle of conservation of baryon number. In gravitational collapse, however, that principle is not directly violated; it is transcended. It is transcended because in collapse one loses the possibility of measuring baryon number, and therefore this

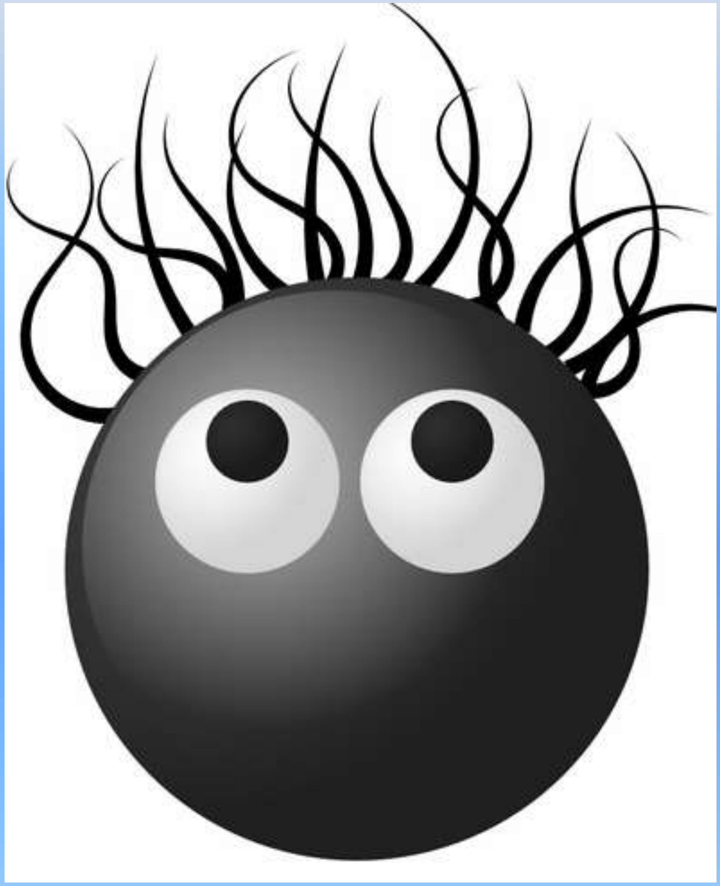


R. Ruffini and John Wheeler, "Introducing the black hole", Physics Today, January 1971, Pages 30-41

"Kerr paradigm"

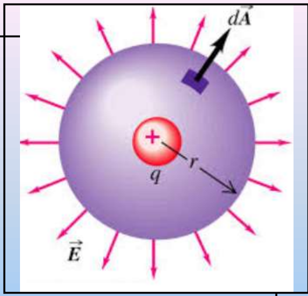
proof?

no hair theorems



a Black Hole is still entirely defined by a set of parameters which are its mass, spin and charge respectively

(saying the black hole has “no hair” is a metaphor for this simplicity)



{ however, **<u>no general proof</u>**
uniqueness and no-hair theorems: ***<u>case by case study</u>*** }

vacuum →
electromagnetic field →
vacuum →
electromagnetic field →

BH Metric	Mass	Charge	Momentum
Schwarzschild	Yes	No	No
Reissner–Nordström	Yes	Yes	No
Kerr	Yes	No	Yes
Kerr–Newman	Yes	Yes	Yes

These lectures:

- **the status of “no hair” conjecture**
- **other fields/beyond GR ?**

Kerr geometry "*provides the **absolutely exact representation** of untold numbers of Black Holes that populate the Universe*" *(*Chandrasekhar)

Black Hole hair: challenging the Kerr paradigm

two main directions:

$$G_{\alpha\beta} = 8\pi G\, T_{\alpha\beta}$$

i) include matter fields (other than Maxwell)

ii) change the theory of gravity

also together

(still) an active area of research…

the meaning of hair:

matter fields ψ

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + L_m(\psi) \right)$$

$$\delta_g S = 0$$ → *Einstein equations*

$$\delta_\psi S = 0$$ → *matter field equations*

the study is done *case by case*

(no general results):

prove a no hair theorem

or

construct the hairy black hole solutions

- ***rich subject***
- ***many interesting results***

some general remarks (i)

- *very few exact solutions*
- *numerical methods* (existence proof sometimes)
- *all configurations here:* no dynamics

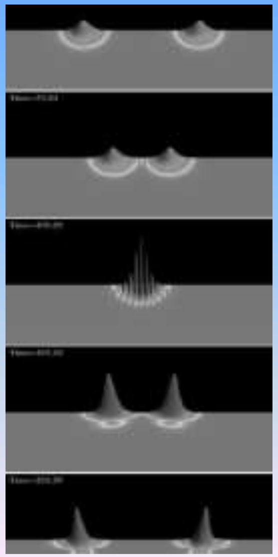
some general remarks (ii)

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + L_m(\psi) \right)$$

matter field(s) ψ

- **probe limit:** ψ in a fixed background → Minkowski; Schwarzschild, Kerr etc

- the concept of **soliton:** *particle-like, no horizon; R^4-topology; globally regular (do not require stability) also, (here) no dynamics*



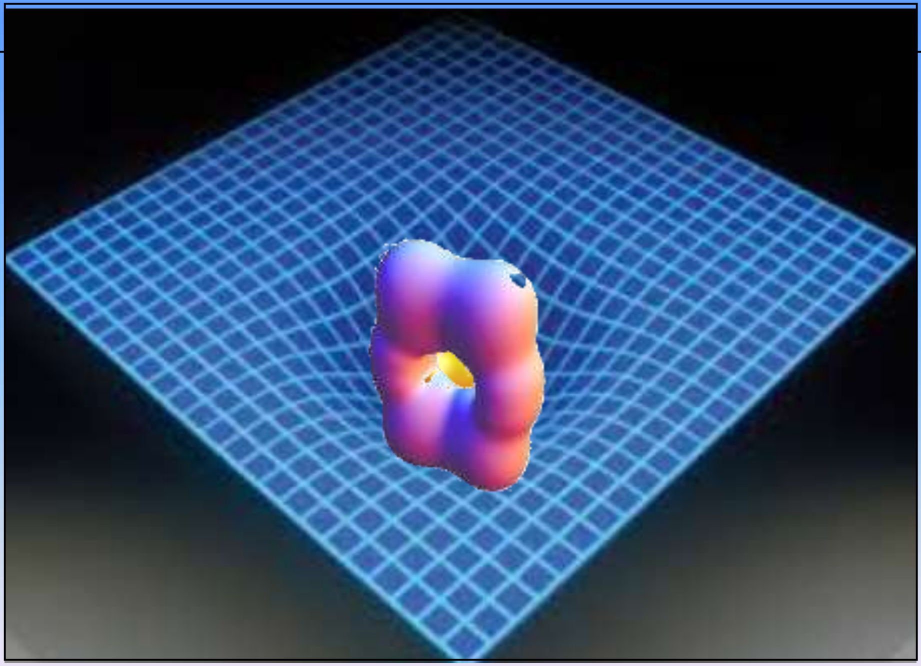
a general mechanism

various such models possess particle-like solutions in a flat spacetime background (solitons) -- supported by non-linear effects

e.g. *Q-balls, monopoles, sphalerons..*

the setup

i) the solutions should survive when including backreaction → Einstein-matter field solitons



a general mechanism

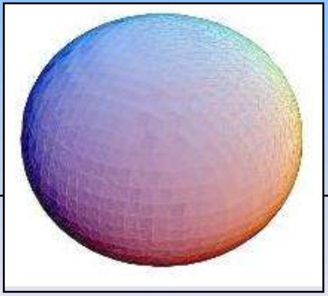
various such models possess particle-like solutions in a flat spacetime background (solitons) -- supported by non-linear effects

e.g. *Q-balls, monopoles, sphalerons..*

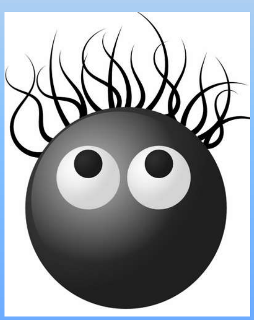
the setup

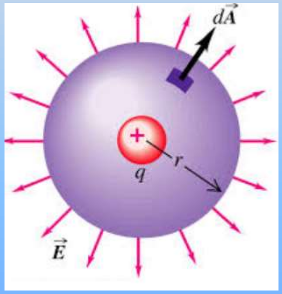
i) the solutions should survive when including backreaction → *gravitating solitons*

ii) moreover, when solitons exist in a given model, bound states of such solitons with an event horizon can ***typically*** be constructed → (hairy) **Black Holes** inside solitons



some general remarks (iii)





the hair

primary
(new global charges)

secondary
(typically)

finally:

it is ***not safe*** *to extrapolate the results to D>4 and/or other spacetime asymptotics*

Black Holes

+

Scalar Fields

why scalar hair?

i) ***Scalar fields are one of the simplest types of "matter"*** often considered by physicists

ii***) Scalar fields may be considered as a proxy to realistic matter,*** since canonical scalar fields can be modeled as perfect fluids with some equation of state

iii) ***There is at least one scalar particle in Nature*** (Higgs boson)

Beyond the Standard Model High Energy Physics models predict many more (also Susy, String Theory,...)

lectures yesterday

- the case of scalar hair is rather special (*more difficult than expected*)
- Mayo and Bekenstein Phys. Rev. D 54 (1996) 5059 [gr-qc/9602057]:

  "the proliferation in the 1990s of stationary black hole solutions with hair of various sorts may give the impression that the principle has fallen by the wayside. However, this is emphatically not the case for scalar field hair."

`simplest case: **real, *massless* scalar field**

Fisher (1948)
Janis, Newman and Winicour (1968) et al.

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi\right)$$

$$ds^2 = -\left[\frac{R-M(\mu-1)}{R+M(\mu+1)}\right]^{1/\mu} dt^2 + \left[\frac{R+M(\mu+1)}{R-M(\mu-1)}\right]^{1/\mu} dR^2 + r(R)^2(d\theta^2 + \sin^2\theta d\phi^2)$$

metric

$$r(R)^2 = [R-M(\mu-1)]^{1-1/\mu}\,[R+M(\mu+1)]^{1+1/\mu}$$

scalar field

$$\Phi(R) = \frac{Q_S}{2M\mu}\ln\left[\frac{R-M(\mu-1)}{R+M(\mu+1)}\right]$$

$$\mu \equiv \sqrt{1+\frac{Q_S^2}{M^2}} > 1$$

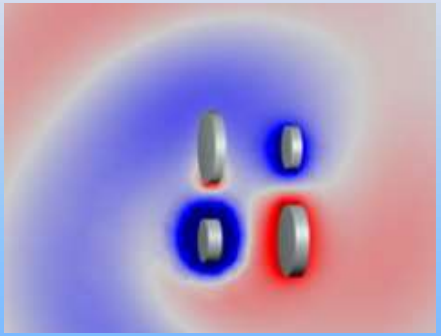
scalar 'monopole' (RN-like)

naked singularity!

(Exercise 1)

thus we need to consider more complicated models…

Also:



scalar field: *multipolar structure*

⇓

BH with e.g. *dipole scalar hair?*

(Exercise 1)

field decomposition

$$\phi\left(r,\theta,\varphi\right)$$

$$\phi\left(r,\theta,\varphi\right)=\sum_{\ell\geq 1}\sum_{m=-\ell}^{m=\ell} c_{\ell m}R_{\ell}(r)Y_{\ell m}(\theta,\varphi)$$

***Minkowski spacetime:*:**

$$R_{\ell}(r)=c_1 r^{\ell}+\frac{c_2}{r^{\ell+1}}$$

(probe limit)

***Schwarzschild Black Hole:*:**

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\frac{dr^2}{1-2M/r}+r^2(d\theta^2+\sin^2\theta d\phi^2)$$

(exercise)

$$R_1\left(r\right)=c_1(r-M)+c_2\left[-1+\left(\frac{1}{2}-\frac{r}{2M}\right)\ln\left(1-\frac{2M}{r}\right)\right]$$

singularities

Black Holes
+
Scalar Fields

- there is some ***tension*** between Black Holes and Scalar fields
- normally, the Black Holes ***do not allow*** for scalar clouds around
- however, the ***no-hair theorems*** can be circumvented
- Black Holes with scalar hair: many ***unusual properties***

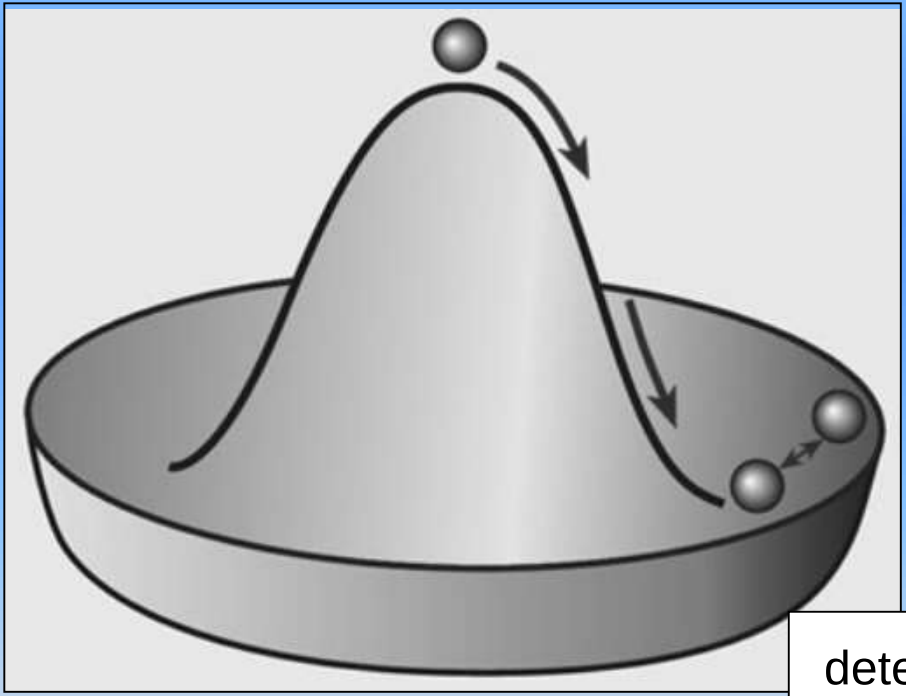
more complex case (e.g. self-interaction)

at least a scalar field exists in Nature:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \underbrace{m^2\phi + \lambda\phi^3}_{\frac{\partial V(\phi)}{\partial \phi}}$$

the Higgs field:

special potential



'Mexican hat' potential

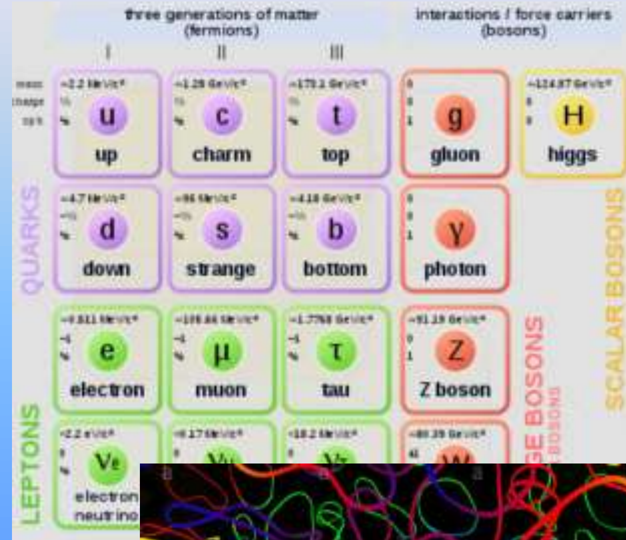
- *non-linear field*
- *interacts with other fields in the Standard Model of Particle Physics*

detected at LHC in CERN (2012)

mass ~ 125 GeV (~10^{-25} Kg)

other scalar fields?

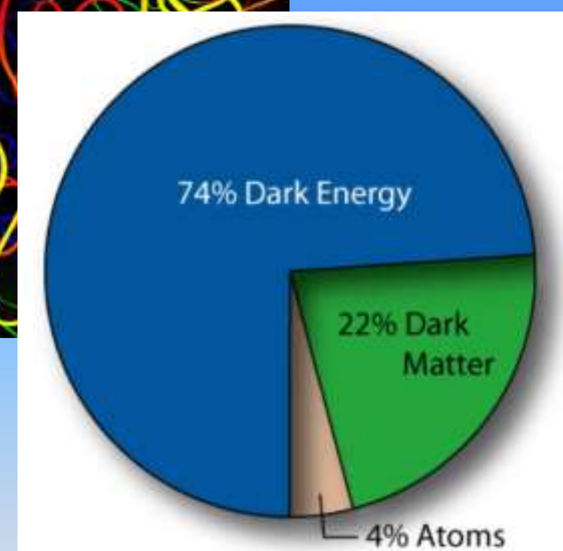
extensions of the Standard Model



hints

String Theory
+extra dims

cosmo/ogy
inflation



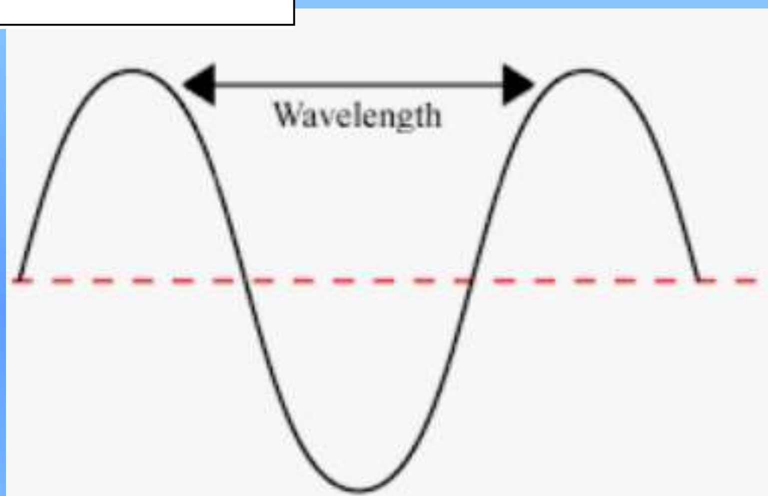
effective description..

no evidence yet…

OBSERVATION: *normally (very) different characteristic scales*

massive scalar field

$$\lambda = \frac{h}{p}$$

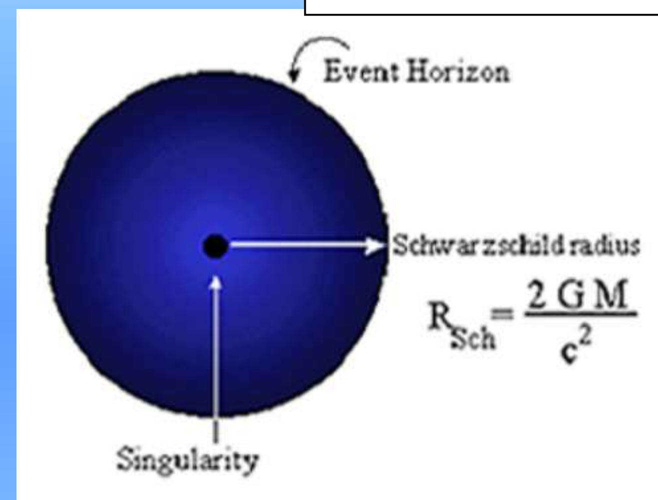


Higgs field:

$$\lambda \sim 10^{-17} m$$

gravity

$$r_h = \frac{2GM}{c^2}$$



Sagittarius A* Black Hole:

r_h ~ 24 million kilometers

(distance Earth-Sun: 47 million kilometers)

thus:

Black Holes

+

Scalar Fields

new physics required: **'light scalars'**

$$\lambda = \frac{h}{mc} \quad \sim \quad r_h = \frac{2GM}{c^2}$$

(macroscopic Black Holes)

for Black Holes with the mass of the Sun: m ~ 10^{-11} eV (~10^{-47} Kg)

(Higgs mass ~ 125 GeV)

still not so simple…

various no-hair theorems:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

<u>Assumption 1</u>: -- canonical and minimally coupled scalar field to Einstein's gravity.

<u>Assumption 2</u>:-- the potential V obeys: Φ(dV/dΦ)≥ 0 everywhere
- other versions as well (V>0)

<u>Assumption 3</u>:-- the scalar field inherits the spacetime symmetries.

$$\partial_t\Phi = 0 = \partial_\phi\Phi$$

more details e.g. in Herdeiro and Radu, **e-Print: 1504.08209**

a (classic) no-hair theorem: (J. Bekenstein 1972)

no (static) scalar field around a Black Hole

Klein-Gordon equation

$$\nabla^2 \phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Leftrightarrow \quad \phi \nabla^2 \phi = \phi \frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi \nabla^2 \phi = \nabla(\phi \nabla \phi) - (\nabla \phi)^2$$

Bekenstein argument:

$$\nabla^2 \phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Longleftrightarrow \quad \phi \nabla^2 \phi = \phi \frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi \nabla^2 \phi = \nabla(\phi \nabla \phi) - (\nabla \phi)^2$$

$$\nabla(\phi \nabla \phi) = (\nabla \phi)^2 + \phi \frac{\partial V(\phi)}{\partial \phi}$$

Bekenstein argument:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial\phi}$$

$$\Longleftrightarrow$$

$$\phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial\phi}$$

Identity:

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$$\nabla(\phi\nabla\phi) - (\nabla\phi)^2 = \phi\frac{\partial V(\phi)}{\partial\phi}$$

$$\Longleftrightarrow$$

$$\nabla(\phi\nabla\phi) = (\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial\phi}$$

Bekenstein argument:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Longleftrightarrow \quad \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial \phi}$$

$$\int d^3x\,\sqrt{-g}\;\nabla(\phi\nabla\phi) = \int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial \phi}\right]$$

$$\underbrace{\oint_\infty (\phi\nabla\phi)}_{=0} - \underbrace{\oint_H (\phi\nabla\phi)}_{=0}$$

$$\Rightarrow \int d^3x\,\sqrt{-g}\underbrace{\left[(\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial \phi}\right]}_{\geq 0} = 0$$

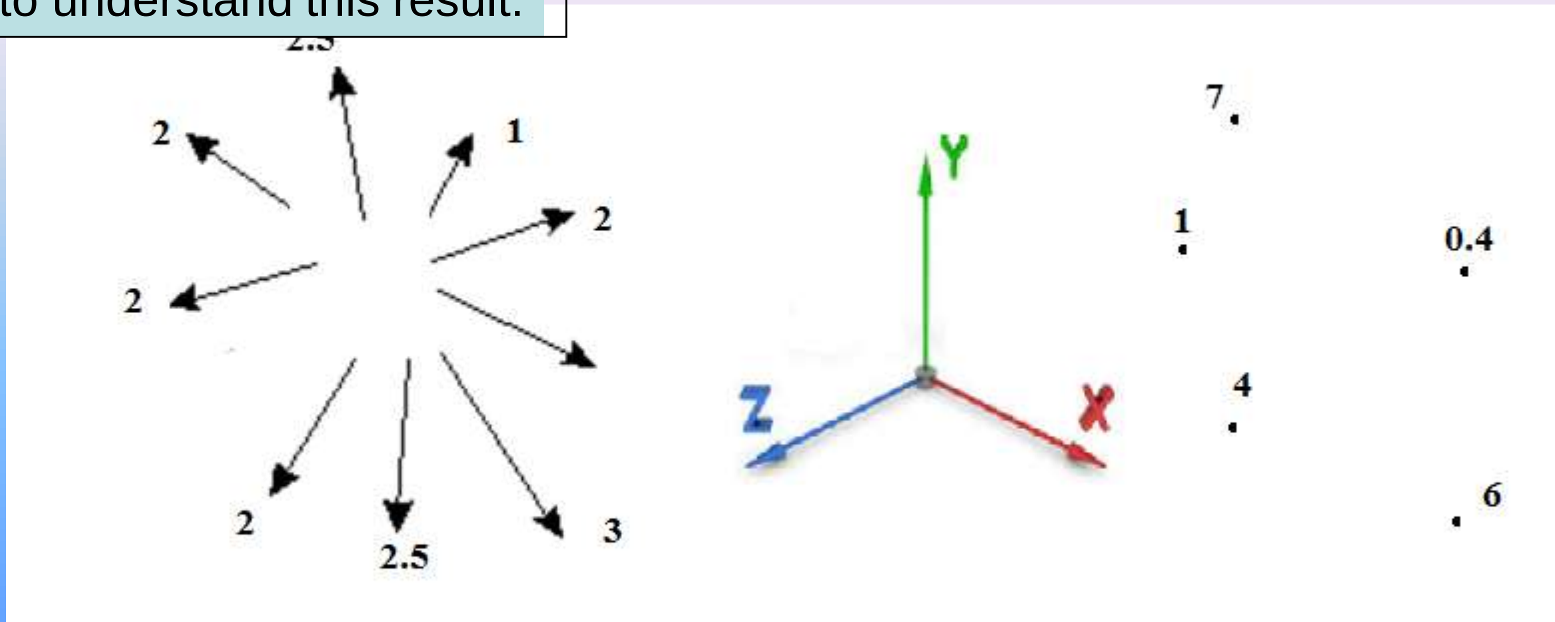
e.g. $V(\phi) = \frac{1}{2}m^2\phi^2$

$$\int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2 + m^2\phi^2\right] = 0$$

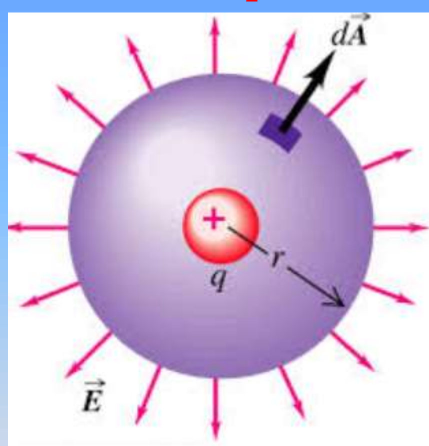
$\phi = 0$

Q.E.D.

how to understand this result:



electric field *vs.* *scalar field*

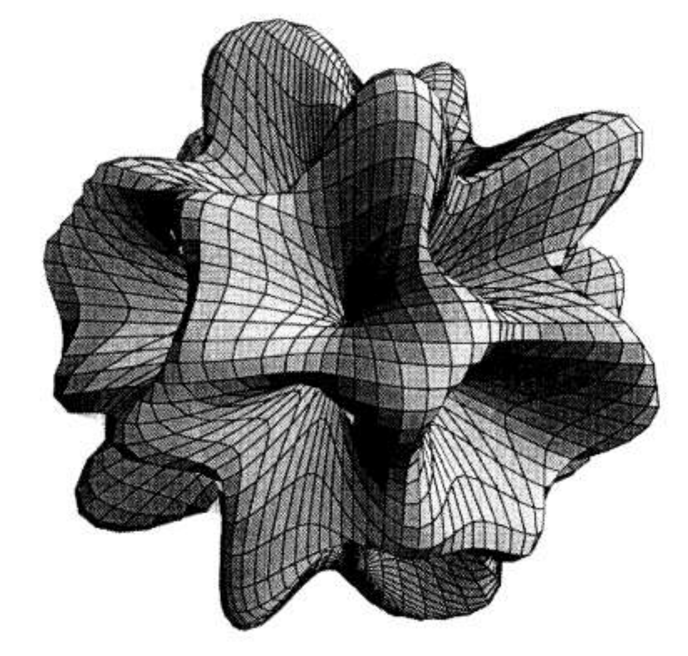


no scalar charge

non-zero flux $\Longrightarrow$ global (electric) charge

LOOPHOLES?

black holes with 'scalar hair'?



Yes – several different mechanisms

- *various recent developments*
- *active field of research*

no-hair theorems:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

Assumption 1: **-- canonical and minimally coupled scalar field to Einstein's gravity.**

Assumption 2:-- the potential V obeys: Φ(dV/dΦ)≥ 0 everywhere

- other versions as well (V>0)

Assumption 3:-- **the scalar field inherits the spacetime symmetries.**

$$\partial_t\Phi = 0 = \partial_\phi\Phi$$

more details in Herdeiro and Radu, **e-Print: 1504.08209**

however, these assumptions can be violated...

no-hair theorems:

Assumption 1: -- canonical and minimally coupled scalar field(s) to Einstein's gravity.

violation: an early example: **Einstein-Skyrme model**

four scalars **satisfying the sigma-model constraint $\Phi^a\Phi^a = 1$**

Skyrmions

the flat space Skyrme model - *effective theory*

(T. Skyrme, 1961) - *active field of research*

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a - \kappa\underbrace{|\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b|^2}_{\text{quartic kinetic term}}\right)$$

- the first physically relevant counterexample to the no-hair conjecture in the literature

Luckock and Moss (1986)

review: ***hep-th/9810070***

no-hair theorems:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

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$$\partial_t\Phi = 0 = \partial_\phi\Phi$$

more details in Herdeiro and Radu, **e-Print: 1504.08209** *single scalar*

LOOPHOLES?

no-hair theorems:

Assumption 2:-- the potential V obeys: Φ(dV/dΦ)≥ 0 everywhere,

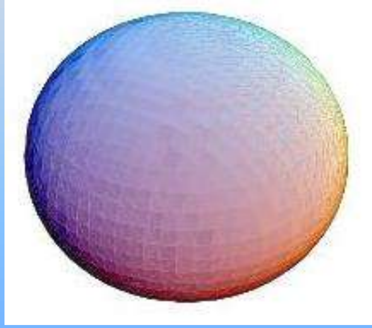
$$\int d^3x \sqrt{-g}\left[(\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial\phi}\right] = 0$$

violation: $\Phi V' < 0$

- *it requires violation of the energy conditions*
- *the scalar fields may possess particle-like solutions*
- *all known solutions are unstable*

such solutions are simple to construct (even in closed form)

Spherical symmetry: $\Phi = \phi(r)$



The KG equation:

(here flat space)

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi}{dr}) = \frac{dV}{d\phi}$$

“potential engineering”

steps towards ***build your own*** hairy BH solution:

exercise 2

- step 1) ***choose your own scalar profile*** **ϕ(r)**
- step 2) ***invert it to get*** **r=r(ϕ)**
- step 3) ***use the Klein-Gordon equation to compute*** **dV/dϕ**
- step 4) ***reconstruct the scalar potential V(ϕ)***

an example:

1504.08209 [gr-qc]

step 1: scalar fields in Schwarzschild BH background

(Klein-Gordon equation only)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \; .$$

simplest (?) example:
(Coulomb field)

$$\Phi(r) = -\frac{Q_S}{r}$$

$$V(\Phi) = -\lambda\Phi^5 < 0$$

(no backreaction)

$$Q_S = -\left(\frac{2M}{5\lambda}\right)^{1/3} < 0$$

exercise!

$$(T^S)^t_t = \frac{14M - 5r}{2^{1/3}5^{5/3}r^5}\left(\frac{M}{\lambda}\right)^{2/3} , \qquad E = \frac{3\pi}{2^{1/3}5^{5/3}}(M\lambda^2)^{-1/3}$$

“potential engineering”

Including the backreaction:

1504.08209 [gr-qc]

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$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$N(r) \equiv 1 - \frac{2m(r)}{r}$$

exercise 3

$$S_{eff} = \int_{r_H}^{\infty} dr\ \sigma(r)\left[m' - \left(\frac{1}{2}Nr^2\Phi'^2 + r^2V(\Phi)\right)\right]$$

prove virial identity

(very useful expression)

an example:

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

Einstein—Klein-Gordon equations:

“potential engineering”

$$\sigma' = \sigma r \phi'^2$$
$$m' = r^2(\frac{1}{2}N\phi'^2 + U)$$
$$\phi'' + \left(\frac{N'}{N} + \frac{\sigma'}{\sigma} + \frac{2}{r}\right)\phi' - \frac{U'}{N} = 0.$$

we postulate:

$$\Phi(r) = -\frac{Q_S}{r}$$

(just a possible choice)

1504.08209 [gr-qc]

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the solution:

$$\Phi(r) = -\frac{Q_S}{r}$$

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Scalar field potential:

"potential engineering"

$$V(\Phi) = -\frac{15}{2}\lambda e^{\frac{\Phi^2}{2}} W + \frac{1}{2Q_S^2}\left((1+2e^{\Phi^2})(\Phi^2-3) + \frac{W^2-27}{\Phi^2-3}\right)$$

$$W \equiv 3\Phi + \sqrt{\frac{\pi}{2}} e^{\frac{\Phi^2}{2}} (\Phi^2-3)\mathrm{Erf}(\frac{\Phi}{\sqrt{2}})$$

not natural:

mass function:

$$m(r) = \frac{r^3}{Q_S^2}\Bigg[1 + \frac{Q_S^2}{r^2} + e^{\frac{Q_S^2}{r^2}}(\frac{225}{8}\lambda^2 Q_S^4 - 1) - \frac{1}{2}e^{\frac{Q_S^2}{r^2}}\left(\frac{15}{2}\lambda Q_S^2 - \frac{Q_S}{r}e^{-\frac{Q_S^2}{2r^2}} + \sqrt{\frac{\pi}{2}}\mathrm{Erf}(\frac{Q_S}{\sqrt{2}r})\right)^2\Bigg],$$

extra metric function:

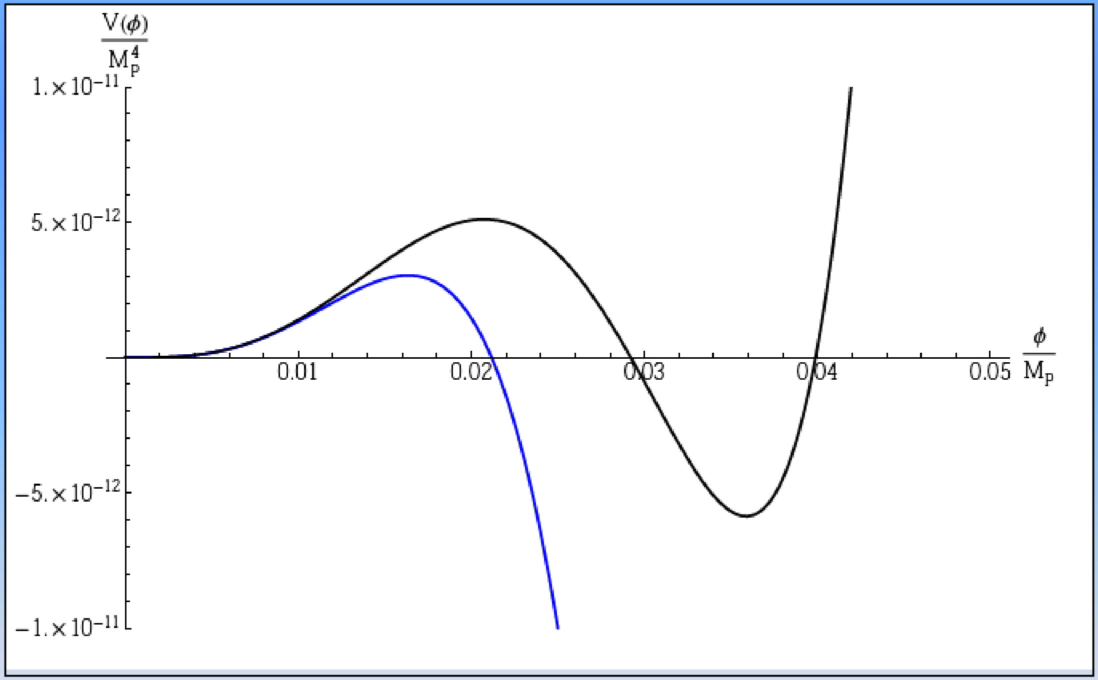
$$\sigma(r) = e^{-\frac{Q_S^2}{2r^2}}$$

with $\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t.$

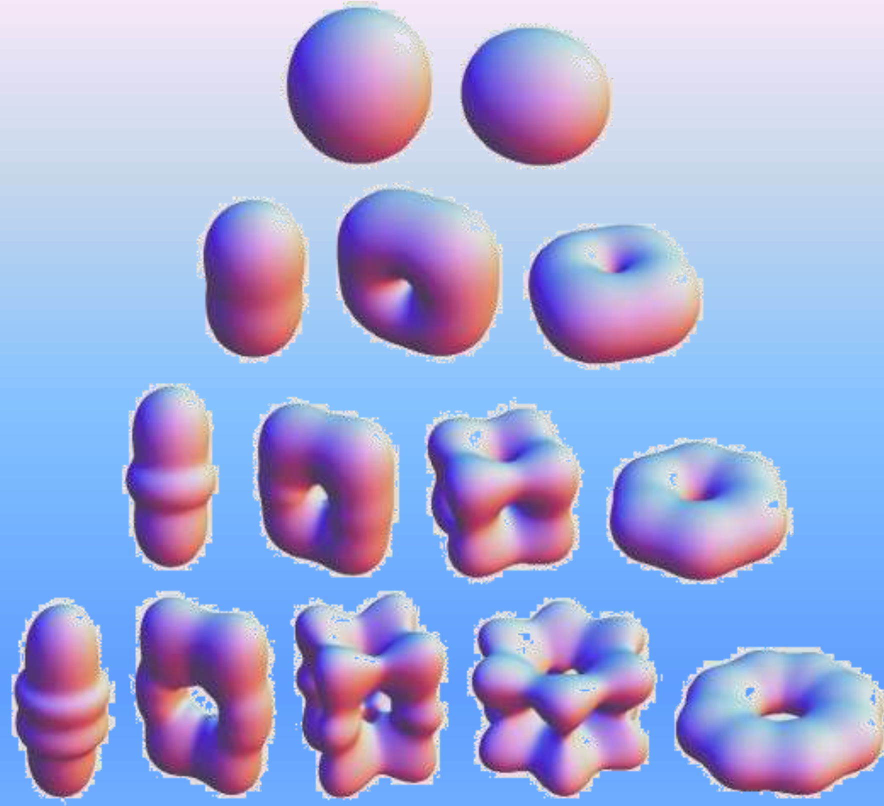
what about the Higgs field?

Burda, Gregory and Moss\
"Gravity and the stability of the Higgs vacuum", Phys.Rev.Lett. 115 (2015) 071303

$$V(\phi) = \lambda_{\text{eff}}(\phi)\frac{\phi^4}{4} + (\delta\lambda)_{\text{bsm}}\frac{\phi^4}{4} + \frac{\lambda_6}{6}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_p^4} + \ldots$$



Black holes with Higgs hair



Many thanks for your attention!

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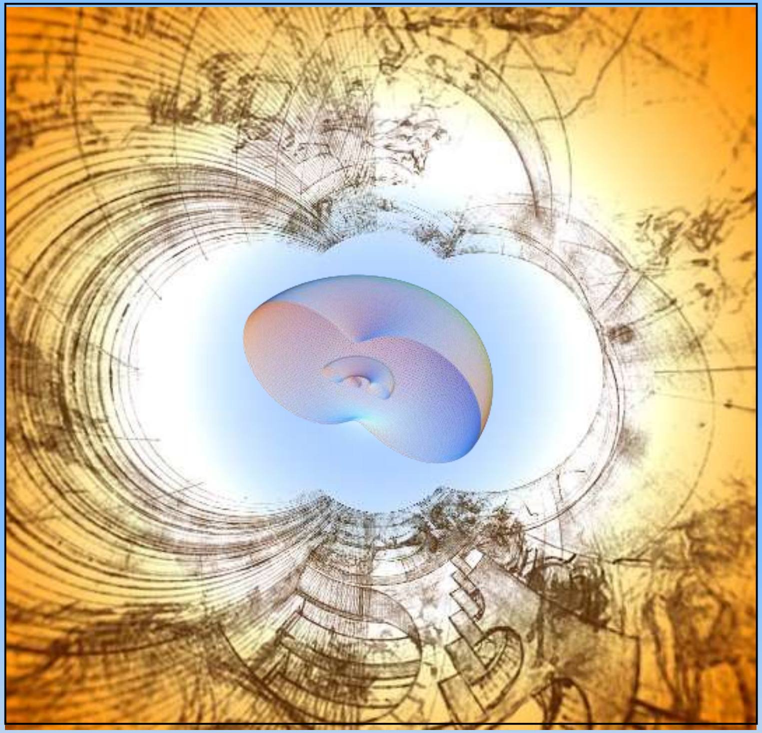
New Horizons for Psi -- Lisbon 2/07/24

Black Hole uniqueness and dirty black holes



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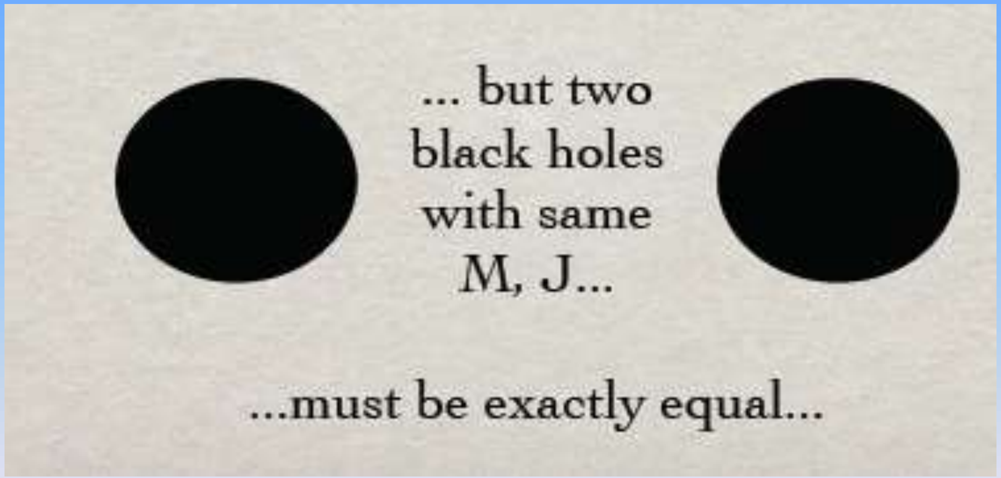
Gr@v

Summary:

The "no-hair" original idea (1971):
collapse leads to equilibrium black holes uniquely determined by (M,J,Q) - asymptotically measured quantities subject to a Gauss law
and no other independent characteristics (hair)

The idea is motivated by the *uniqueness theorems*
and indicates that black holes are **very special objects**



Black Holes

+

Scalar Fields

no-hair theorems:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

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Assumption 2:-- the potential V obeys: Φ(dV/dΦ)≥ 0 everywhere,

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$$\partial_t\Phi = 0 = \partial_\phi\Phi$$

more details in Herdeiro and Radu, **e-Print: 1504.08209**

however, these assumptions can be violated...

LOOPHOLES?

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violation: an early example: **Einstein-Skyrme model**

four scalars **satisfying the sigma-model constraint $\Phi^a \Phi^a = 1$**

Skyrmions

$$\mathcal{S} = \frac{1}{4\pi} \int d^4x \sqrt{-g} \left(\frac{R}{4} - \frac{1}{2} \nabla_\mu \Phi^a \nabla^\mu \Phi^a - \kappa \underbrace{|\nabla_{[\mu} \Phi^a \nabla_{\nu]} \Phi^b|^2}_{} \right)$$

quartic kinetic term

rather exotic example

single scalar

LOOPHOLES?

no-hair theorems:

Assumption 2:-- the potential V obeys: $\Phi(dV/d\Phi) \geq 0$ everywhere,

violation:

$$\Phi V' < 0$$

- *it requires violations of the energy conditions*
- *such models typically possess particle-like solutions even in a flat space background*
- *the solutions are unstable (?)*

"potential engineering"

rather artificial models

LOOPHOLES?

no-hair theorems:

Assumption 3:-- the scalar field inherits the spacetime symmetries.

violation:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\frac{1}{2}g^{ab}\left(\Psi^*_{,a}\Psi_{,b}+\Psi^*_{,b}\Psi_{,a}\right)-\mu^2\Psi^*\Psi\right]$$

Ψ : a complex scalar field with wave features

(no quantum effects) $\Psi\sim e^{i(m\varphi-\omega t)}$

Kerr black hole with scalar hair:

(e-Print: 1403.2757)

more generally…

spin 0: *arXiv: 1501.04319*

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}g^{ab}\left(\Psi^*_{,a}\Psi_{,b} + \Psi^*_{,b}\Psi_{,a}\right) - \mu^2\Psi^*\Psi\right]$$

spin 1: *arXiv: 1603.02687*

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha\right)$$

basic ingredients

- i) **complex field:** Ψ, A_α,... $\sim e^{i(m\varphi-\omega t)}$
- **ii) mass term:** μ (*however*, box&AdS)

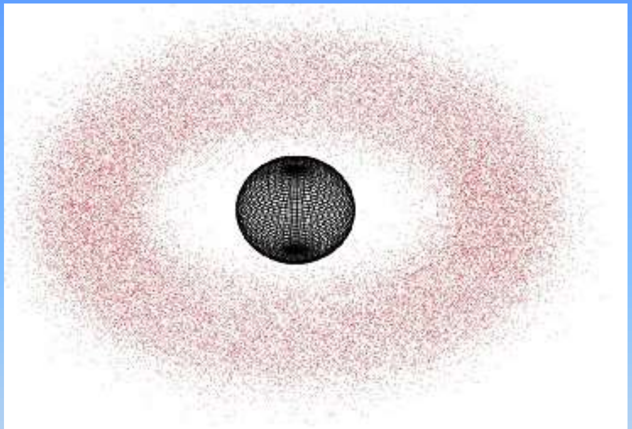
spin 2: ?

a very rich subject

two (complementary) viewpoints:

- **Boson/Proca stars**:- one can add a BH at the center
- **Kerr black holes**: - branching towards a new family of solutions due to superradiant instability

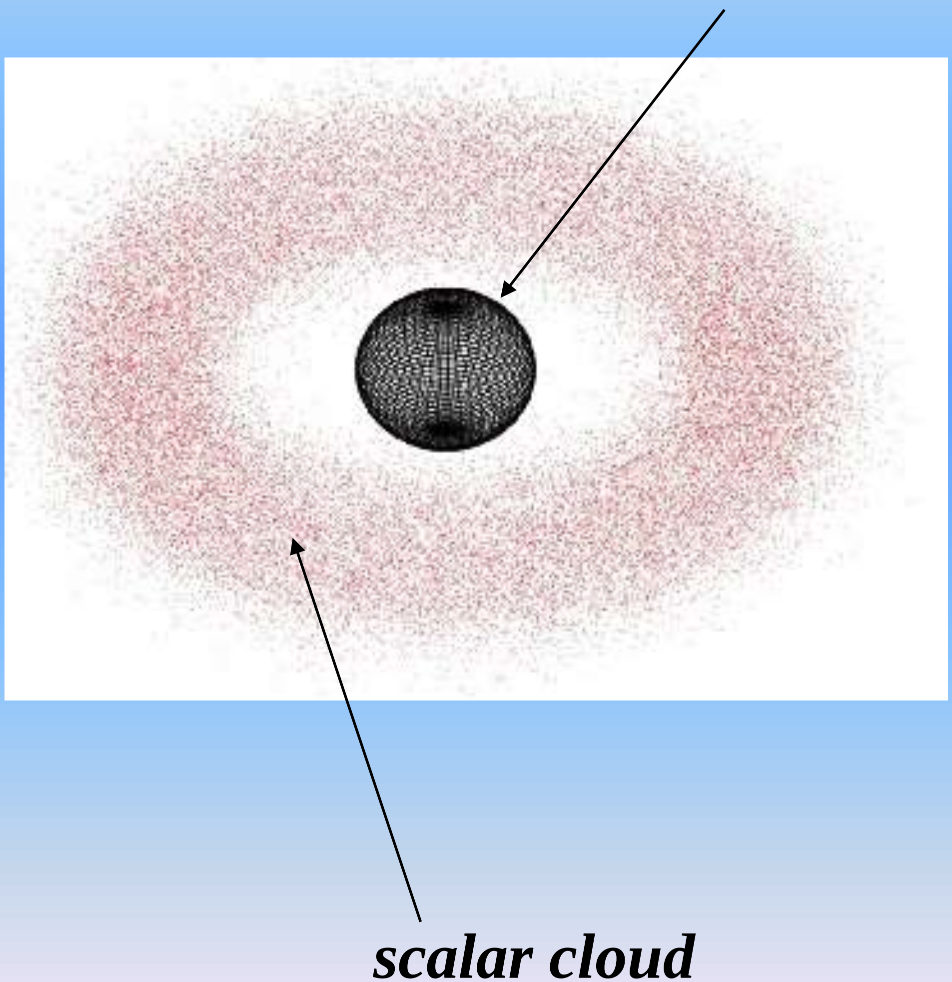
S. Dolan's lecture



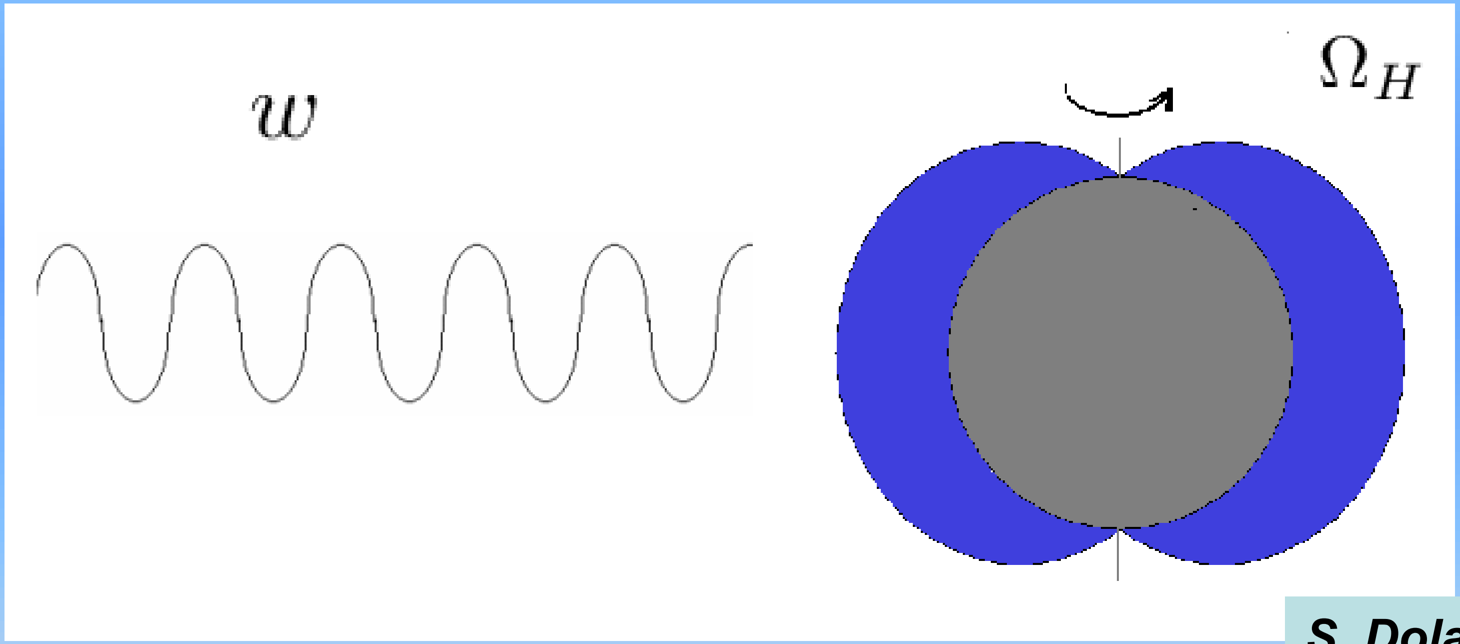
hairy black hole*:* bound state *soliton* + *Kerr horizon*

Klein-Gordon equation
in a rotating Black Hole background
possesses 'scalar cloud 'solutions
(even closed form (Hod 2012))

$$\nabla^2 \phi - \mu^2 \phi = 0$$



wave analogue of Penrose process: → **superradiant instability**



R>1

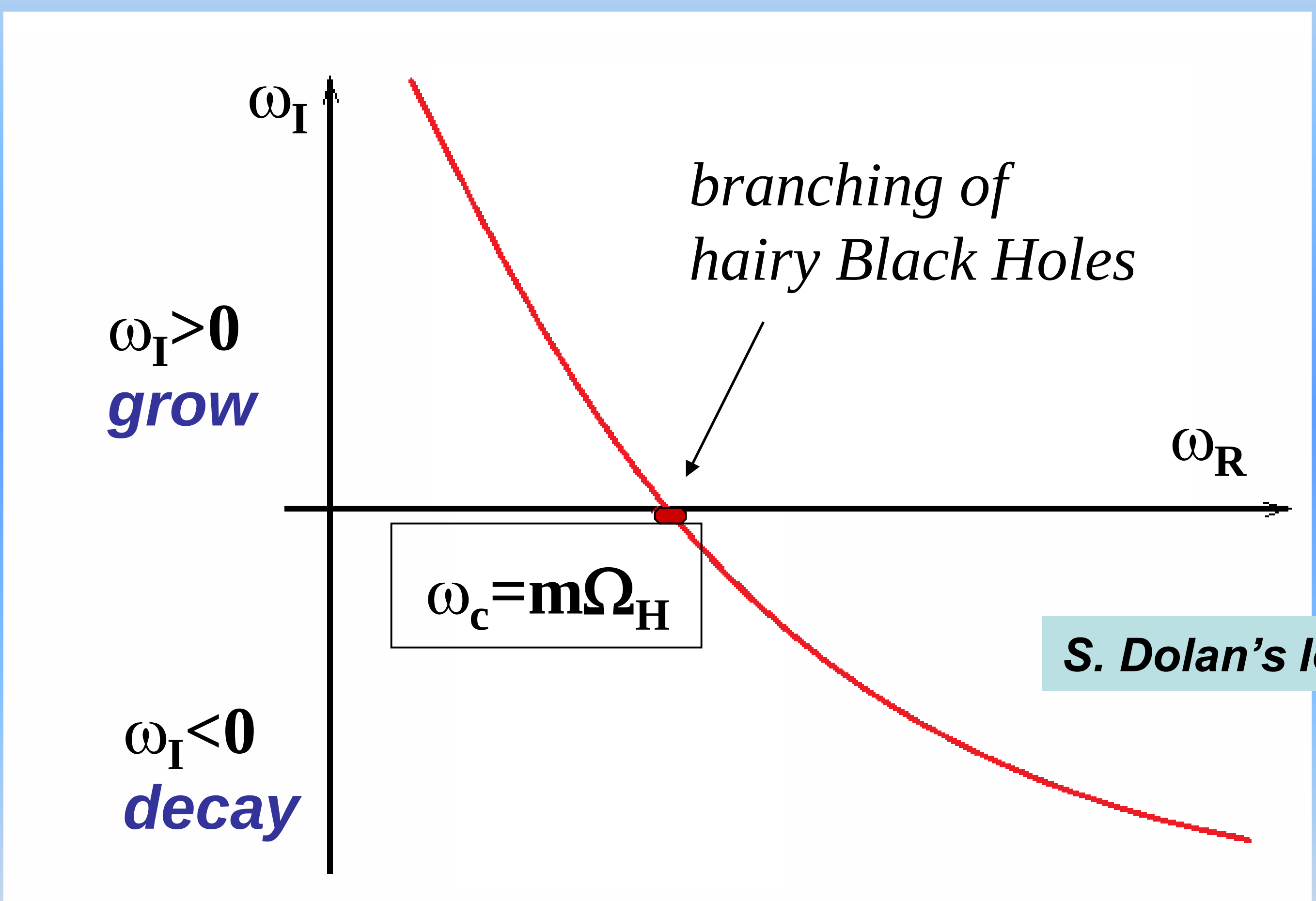
S. Dolan's lecture

Zeldovich (1971) + many studies…

review: arXiv:1501.06570 *(Brito, Cardoso and Pani)* bosonic field

$$\Psi \sim e^{i(m\varphi - \omega t)}$$

$$\omega = \omega_R + i\omega_I$$



S. Dolan's lecture

(no quantum effects)

$$\Psi = \phi(r,\theta) e^{i(m\varphi - wt)}$$

scalar:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2} g^{ab} \left(\Psi^*_{,a} \Psi_{,b} + \Psi^*_{,b} \Psi_{,a} \right) - \mu^2 \Psi^* \Psi \right]$$

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2} r^2 \sin^2\theta \, (d\varphi - W dt)^2 \ , \qquad N \equiv 1 - \frac{r_H}{r}$$

vector:

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{4} \mathcal{F}_{\alpha\beta} \bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2} \mu^2 \mathcal{A}_\alpha \bar{\mathcal{A}}^\alpha \right)$$

$$\mathcal{A} = e^{i(m\varphi - wt)} \left(iV dt + H_1 dr + H_2 d\theta + i H_3 \sin\theta d\varphi \right)$$

stationary configurations

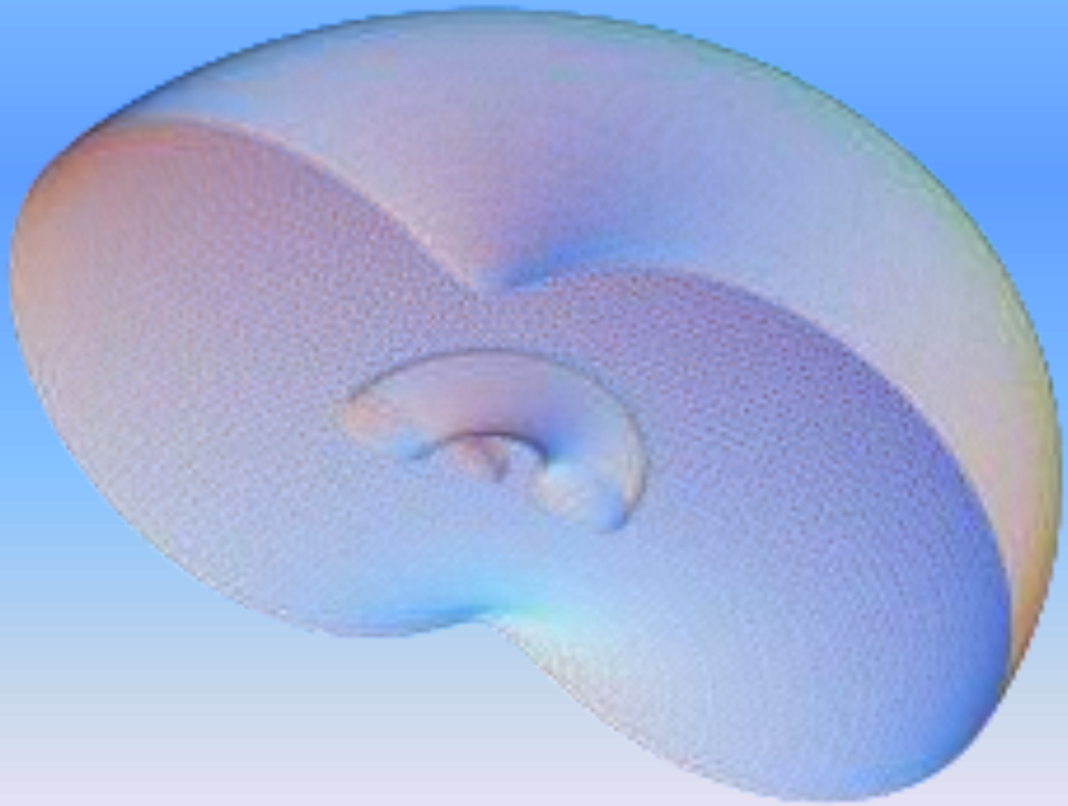
numerics

Kerr black holes with synchronised hair

existence proof:
arXiv:1510.08025

Chodosh&Shlapentokh-Rothman

solutions regular on and outside the horizon



spin-one

naively, such solutions should be simpler than Kerr-Newman:

However:

different pattern from Kerr(-Newman) !

synchronization condition
: (circumvent no-hair theorems)

• **no static limit** $w = m\Omega_H$

with $\Phi \sim e^{i(m\varphi - wt)}$

counterexample to all we knew

general properties:

different pattern from Kerr

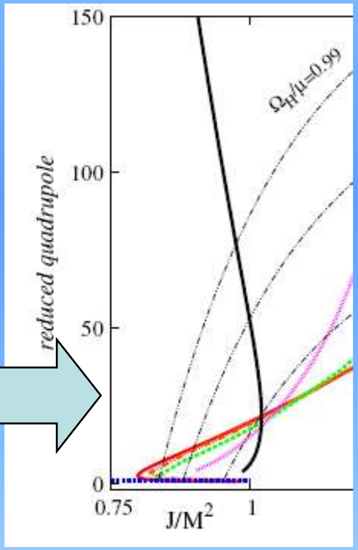
- **no static limit**
- **can violate Kerr bound** $J/M^2 > 1$

counterexample to all we knew

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**



counterexample to all we knew

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**

counterexample to all we knew

general properties:

different pattern from Kerr

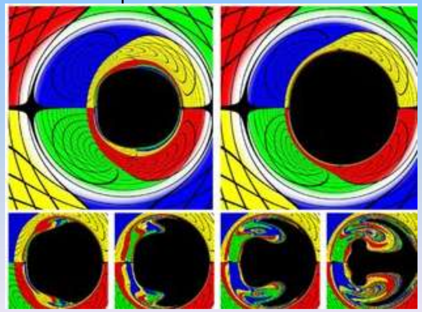
- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**

counterexample to all we knew

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**
- **different shadows**

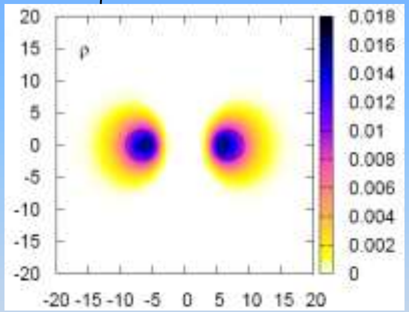


counterexample to all we knew

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**
- **different shadows**
- **solitonic limit**



counterexample to all we knew

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
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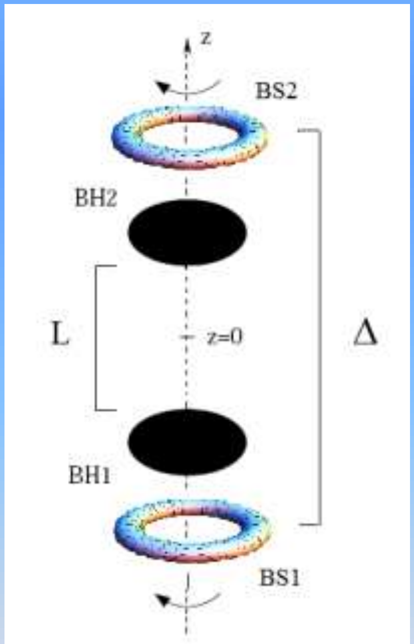
- **ergo-Saturns**
- **different shadows**
- **solitonic limit**



•*a recent result: two BHs balanced by scalar hair*

arXiv: 2305.15467

-- *endpoint of superradiant instablity?*

-- *“black hole bomb”*

(Press and Teukolsky -- 1972)

important result:

Selected for a Viewpoint in *Physics*

PRL **119,** 041101 (2017) PHYSICAL REVIEW LETTERS week ending 28 JULY 2017

Superradiant Instability and Backreaction of Massive Vector Fields around Kerr Black Holes

William E. East[1] and Frans Pretorius[2]

•***the endpoints of evolution <u>matches</u> the known hairy black holes***

Kerr geometry *"provides the **absolutely exact representation** of untold numbers of Black Holes that populate the Universe"* (Chandrasekhar)

challenging the Kerr paradigm

two main directions:

$$G_{\alpha\beta} = 8\pi G\, T_{\alpha\beta}$$

i) include matter fields (other than Maxwell)

ii) change the theory of gravity

also together

still an active area of research…

Black Holes

+

gauge fields

<u>spin-1 fields</u> *(other than Proca)*

Einstein-Maxwell system:

simple properties:

- *Reissner-Nordstrom/Kerr-Newman: unique solution*
- *the electric (magnetic) charge:* the only new parameter

<u>non-Abelian gauge fields</u>: part of the nature

- *is the same picture valid?*

- *for many years, this was the consensus in the literature…*
- *a few no-hair theorems..*

known non-Abelian solutions before EYM:
-- magnetic monopoles (1974)
-- sphalerons (1984)

they exist in flat space already: YM+Higgs: no need for gravity

Detour: Einstein vs Yang-Mills

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichenrowitz: there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Pure Yang-Mills (attraction/repulsion)

$$L = \frac{1}{2}\mathrm{Tr}\ F_{\mu\nu}^2$$

Deser, Coleman: Classical Yang-Mills theory in 3+1 dim is scale invariant - there is no soliton solution

Israel's theorem: Static Einstein-Maxwell black holes are spherically symmetric

'No-hair' theorem: Stationary black holes are completely characterized by their mass **M**, charge **Q** and angular momentum **J**

however…

VOLUME 61, NUMBER 2 PHYSICAL REVIEW LETTERS 11 JULY 1988

Particlelike Solutions of the Einstein-Yang-Mills Equations

Robert Bartnik and John McKinnon

Centre for Mathematical Analysis, Australian National University, Canberra, A.C.T. 2601, Australia

(Received 5 February 1988)

We study the static spherically symmetric Einstein-Yang-Mills equations with SU(2) gauge group and find numerical solutions which are nonsingular and asymptotically flat. These solutions have a high-density interior region with sharp boundary, a near-field region where the metric is approximately Reissner-Nørdstrom with Dirac monopole curvature source, and a far-field region where the metric is approximately Schwarzschild.

the action: $$I = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\mathcal{L}$$

balance

$$\mathcal{L} = R - \frac{1}{4}(F^a_{\mu\nu})^2$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$$

⇒ ***nonlinearity***

(the origin of all new features)

$$A = A^a_\mu \tau^a dx^\mu \qquad \tau^a = \sigma^a/2i \qquad [\tau^a, \tau^b] = \epsilon^{abc}\tau^c$$

SU(2) gauge group

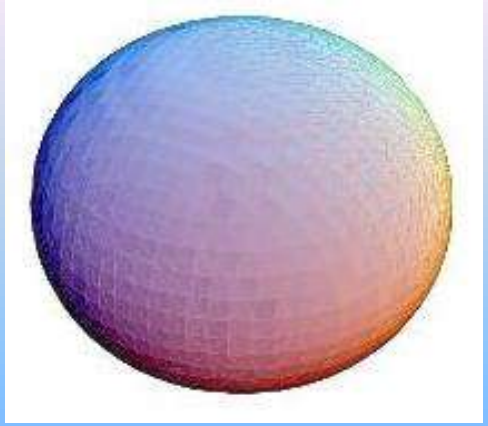
(however, not important)

however…

Bartnik and McKinnon (1988):

EYM solitons (no horizon)

regular everywhere



Galtsov and Volkov,
Kuenzle, Bizon (1989, 1990)

EYM black holes

'coloured' Black Holes

the no-hair conjecture is violated in Einstein-Yang-Mills model

a review of
such hairy solutions:
Volkov-Galtsov
hep-th/9810070

(*Exercise 4*)

-- no exact solutions…

-- however, rigorous existence proof

(several mathematicians in the ’90s)

-- many interesting properties

-- various generalizations

-- in the static case, the only global chare is the mass M

(no magnetic/electric charge)

however:

-- ***all solutions are unstable***

-- *also, the Schwarzschild solution maximize the entropy (for a given mass)*

-- *no important physical applications*

Mass~M_{Pl}

spin 0 + spin 1:

Einstein-Maxwell-dilaton system:

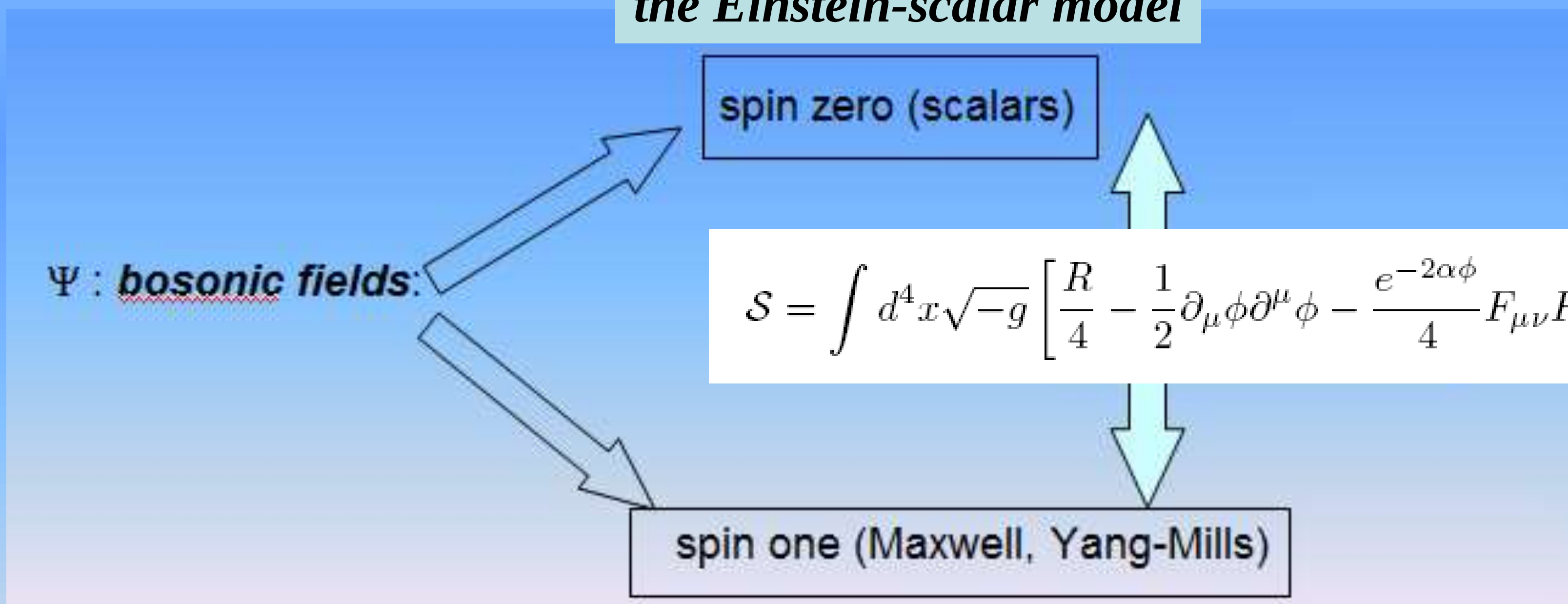
- *an early example of hairy Black Holes*
- *secondary hair*

$$\phi = -D/r + \ldots$$

$\{$***M,Q***$\}$

large literature

the Einstein-scalar model



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{4} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{e^{-2\alpha\phi}}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

the Einstein-Maxwell/YM model

violations of the no hair conjecture

vacuum:
Kerr solution
R

spin 0 **single (massive, complex) scalar field**

$(\partial_\nu \Phi^*)(\partial^\nu \Phi)$

Φ

('synchronized hair)'

spin 1

- *gauge fields:*

U(1): $F_{\mu\nu}F^{\mu\nu}$

SU(2): $F^{(a)}_{\mu\nu}F^{(a)\mu\nu}$

- *Abelian Proca field:* $\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} + \frac{1}{2}\mu^2 \mathcal{A}_\alpha \bar{\mathcal{A}}^\alpha$

('synchronized hair)'

spin 0+1

-*gauged scalar fields* $D_\alpha \Psi \equiv \partial_\alpha \Psi + iqA_\alpha \Psi$

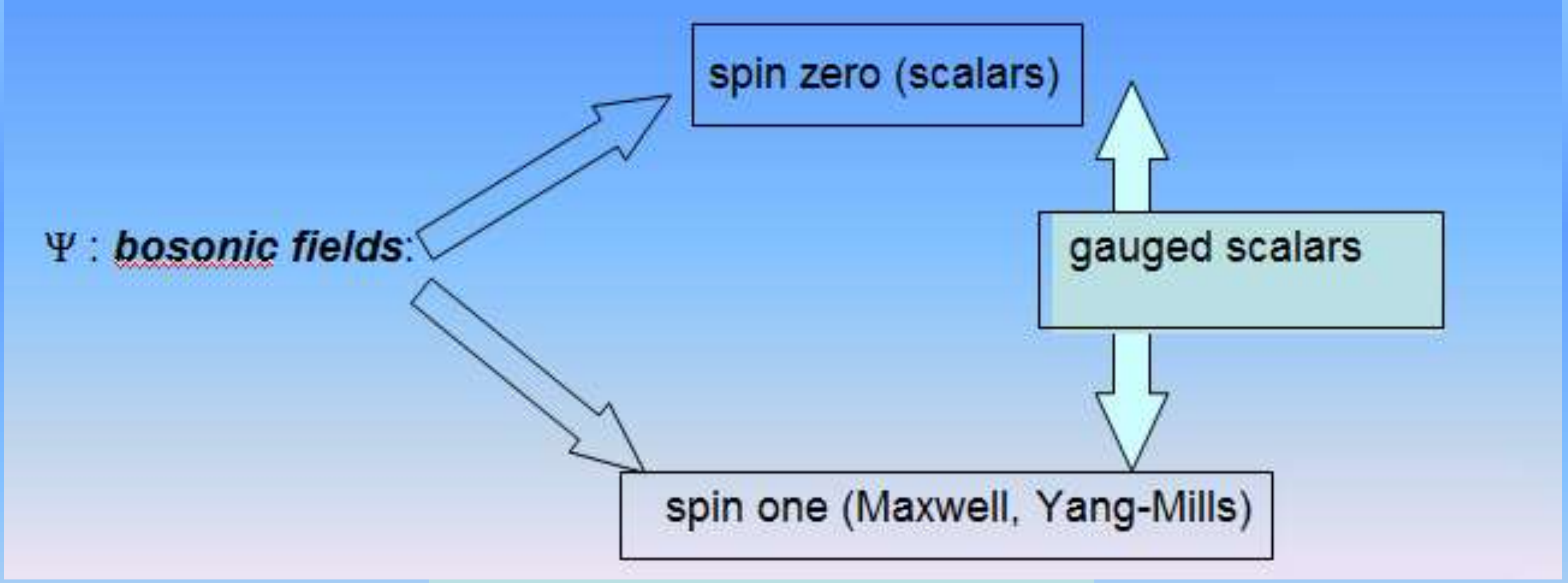
?

`natural` models

spin 0 + spin 1:

physical model :

the Einstein-scalar model



the Einstein-Maxwell/YM model

the Einstein-scalar model

mass term

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \partial_\alpha \Psi^* \partial^\alpha \Psi - U(|\Psi|) \right)$$

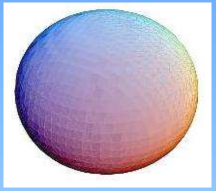
$$\mu^2 \equiv (d^2U/d|\Psi|^2)\big|_{\Psi=0},$$

global U(1) invariance: $\Psi \to \Psi e^{-i\chi}$

constant phase

spherically symmetric sector:

boson stars



no black holes

(Pena and Sudarsly theorem)

next step:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \partial_\alpha \Psi^* \partial^\alpha \Psi - U(|\Psi|) \right)$$

local U(1) invariance: $\Psi \rightarrow \Psi e^{-i\chi(x^\alpha)}$

the Einstein-Maxwell-scalar model

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|) \right]$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$D_\alpha \Psi \equiv \partial_\alpha \Psi + iqA_\alpha \Psi$$

gauge coupling constant (q)

- **more realistic (standard model)**

the Einstein-scalar model

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \partial_\alpha \Psi^* \partial^\alpha \Psi - U(|\Psi|) \right)$$

spherically symmetric sector:

→ boson stars

→ no black holes (Pena and Sudarsly theorem)

---

the Einstein-Maxwell-scalar model

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|) \right]$$

spherically symmetric sector:

→ gauged boson stars

→ *(expectation)* **no black holes** (Mayo-Bekenstein theorem)

the Einstein-Maxwell-scalar model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha\Psi^* D^\alpha\Psi - U(|\Psi|)\right]$$

gauged boson stars

spherically symmetric sector:

no black holes

(Mayo-Bekenstein theorem)

however, loophole!

e-Print: 1907.04982

2004.00336

recent result

static, spherically symmetric black holes

resonance condition

$$\omega = q\Phi$$

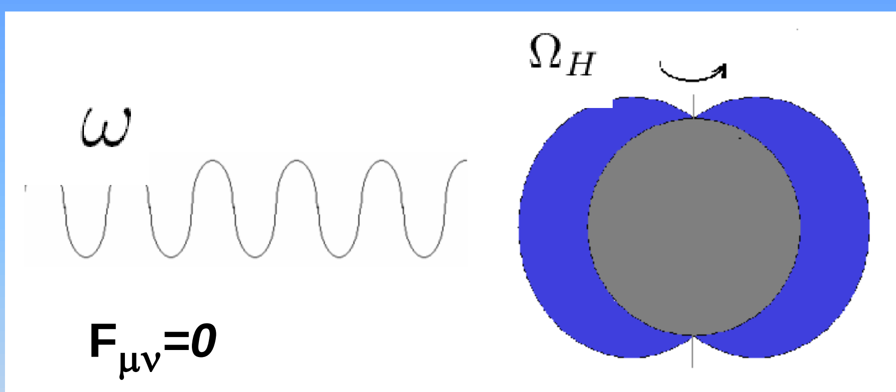
the Einstein-Maxwell-scalar model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha\Psi^* D^\alpha\Psi - U(|\Psi|)\right]$$

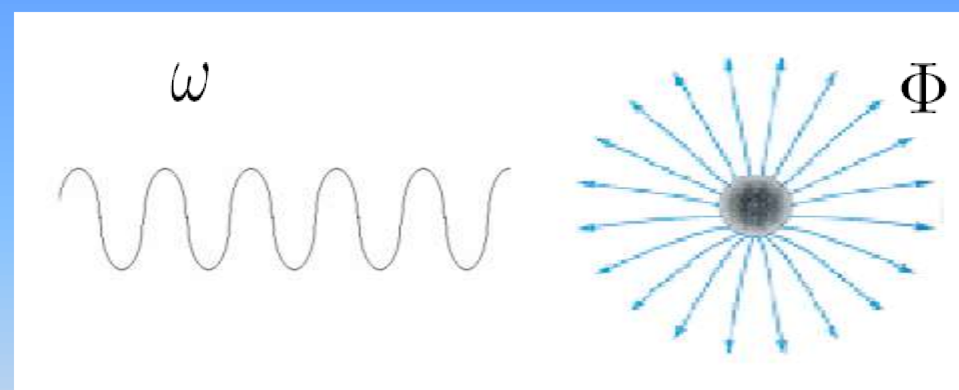
e-Print: 2312.02280

Rotation ⟺ Electric charge

rotating black hole with scalar hair



charged black hole with scalar hair



synchronization condition:

$$w = m\Omega_H$$

⟺

resonance condition:

$$\omega = q\Phi$$

include matter fields

what we know it exists
***In Nature*:**

the Lagrangian of the Standard Model:

Microcosm

$$\mathcal{L} = \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu})}_{\text{U(1), SU(2) and SU(3) gauge terms}}$$

$$\underbrace{+(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^{\mu} iD_{\mu}\begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \bar{e}_R\sigma^{\mu}iD_{\mu}e_R + \bar{\nu}_R\sigma^{\mu}iD_{\mu}\nu_R + \text{Hermitian conjugate}}_{\text{lepton dynamical term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R\bar{M}^e\bar{\phi}\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\right]}_{\text{electron, muon, tauon mass term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^{\nu}\nu_R + \bar{\nu}_R\bar{M}^{\nu}\phi^T\begin{pmatrix}-e_L\\ \nu_L\end{pmatrix}\right]}_{\text{neutrino mass term}}$$

$$\underbrace{+(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^{\mu} iD_{\mu}\begin{pmatrix}u_L\\ d_L\end{pmatrix} + \bar{u}_R\sigma^{\mu}iD_{\mu}u_R + \bar{d}_R\sigma^{\mu}iD_{\mu}d_R + \text{Hermitian conjugate}}_{\text{quark dynamical term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R\bar{M}^d\bar{\phi}\begin{pmatrix}u_L\\ d_L\end{pmatrix}\right]}_{\text{down, strange, bottom mass term}}$$

$$\underbrace{-\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R\bar{M}^u\phi^T\begin{pmatrix}-d_L\\ u_L\end{pmatrix}\right]}_{\text{up, charmed, top mass term}}$$

$$\underbrace{+\overline{(D_{\mu}\phi)}D^{\mu}\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2.}_{\text{Higgs dynamical and mass term}}$$

Spin 0: ***scalars***
Higgs field

Spin 1/2: ***spinors***

Spin 1: ***vector fields***
Maxwell + non-Abelian
(SU(2), SU(3))

$$T_{\alpha\beta} \neq 0$$

Standard Model: SU(2)xU(1) gauged Higgs solitons

$$\Phi(x) = \begin{pmatrix} \psi_2(x) + i\psi_1(x) \\ \phi(x) - i\psi_3(x) \end{pmatrix}.$$

sphalerons (monopoles?)

$$D_\mu \Phi = (\partial_\mu + A_\mu)\Phi$$

mass ~10 TeV

Hairy Black Holes inside sphalerons/monopoles

Georgi-Glashow model: SU(2) gauged Higgs solitons

(also GUT)

$$\Phi(x) = \Phi^a \tau_a$$

minimal mass - 1GeV

$$D_\mu \Phi = \partial_\mu \Phi + i[A_\mu, \Phi]$$

(no evidence)

Kerr geometry "*provides the **absolutely exact representation** of untold numbers of Black Holes that populate the Universe*" (Chandrasekhar)

challenging the Kerr paradigm

two main directions:

$$G_{\alpha\beta} = 8\pi G \, T_{\alpha\beta}$$

i) include matter fields (other than Maxwell)

ii) change the theory of gravity

also together

still an active area of research…

Hairy Black Holes
beyond General Relativity

a classic example

stringy model

Einstein-Gauss-Bonnet-dilaton model

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\phi)^2 + \alpha e^{-\gamma\phi}R^2_{\mathrm{GB}}\right]$$

dilaton

$$R^2_{\mathrm{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 .$$

Extensively studied over the last decades

- a lot of interesting features

- (perhaps) the most unusual property: *the existence of a minimal size for the black holes*

- the Kerr bound can also be violated

- no solitons

$$\Box\phi = \alpha\gamma e^{-\gamma\phi}R^2_{\mathrm{GB}}$$

a ‘cousin’ model:

the Chern-Simons modified gravity

$$I = \int d^4x \sqrt{-g} \left(\kappa R + \frac{\alpha}{4} \phi \, {}^*R\, R - \frac{1}{2} g^{ab} (\nabla_a \phi)(\nabla_b \phi) - V(\phi) \right)$$

$${}^*R\, R = {}^*R^a{}_b{}^{cd} R^b{}_{acd} \,, \quad \text{with} \quad {}^*R^a{}_b{}^{cd} = \frac{1}{2} \epsilon^{cdef} R^a{}_{bef}$$

- perturbative solutions mainly

- Schwarzschild BH is a solution; Kerr is not!
(CS term affects only the spinning solutions)

- third order equations of motion…

another mechanism

Black hole scalarisation

not a new idea
Damour and E-Farese
(neutron stars)

$$\mathcal{S}_\phi = -\int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2 + f(\phi)\mathcal{I}(\psi; g)\right]$$

$$\Box\phi = \frac{\partial f}{\partial\phi}\mathcal{I}$$

$$\frac{\partial f}{\partial \phi}\Big|_{\phi=\phi_0} = 0$$

$$f(\phi) = f|_{\phi=0} + \frac{1}{2}\frac{\partial^2 f}{\partial\phi^2}\Big|_{\phi=0}\delta\phi^2 + \mathcal{O}(\delta\phi^3)$$

$$f(\phi) = e^{-2\alpha\phi}$$

different case

$$(\nabla^2 - \mu_{eff}^2)\phi = 0, \quad \text{where} \quad \frac{1}{2}\mu_{eff}^2 = \frac{\partial^2 f}{\partial\phi^2}\Big|_{\phi=0}\mathcal{I}$$

e.g.

$$f(\phi) = e^{-\alpha\phi^2}$$

Fundamental solutions:

$\phi = 0$ $(\psi; g)$ *(ground state of the system)*

usually thermodynamically **favoured**

Scalarised solutions:

$\phi \neq 0$ $(\psi; g)$ *(excited state)*

possible models

$$\mathcal{S}_\phi = -\int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2 + f(\phi)\mathcal{I}(\psi;g)\right]$$

$$\mathcal{I} = \mathcal{L}_{GB} \equiv R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$$

arXiv:1711.01187, 1711.02080, 1711.03390

$$\mathcal{I} = \mathcal{L}_{CS} \equiv R\tilde{R} = {}^*R^a{}_b{}^{cd}R^b{}_{acd}$$

requires rotation
(1810.09560)

$$\mathcal{I} = R$$

1901.02953

$$\mathcal{I} = F_{\mu\nu}F^{\mu\nu}$$

1806.05190

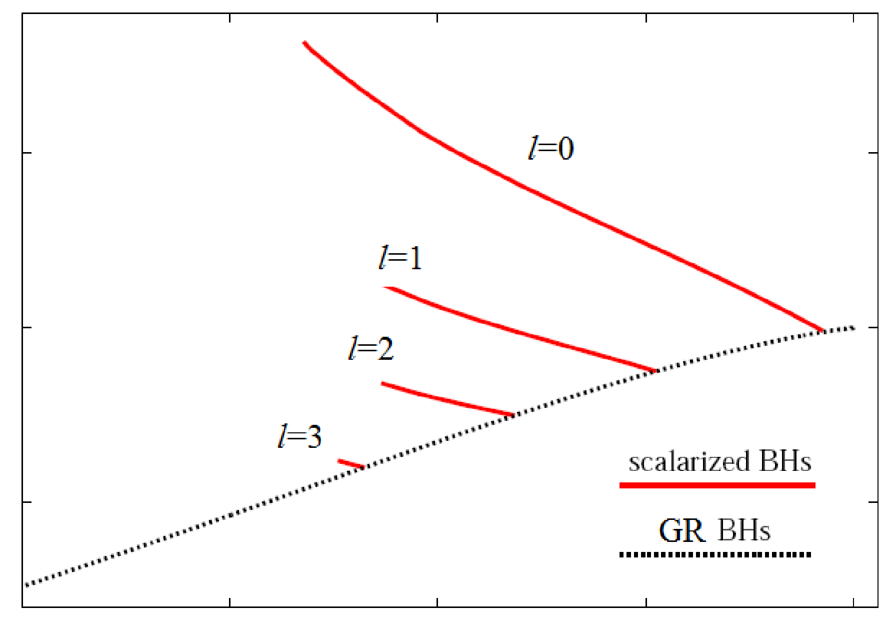
- vectorization
- spinorization…

Generic features of scalarized Black Holes:

i)

$$\phi(r,\theta,\varphi) = \sum_{\ell m} Y_{\ell m}(\theta,\varphi) U_\ell(r)$$

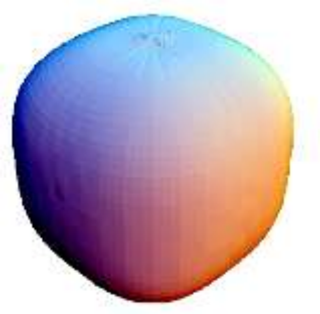
all modes are relevant



the nonlinear continuation of zero modes

l>0

horizon shape



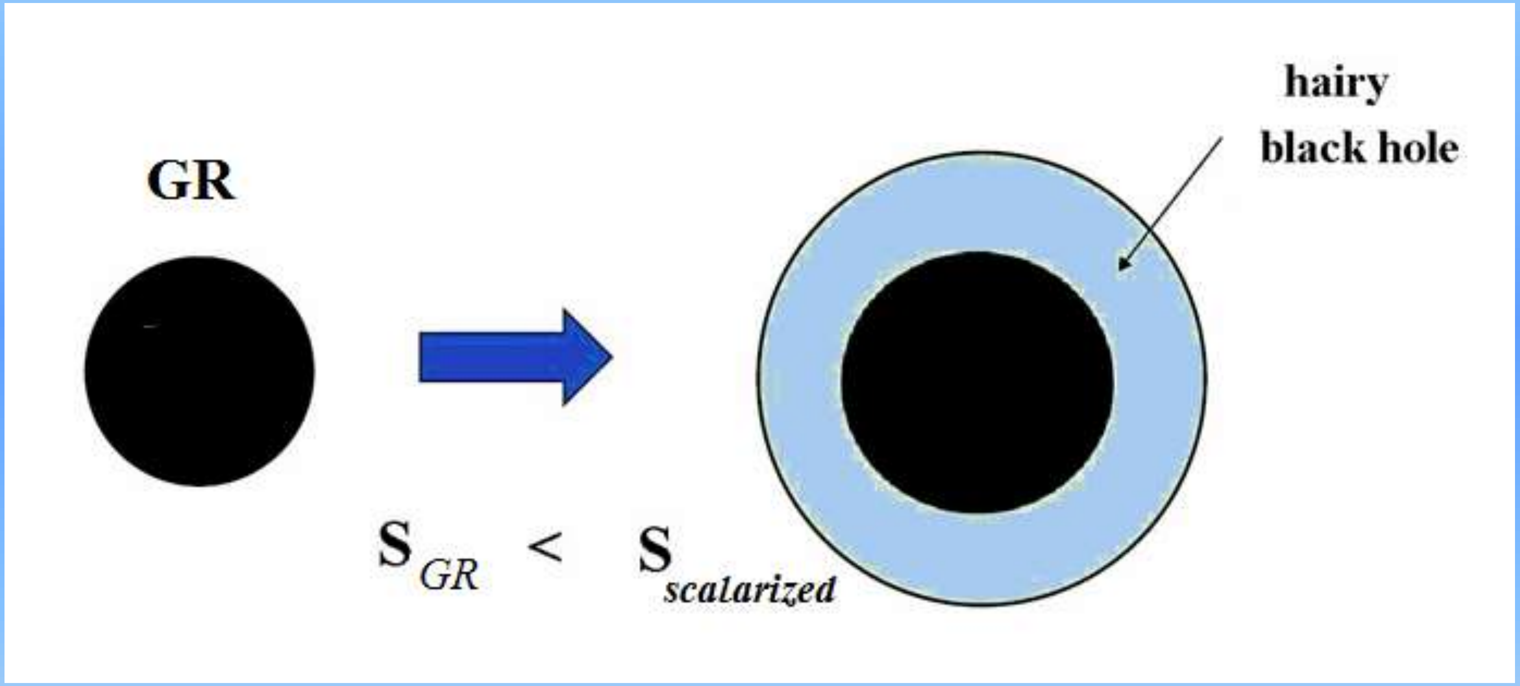
- *static and not spherically symmetric*
- *Black Holes without isometries*

very different from the Einstein(-Maxwell –(dilaton)) case

Generic features:

ii)

$$\phi(r,\theta,\varphi) = \sum_{\ell m} Y_{\ell m}(\theta,\varphi) U_\ell(r)$$

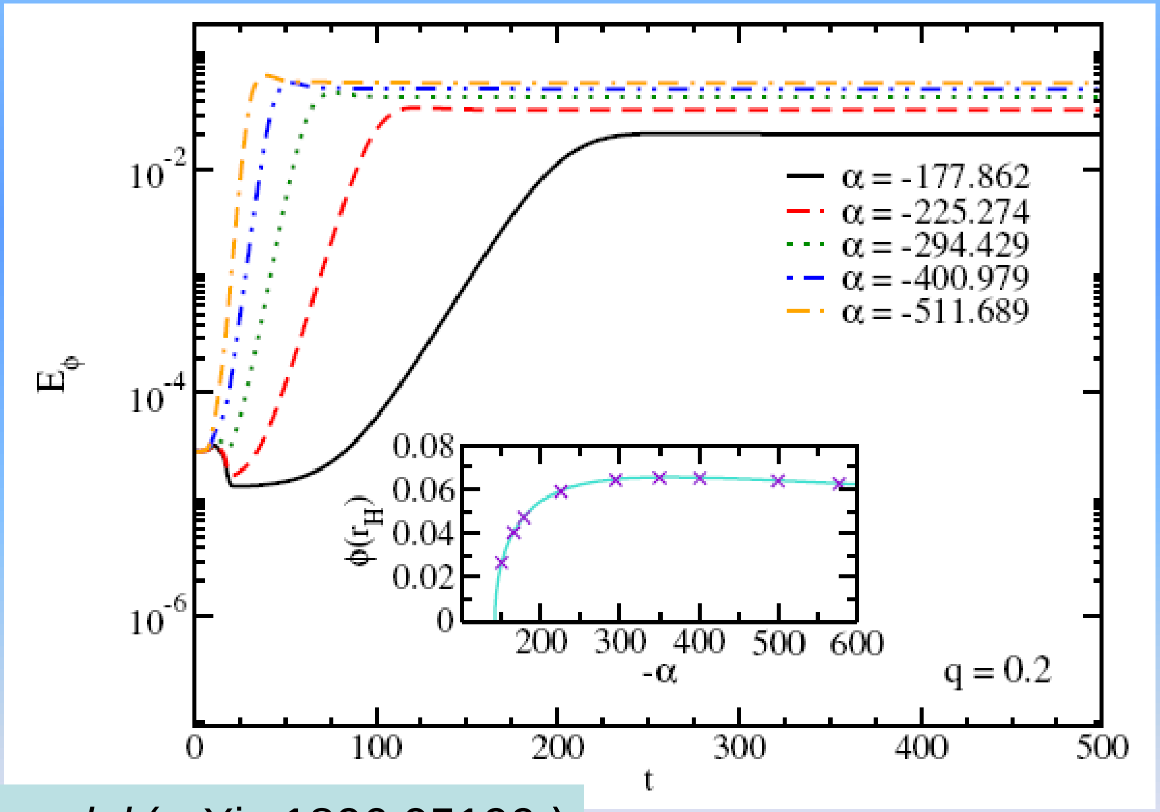


the *l*=0 scalarized Black Holes are usually entropically favoured

Generic features:

iii)

- **the scalarized Black Holes in some models were shown to be stable**
- **also, they can form dynamically**



here: the EMs model (arXiv:1806.05190)

$$\mathcal{I} = F_{\mu\nu}F^{\mu\nu}$$

another example

$$\mathcal{I} = R$$

arXiv: 1901.02953

better motivated

$$\mathcal{L}_\phi = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\xi\phi^2 R$$

$R \neq 0$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{4} + \mathcal{L}_0(\Psi)\right] + \mathcal{S}_\phi$$

$$\mathcal{L}_0(A,\psi) = -\frac{1}{2}(\nabla\psi)^2 - \frac{1}{4}e^{2a\psi}F^2$$

$$\mathcal{L}_0(A) = \frac{1}{4}F^2 + \frac{\alpha}{16}F^4$$

$$F^4 \equiv [F_{\mu\nu}(\star F)^{\mu\nu}]^2$$

NCG inspired modified Schwarzschild BHs (Nicolini solution)

spin 2:

Hairy Black Holes even in the absence of matter fields

Lu, Perkins, Pope and Stelle, “**Black Holes in Higher Derivative Gravity**“, **Phys. Rev. Lett. 114, 171601 (2015)**

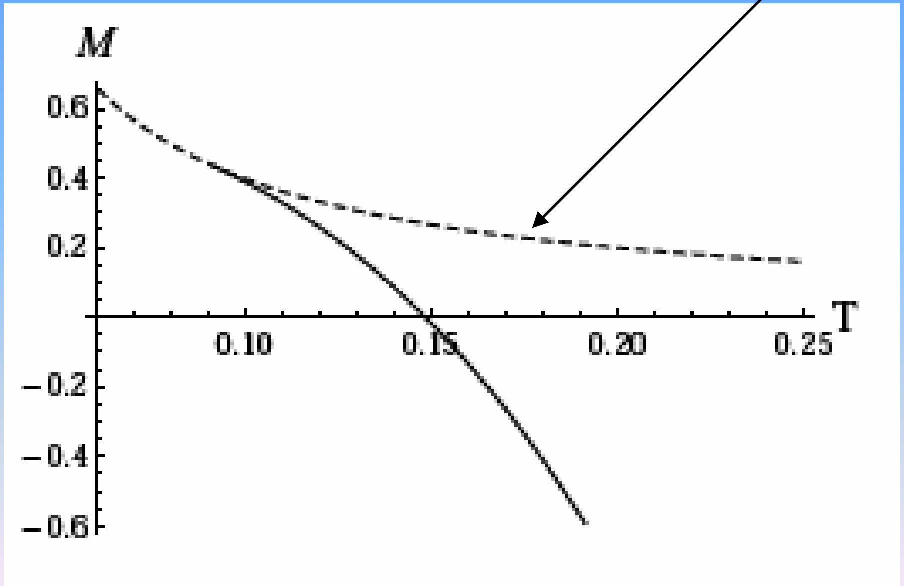
$$I = \int d^4x \sqrt{-g}\left(\gamma R - \alpha C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}\right)$$

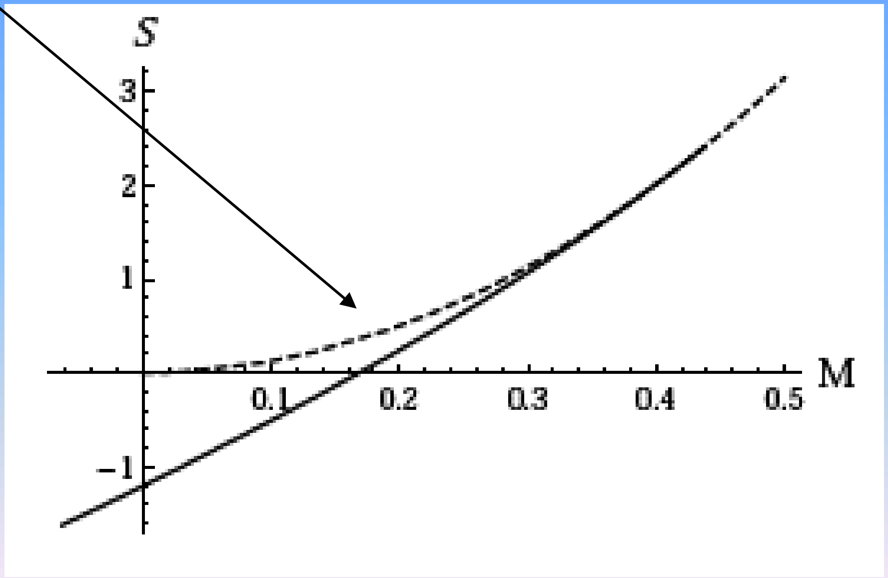
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f(r) \neq h(r)$$

new solutions bifurcating from Schwarzschild black hole

‘tensorization’





open problem

vacuum:
Kerr solution

R

spin 0 **single (massive, complex) scalar field**

$(\partial_\nu \Phi^*)(\partial^\nu \Phi)$

Φ

('synchronized hair)'

• ***fermions ?***

spin 1

- *gauge fields:*

U(1): $F_{\mu\nu}F^{\mu\nu}$

SU(2): $F^{(a)}_{\mu\nu}F^{(a)\mu\nu}$

- *Abelian Proca field:* $\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} + \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha$

('synchronized hair)'

spin 0+1

-gauged scalar fields $D_\alpha\Psi \equiv \partial_\alpha\Psi + iqA_\alpha\Psi$

what about spin 1/2?

no BH with fermionic hair: <u>the only exception</u> *(so far)*

Overview

We have seen that there are ***uniqueness theorems***, in vacuum, proving that the most general black hole solution (regular on and outside an event horizon) is the Kerr solution.

These theorems generalize to electro-vacuum: the most general black hole solution (regular on and outside an event horizon) is the Kerr-Newman solution

These uniqueness theorems motivated the ***no-hair conjecture*** stating that in general (i.e even for more general matter fields) ***the final state of gravitational collapse is a black holes characterized by conserved charges M,J,Q,***
all of them associated to a Gauss law, and no further parameters, to which "***hair***" provides a metaphor.

To support this idea, the community established various ***no-hair theorems*** applying to different models and with different assumptions.

Nevertheless, solutions with hair have been found in various models

Standard Model matter: **there are Black Holes beyond Kerr**

`NO HAIR` CONJECTURE IS NOT VALID

however:

- ***the spirit of 'no hair' theorems is respected***
- ***the solutions are typically unstable***
- ***these are not macroscopic configurations***
- ***(possibly) relevant at microscopic scales, only***

Macrocosm/beyond Standard Model:
even more complicated picture

two recent results:

first mechanism

superradiant instability ⟹ ***Kerr black hole with synchronized hair***

second mechanism

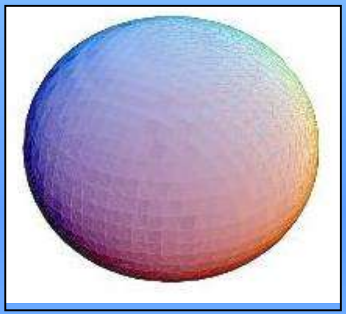
tachyonic instability ⟹ ***scalarized black holes (also static)***

a byproduct of this study:

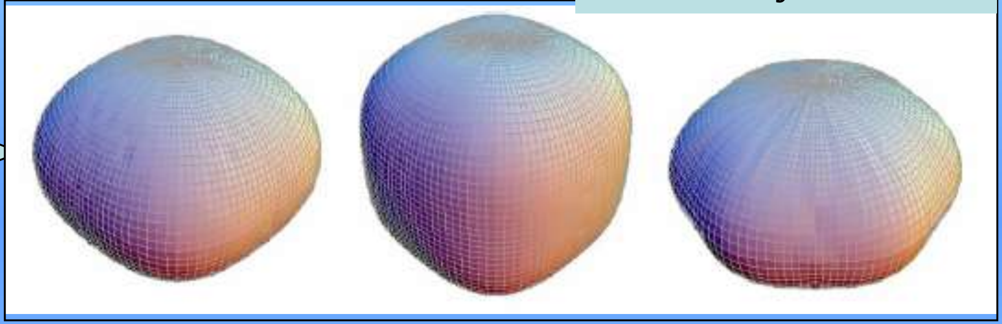
some new theory results as compared to electro-vacuum General Relativity
(one cannot safely extrapolate the Kerr results)

example *(i):* **the shape of the event horizon**
("Black Hole shape')

discrete symmetries



Schwarzschild/RN Black Hole



Black Holes in some Einstein-Maxwell-scalar model

example *(ii):* **the issue of :maximal rotation**

usual Black holes ***cannot*** *rotate 'too fast"*

Kerr bound: $J \leq M^2$

not a universal bound!

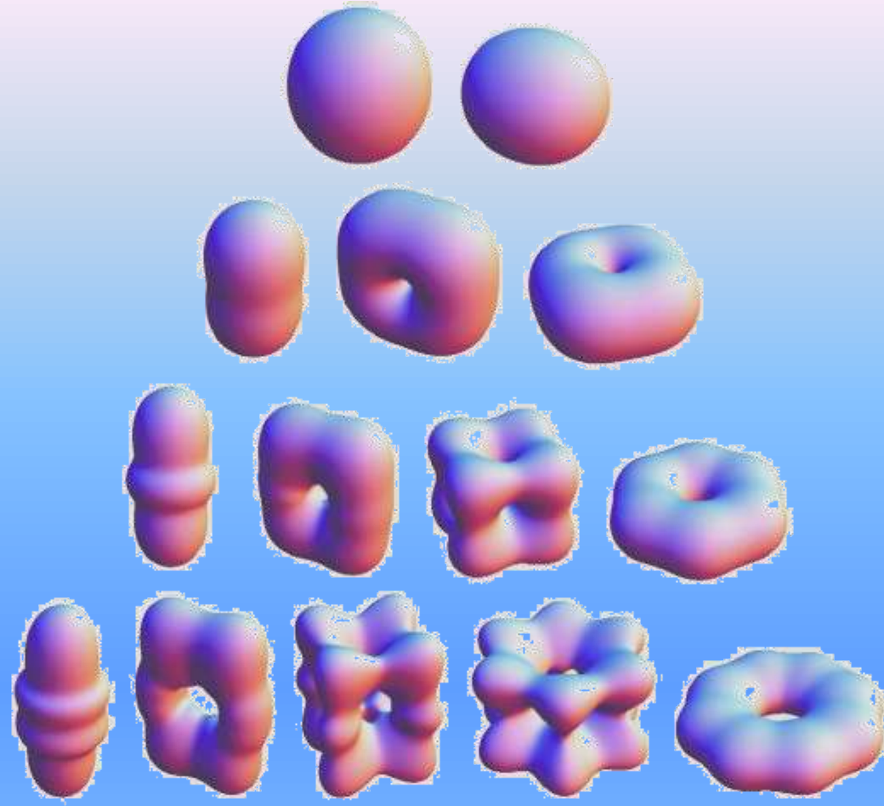
to summarize:

the GR black hole **possesses** hairy generalizations

….however, still a lot of work to be done

- ***find realistic models***
- ***establish stability/uniqueness***
- ***rotating Black Holes***

Q: ***are there any fields in Nature that can be relevant in the physics of astrophysical black holes?***



Many thanks for your attention!

Gr@v

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