Impact of self-interaction on the axion cloud in the relativistic regime

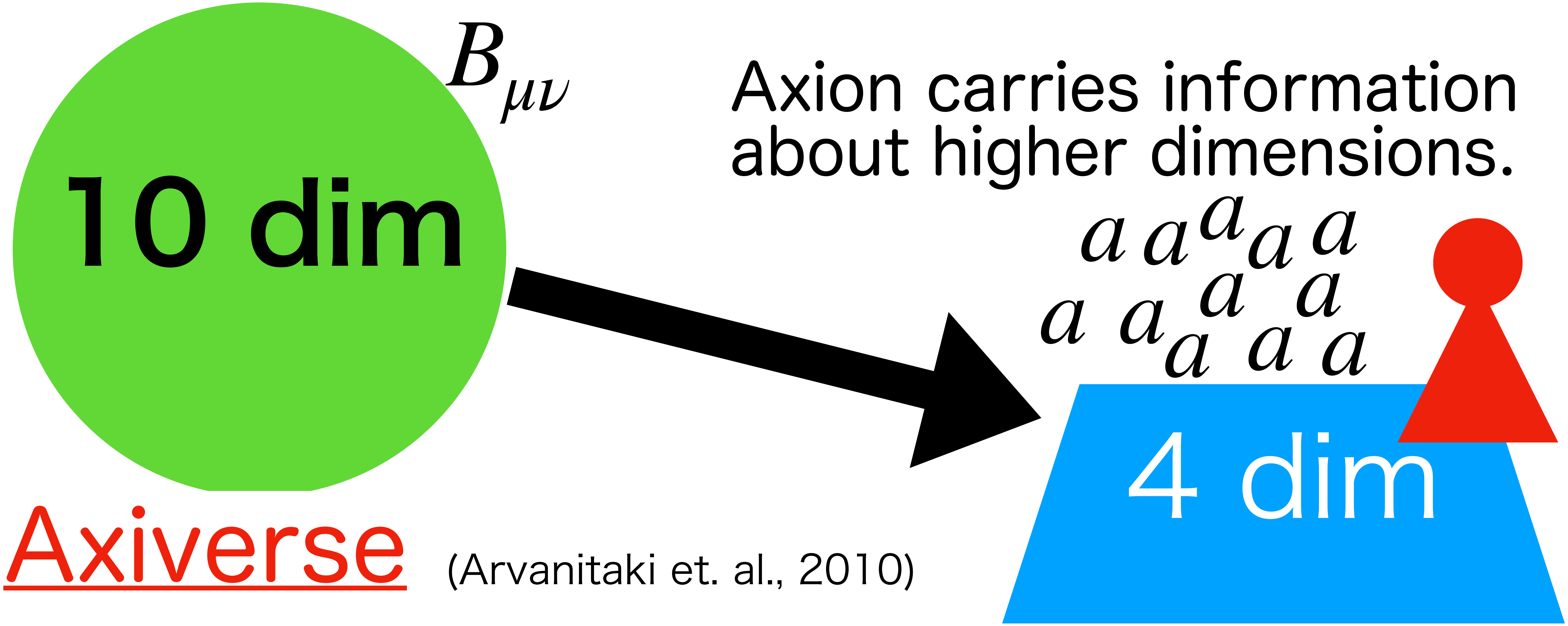
Hidetoshi Omiya(Kyoto U)

Work with Takuya Takahashi, Takahiro Tanaka,
and Hirotaka Yoshino

July 3rd@New Physics from Psi

Why Axion?

Search string theory through axion



Axion carries information about higher dimensions.

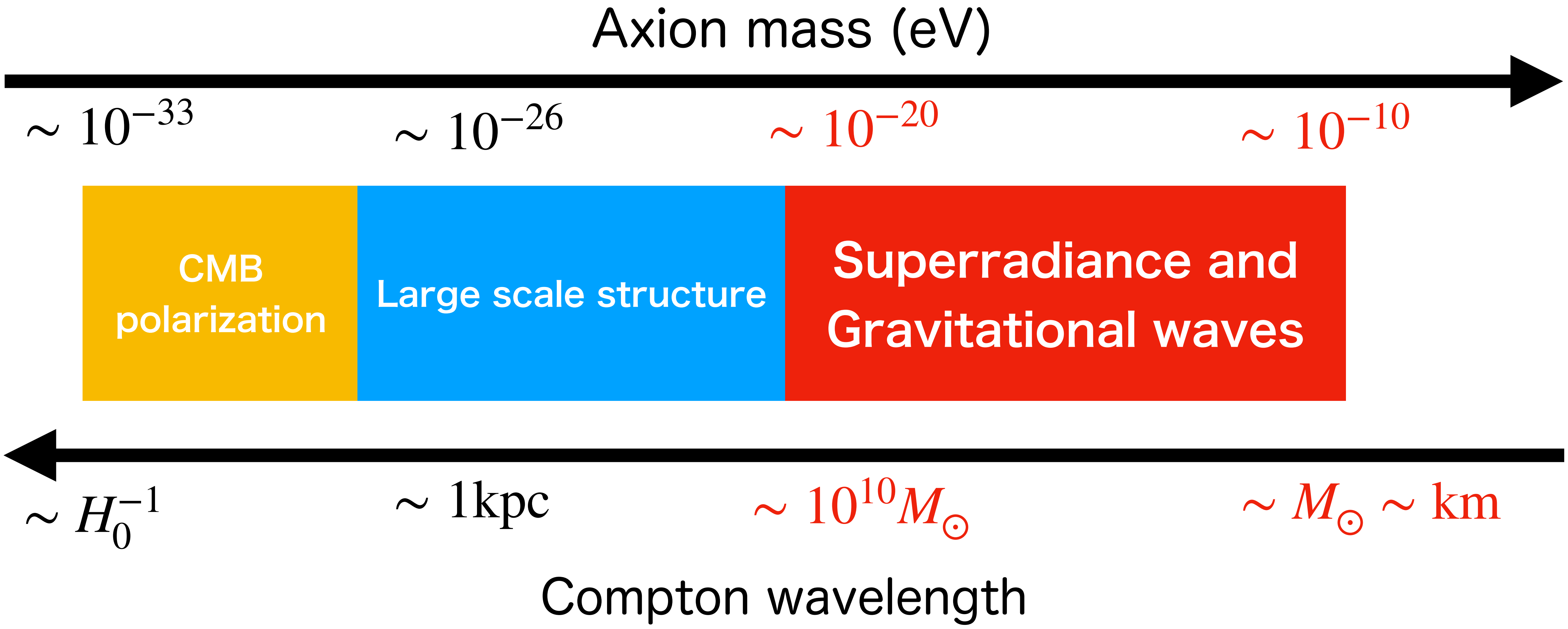
Axiverse (Arvanitaki et. al., 2010)

Plentitude of ultra-light and weakly interacting axions. $\mu \lesssim 10^{-10}\mathrm{eV}$ $F_a \sim 10^{16}\mathrm{GeV}$

(See for example, 2103.06812)

More on axion

- Other interests on axion
  - Solve strong CP problem
  - Candidate of the dark matter
  - **Can be observed by the astrophysical phenomena**



Axion mass (eV)

$\sim 10^{-33}$ $\sim 10^{-26}$ $\sim 10^{-20}$ $\sim 10^{-10}$

CMB polarization

Large scale structure

Superradiance and Gravitational waves

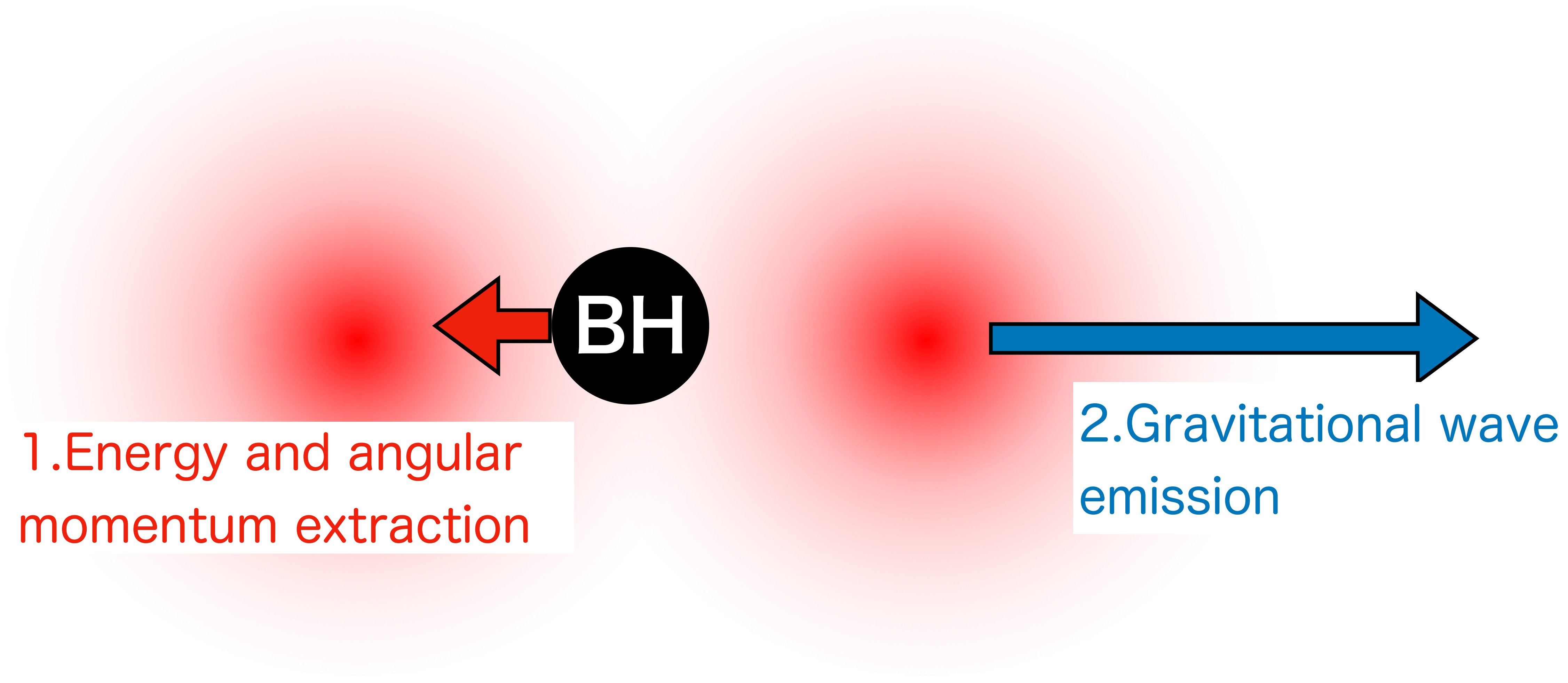
$\sim H_0^{-1}$ $\sim 1\mathrm{kpc}$ $\sim 10^{10}M_\odot$ $\sim M_\odot \sim \mathrm{km}$

Compton wavelength

Supperradiance and axion

Highly spinning black holes + axion

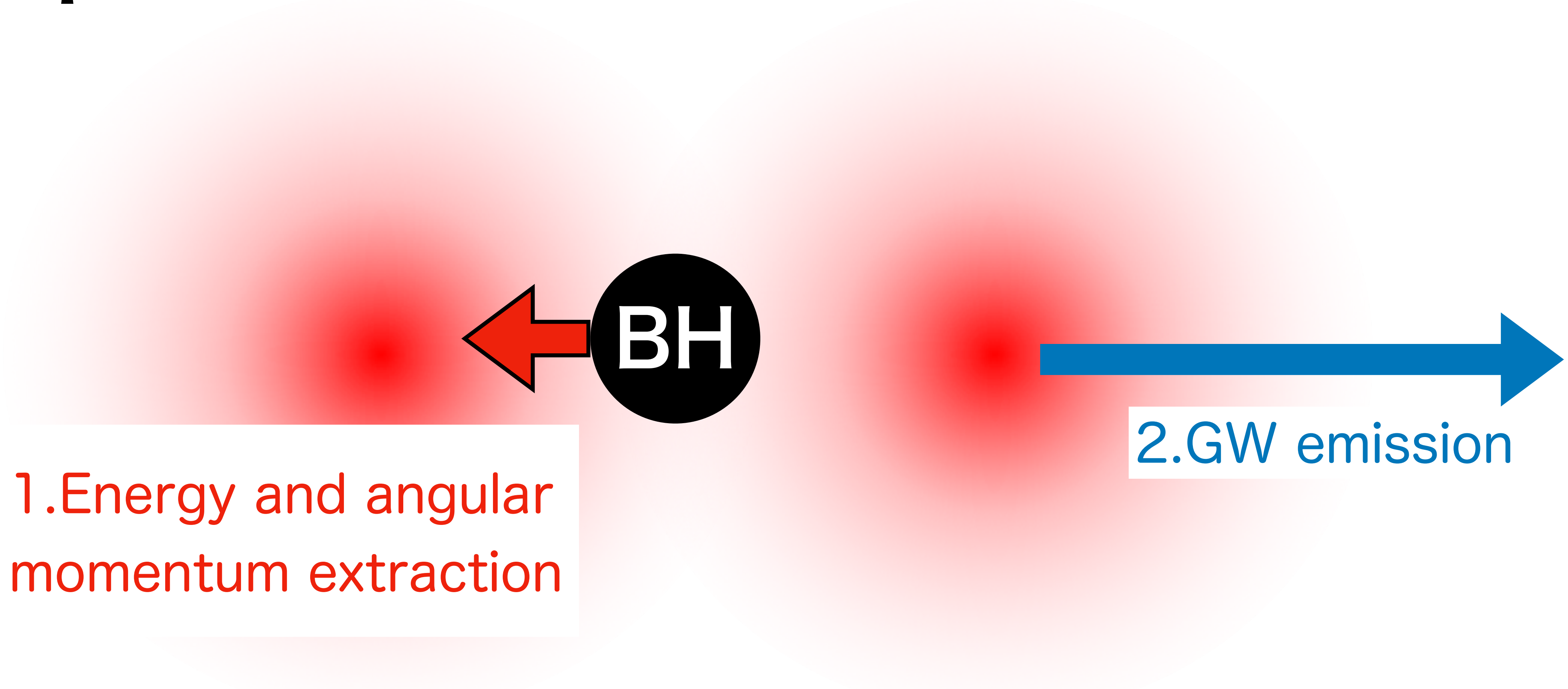
$\xrightarrow[\text{Superradiance}]{}$ Spontaneous formation of the macroscopic condensate of the axion



1.Energy and angular momentum extraction

2.Gravitational wave emission

Supperradiance and axion



1.Energy and angular momentum extraction

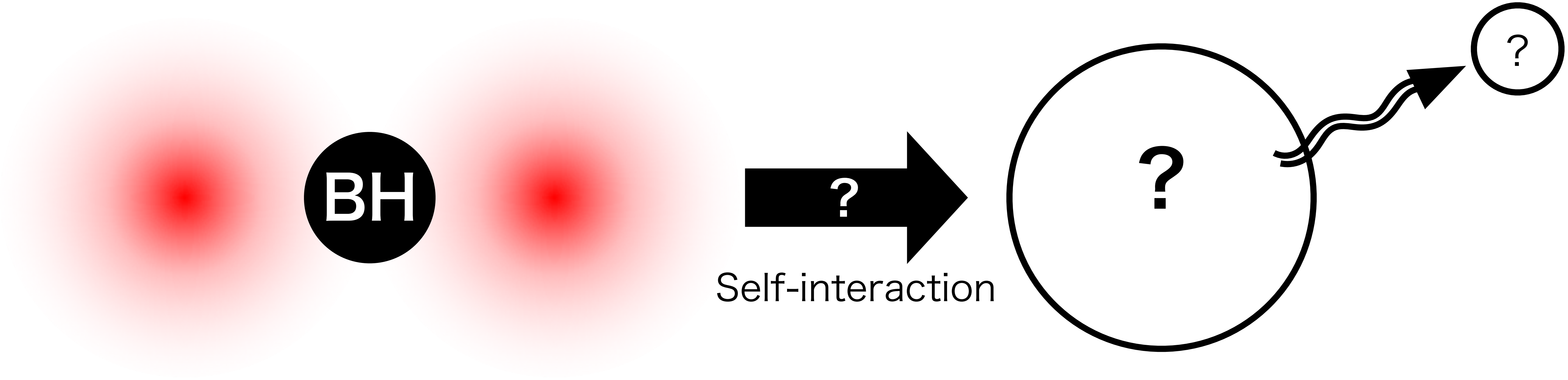
2.GW emission

- To give a precise constraint on axion from observation, precise understanding on the evolution of cloud is necessary.

Important effects

  - Self-interaction
  - Tidal effect from the companion
  - Coupling to other fields

Main message of the talk



Q. How does the cloud evolve under the self-interaction?

A. Higher multipole modes would excite.
No Bosenova.
Emit gravitational waves at several frequencies.

Self-interaction

$$S = F_a^2 \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial_\mu \phi)^2 - \mu^2 \left(1 - \cos\phi\right) \right]$$

$$V(\phi) = \mu^2 (1 - \cos\phi) \sim \frac{1}{2} \mu^2 \phi^2 \boxed{- \frac{1}{4!} \mu^2 \phi^4 + \cdots}$$

Attractive

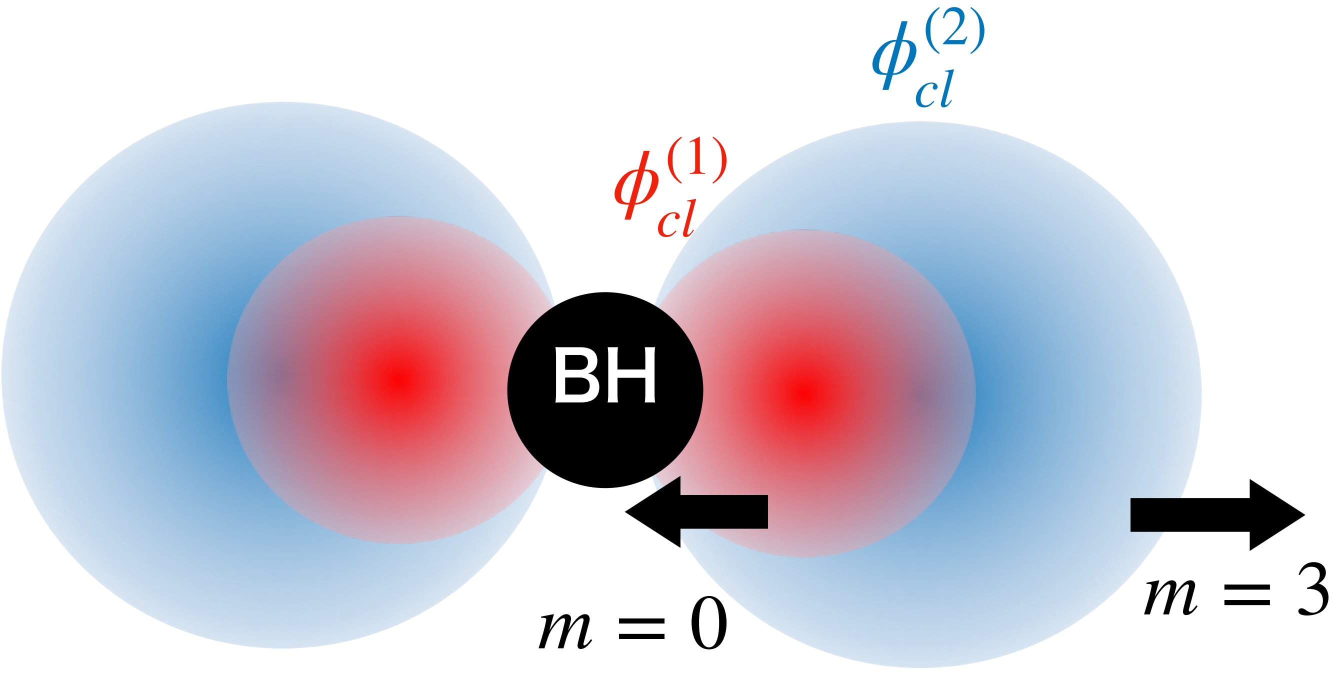
As cloud grows, self-interaction starts to work

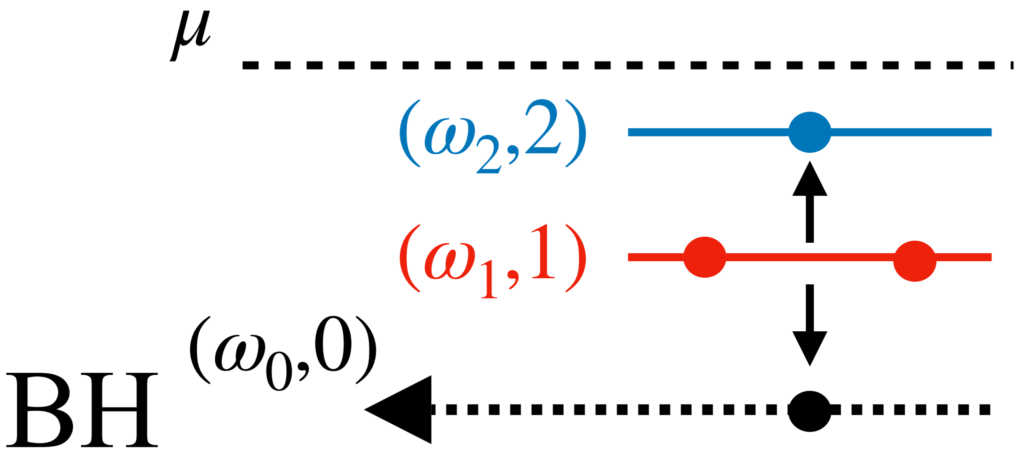
1. Dissipation of the cloud due to scattering
2. Deformation and collapse of the cloud

1.is more important, and dominates the evolution.

Dissipation by mode coupling

(Baryakhtar+, 2020)



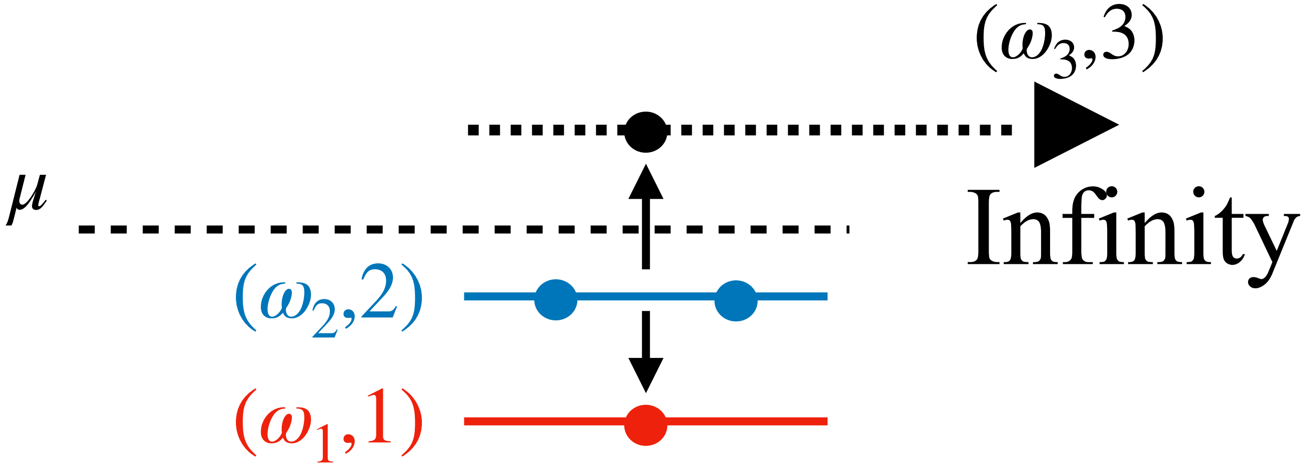


$l = m = 1$ transit to $l = m = 2$.

Dissipate energy through $m = 0$.

$\phi_{cl}^{(1)} : l = m = 1, \omega_R^{(1)} < \mu$

$\phi_{cl}^{(2)} : l = m = 2, \omega_R^{(1)} < \omega_R^{(2)} < \mu$



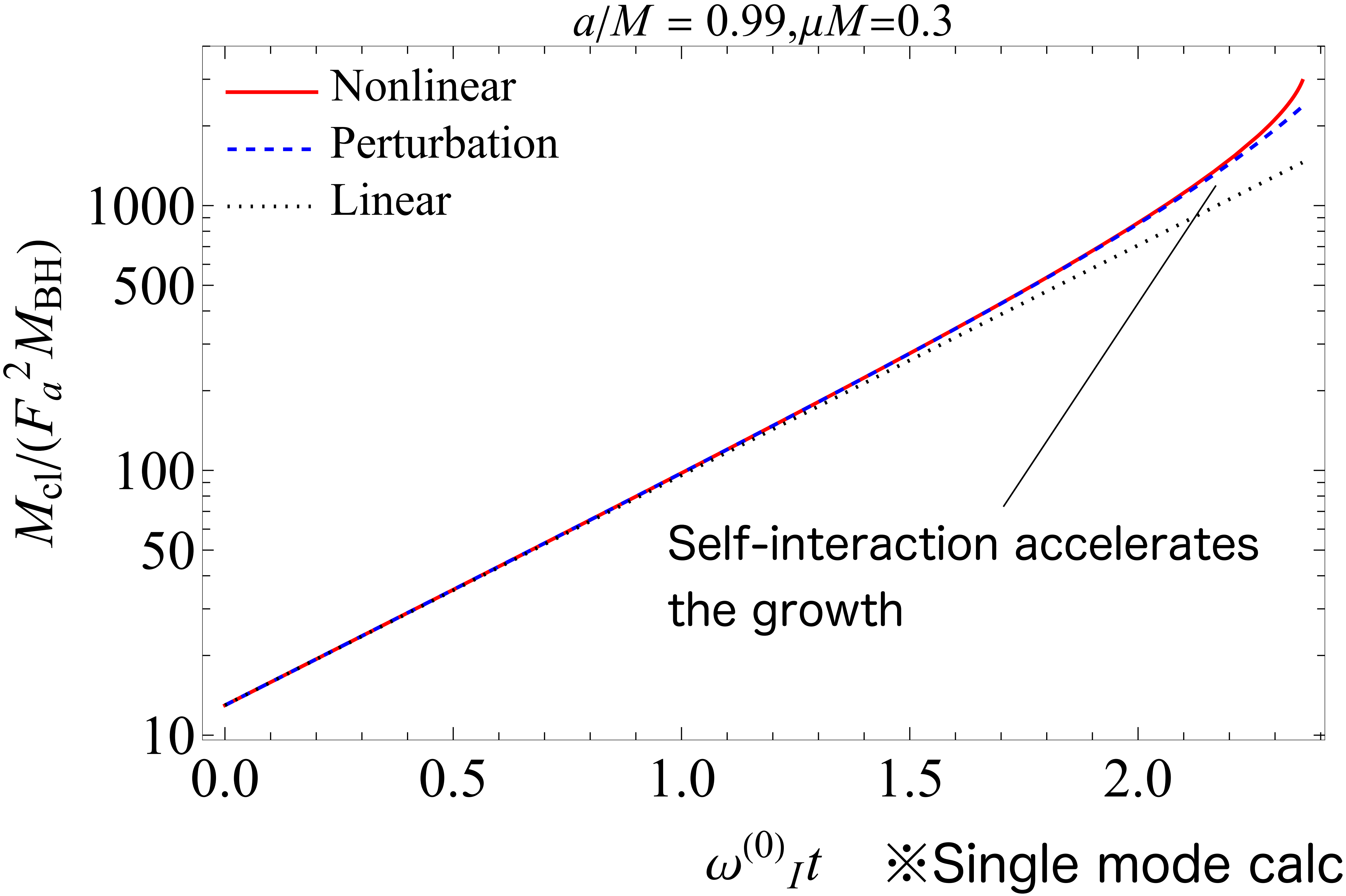
$l = m = 2$ transit to $l = m = 1$

Dissipate through $m = 3$.

Deformation of the cloud

(HO+, 2022)

Real and imaginary part of the frequency is modified.



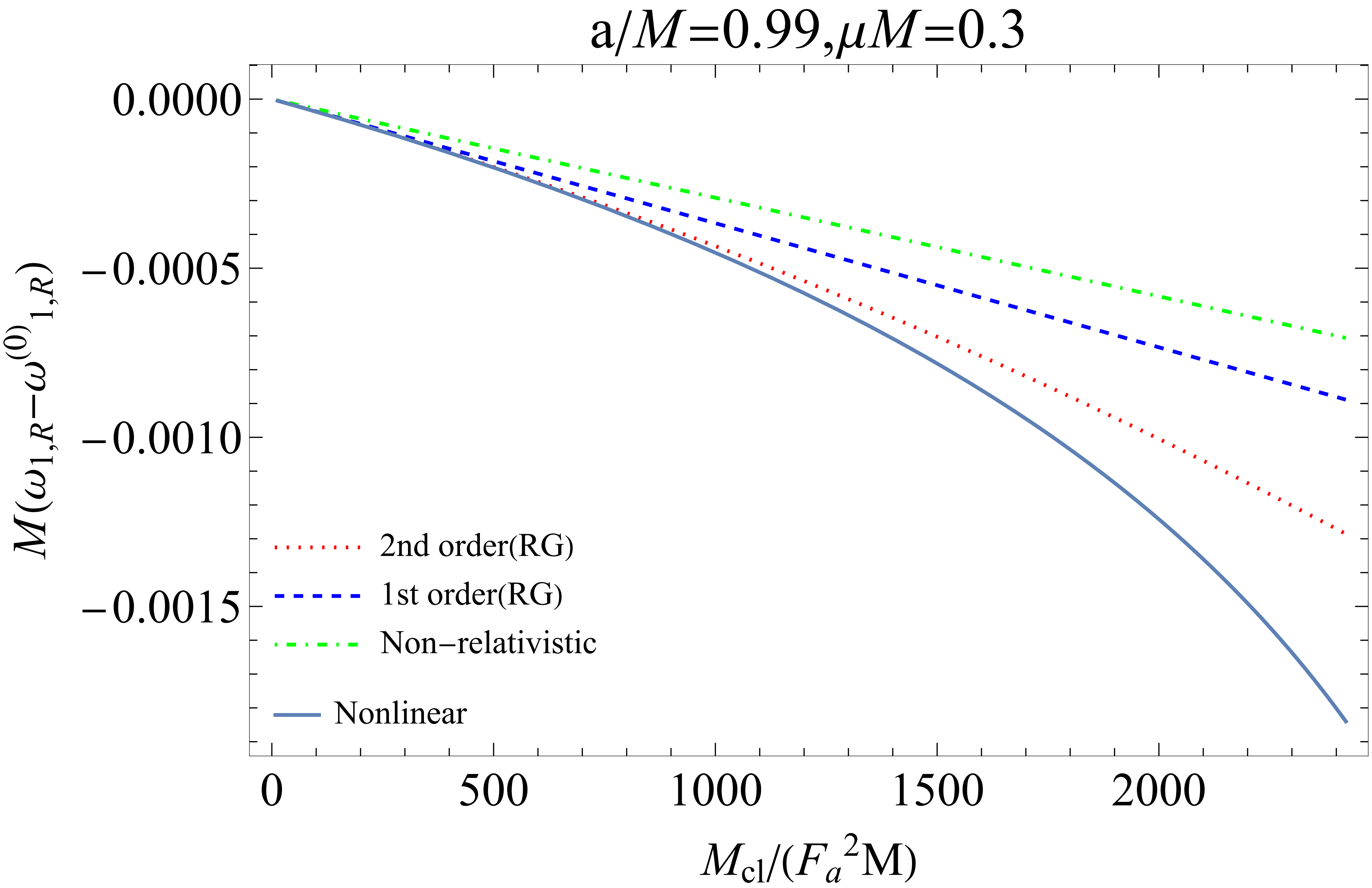
Self-interaction accelerates the growth

※Single mode calculation

Deformation of the cloud

(HO+, 2022)

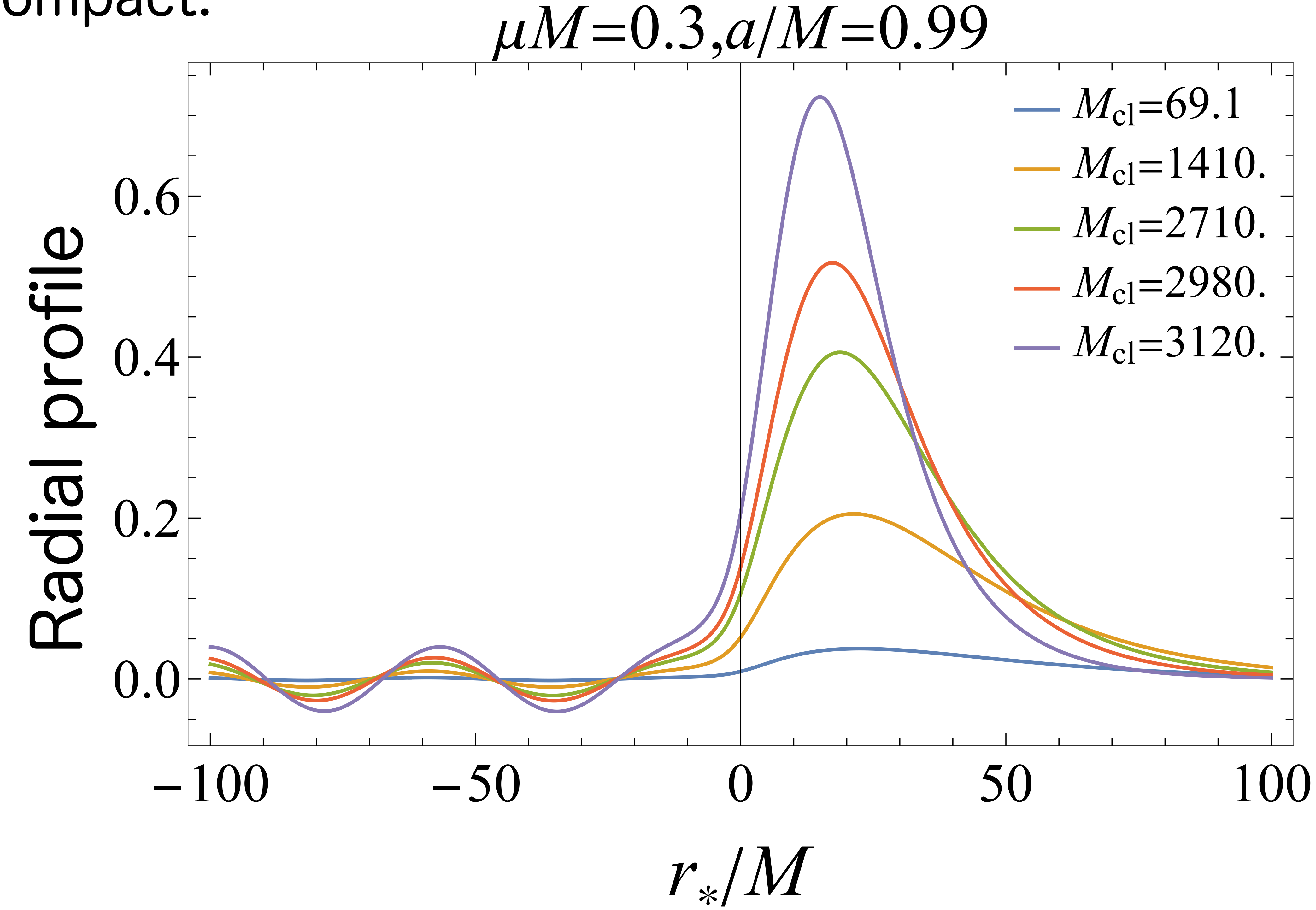
Real and imaginary part of the frequency is also modified.



Deformation of the cloud

(HO+, 2022)

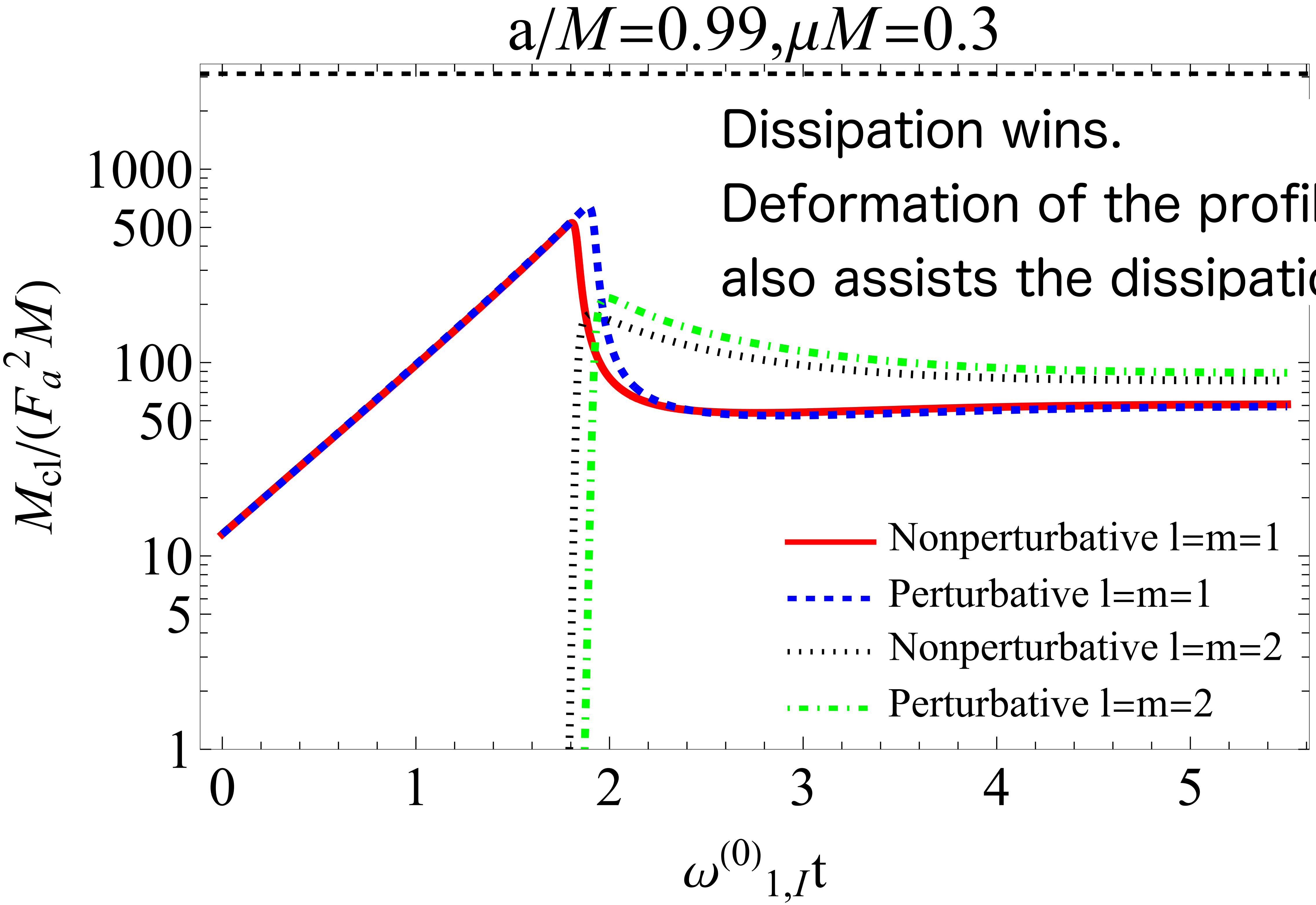
Axions attract each other to make the configuration more compact.



Evolution with self-interaction

(HO+, 2023)

Taking into account both effects……

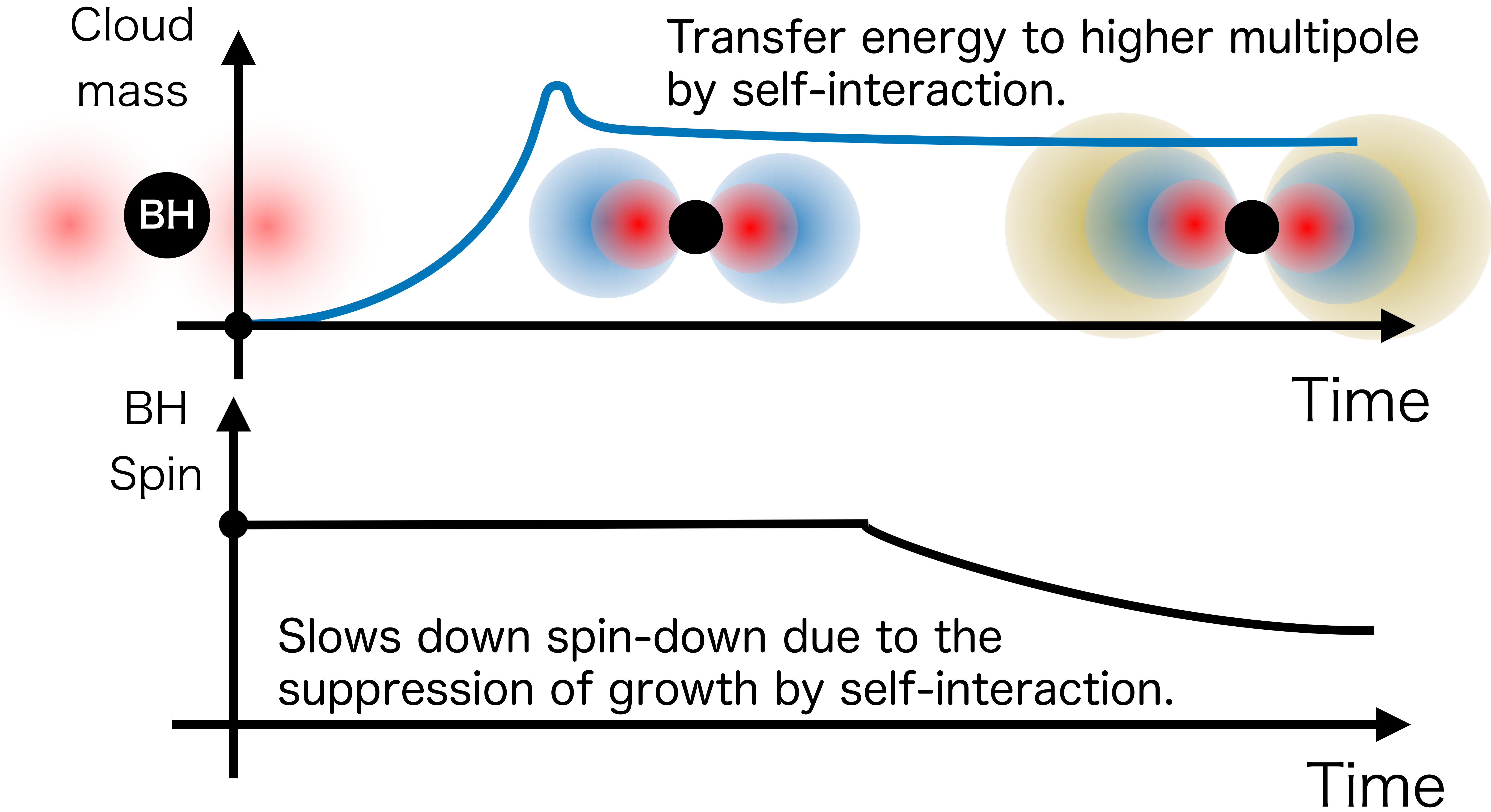


Dissipation wins.

Deformation of the profile also assists the dissipation.

Long term evolution

For observation, long term evolution is needed
→Higher harmonics and spin-down



Transfer energy to higher multipole by self-interaction.

Slows down spin-down due to the suppression of growth by self-interaction.

Evolution equations

(HO+, 2024)

Include up to $l = m = 4$, and evolve adiabatically.

$$\frac{dM_1}{dt} = 2\omega_{1,I} M_1 - 2F_0 M_1^2 M_2 + F_3 M_1 M_2^2 + \cdots \ ,$$

$$\frac{dM_2}{dt} = 2\omega_{2,I} M_2 + F_0 M_1 M_2^2 - 2F_3 M_1 M_2^2 + \cdots \ ,$$

$$\frac{dM_3}{dt} = 2\omega_{3,I} M_3 + \cdots \ , \quad \frac{dM_4}{dt} = 2\omega_{4,I} M_4 + \cdots \ ,$$

Evolution of the cloud

$$\frac{dM}{dt} = -F_a^2 \left(2\omega_{1,I} M_1 + \cdots \right) \ ,$$

$$\frac{dJ}{dt} = -F_a^2 \left(2\omega_{1,I} \frac{m_1 M_1}{\omega_{1,R}} + \cdots \right) \ ,$$

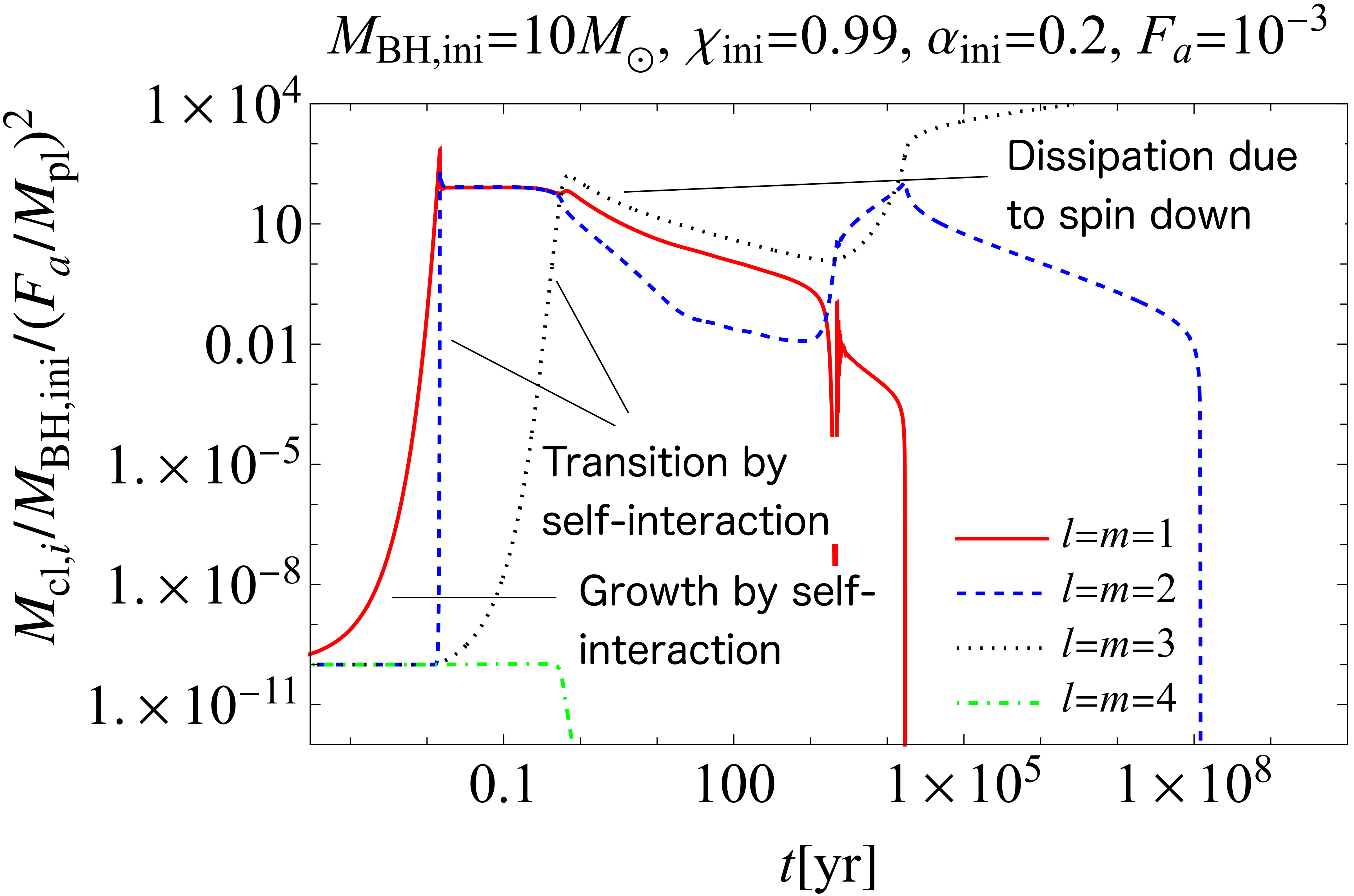
Evolution of the black hole

$M_{1,2,3,4}$: Mass of $l = m = 1,2,3,4$ cloud

M, J: Mass and Angular momentum of the black hole

Example of time evolution

(HO+, 2024)



$M_{\mathrm{BH,ini}}=10M_{\odot}$, $\chi_{\mathrm{ini}}=0.99$, $\alpha_{\mathrm{ini}}=0.2$, $F_a=10^{-3}$

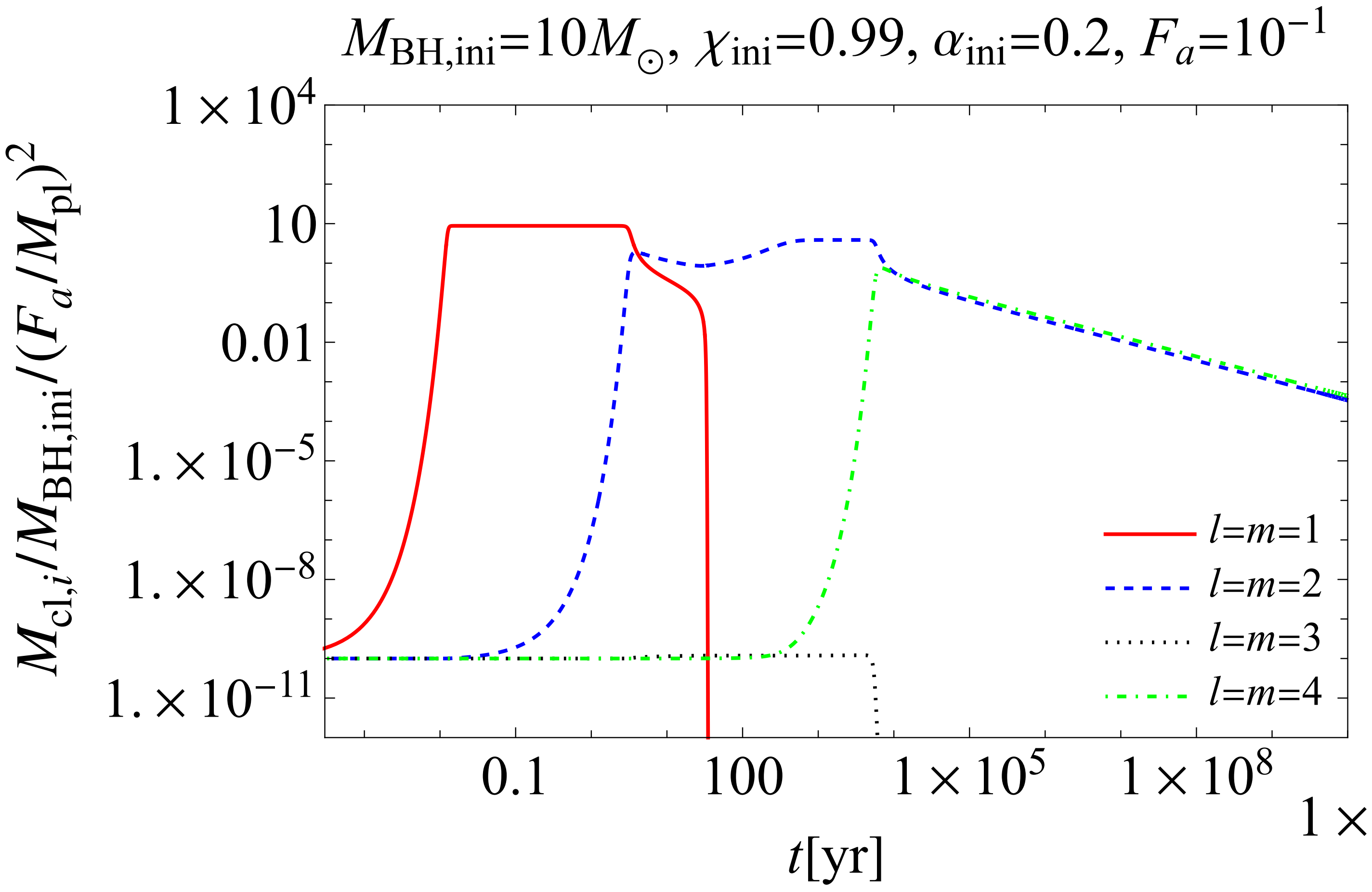
Dissipation due to spin down

Transition by self-interaction

Growth by self-interaction

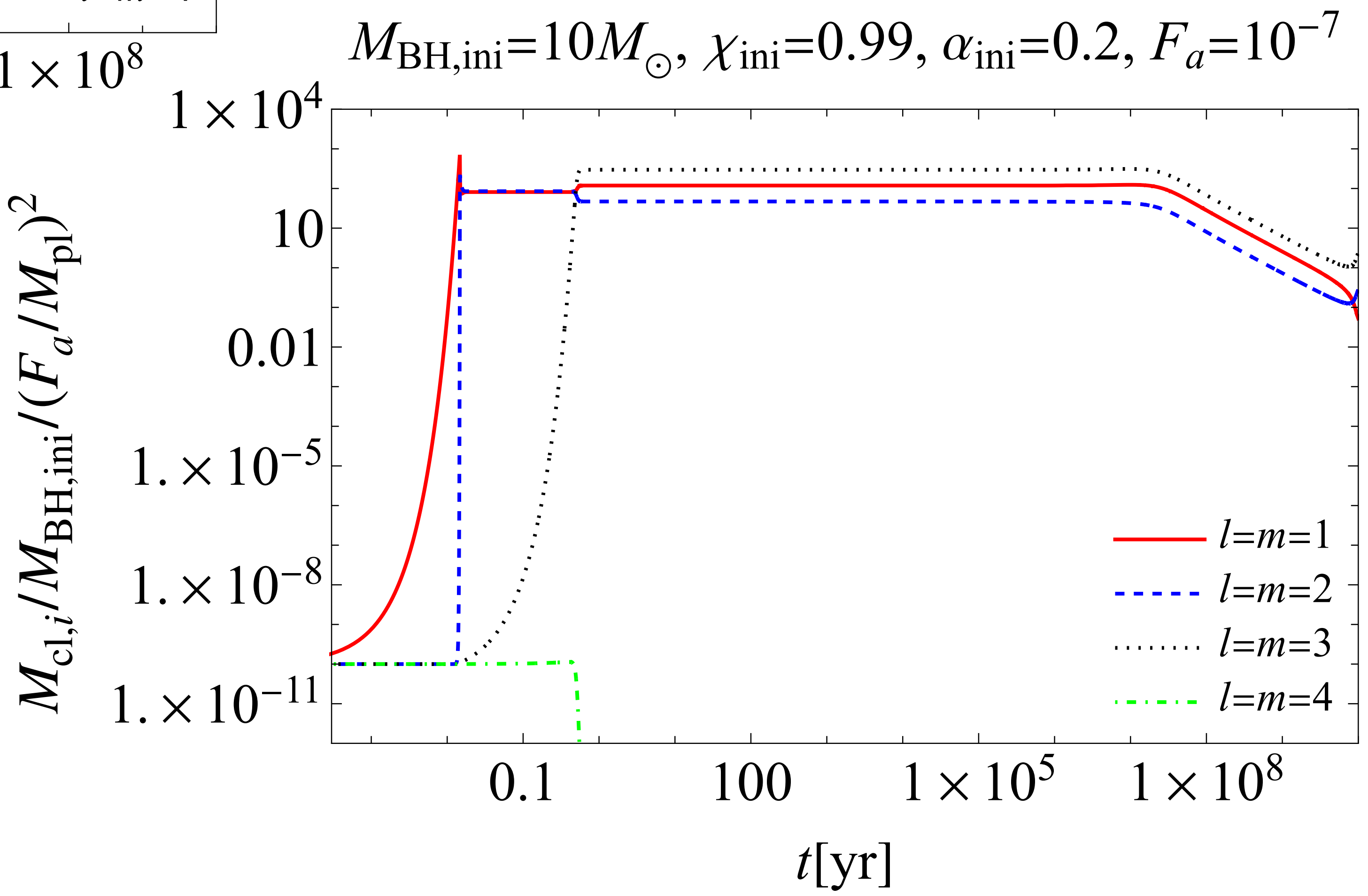
Change decay constant

(HO+, 2024)



Speed up of the spin-down.

Growth saturate before the self-interaction works efficiently.

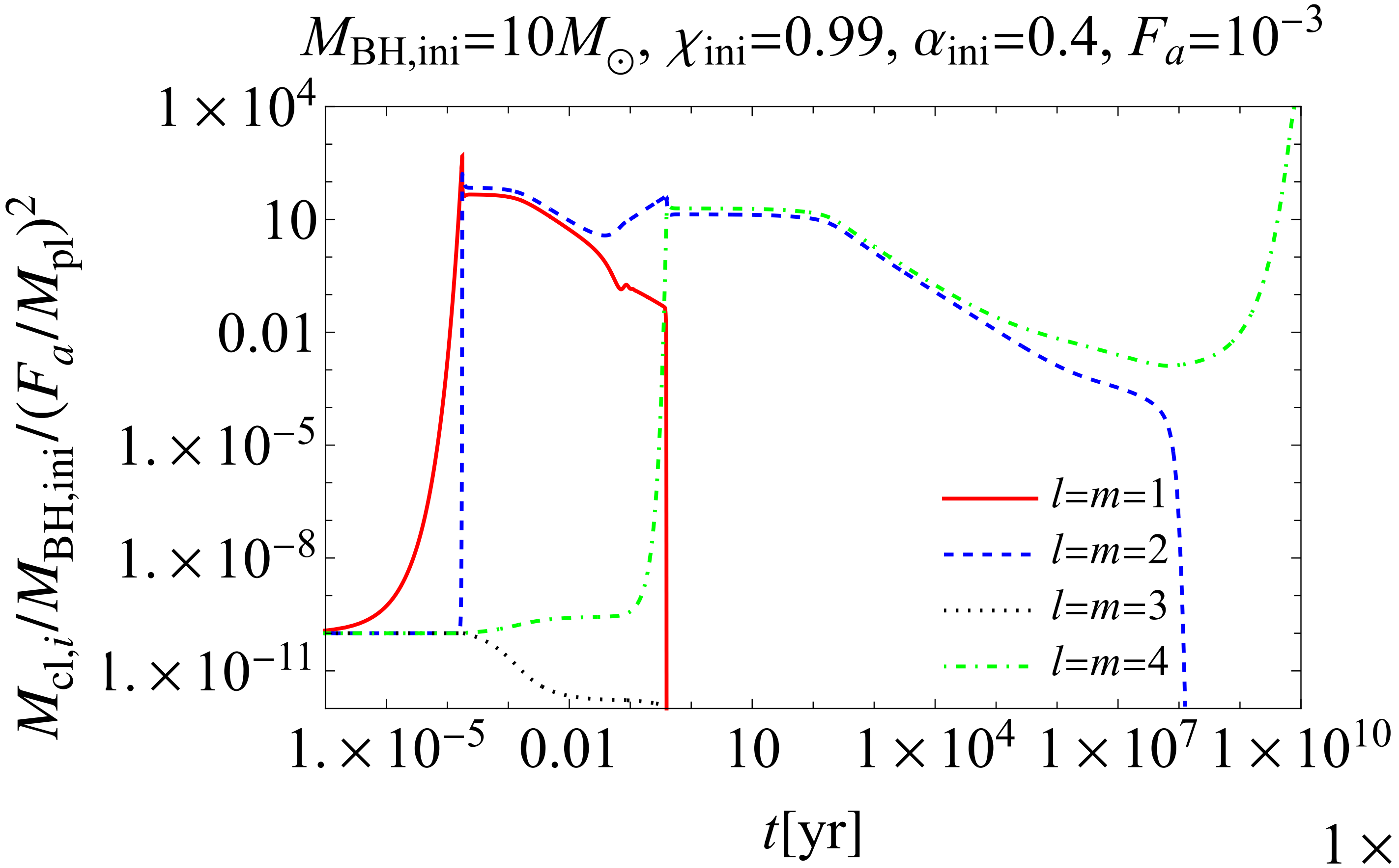


Suppression of the spin-down.

Cloud remains for long time.

Change axion mass

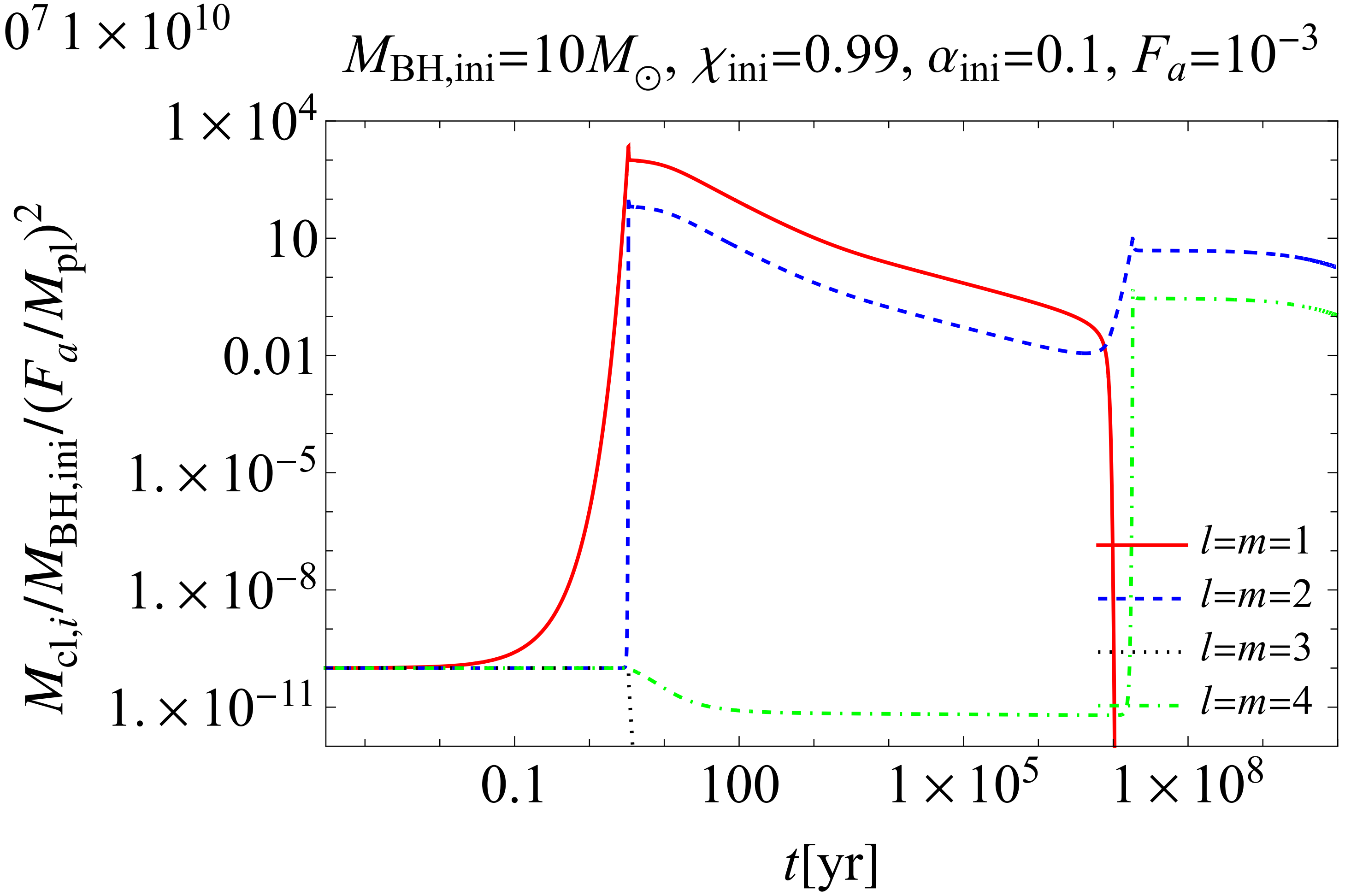
(HO+, 2024)



$M_{\rm BH,ini}=10M_\odot$, $\chi_{\rm ini}=0.99$, $\alpha_{\rm ini}=0.4$, $F_a=10^{-3}$

Speed up of the spin-down.

$l = m = 1$ mode decays before the saturation.

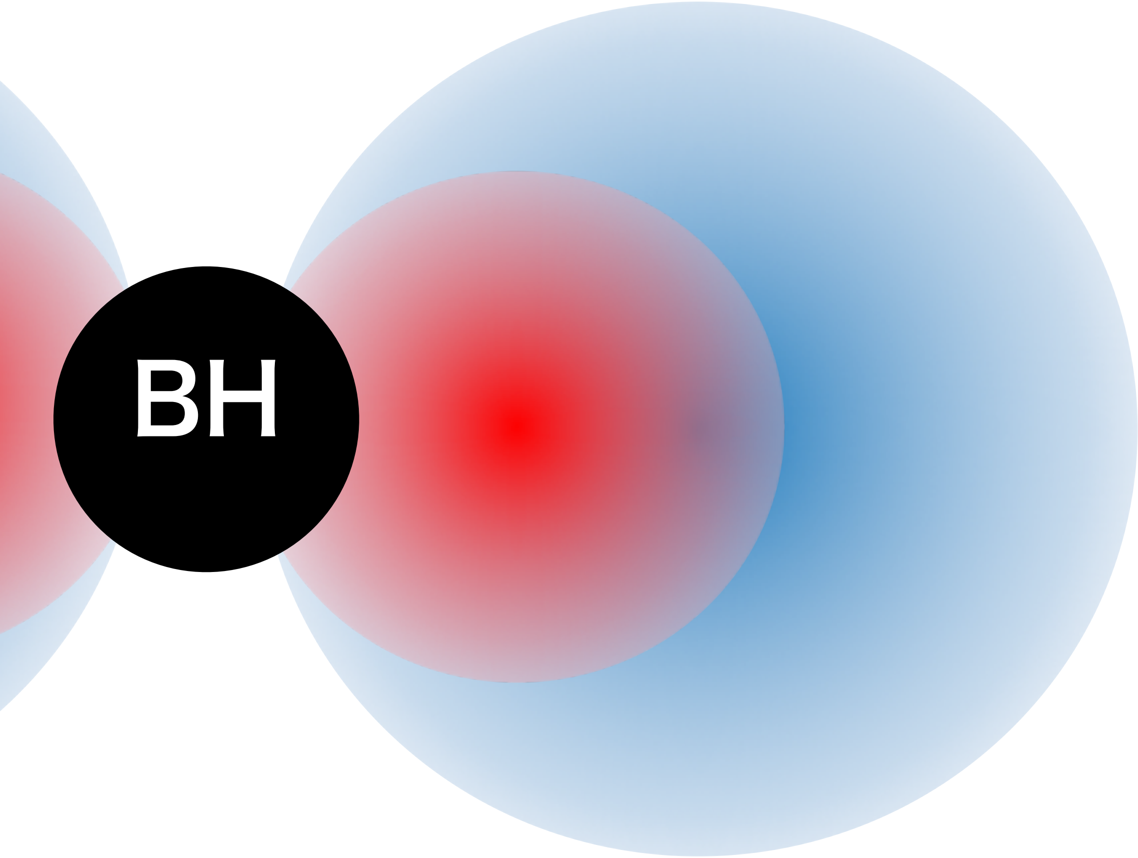


$M_{\rm BH,ini}=10M_\odot$, $\chi_{\rm ini}=0.99$, $\alpha_{\rm ini}=0.1$, $F_a=10^{-3}$

Higher multipole modes do not excite for the small axion mass.

Gravitational waves

How about gravitational wave signals？



$$\phi \sim e^{-i(\omega_1 t - m_1 \varphi)} \psi_1 + \mathrm{c.c.}$$

$$+ e^{-i(\omega_2 t - m_2 \varphi)} \psi_2 + \mathrm{c.c.}$$

$$T_{\mu\nu} \sim \partial_\mu \phi \partial_\nu \phi$$

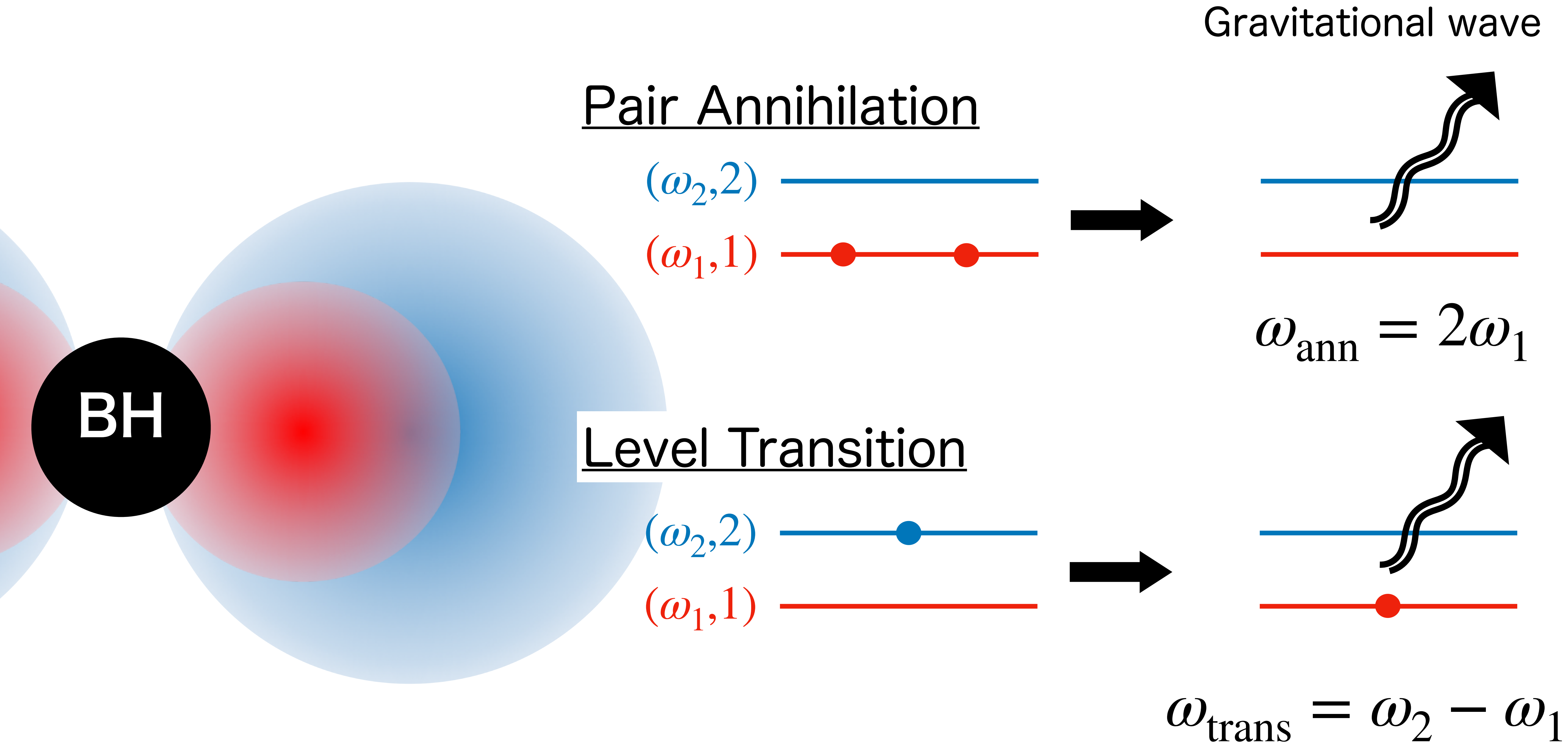
Time dependent part of energy-momentum tensor

$$e^{-i(2\omega_i t - 2m_i \varphi)} \; ,$$

$$e^{-i((\omega_2 - \omega_1)t - (m_2 - m_1)\varphi)} \; ,$$

Oscillates with sum and difference of the frequencies

Gravitational waves



Pair Annihilation

$$\omega_{\rm ann} = 2\omega_1$$

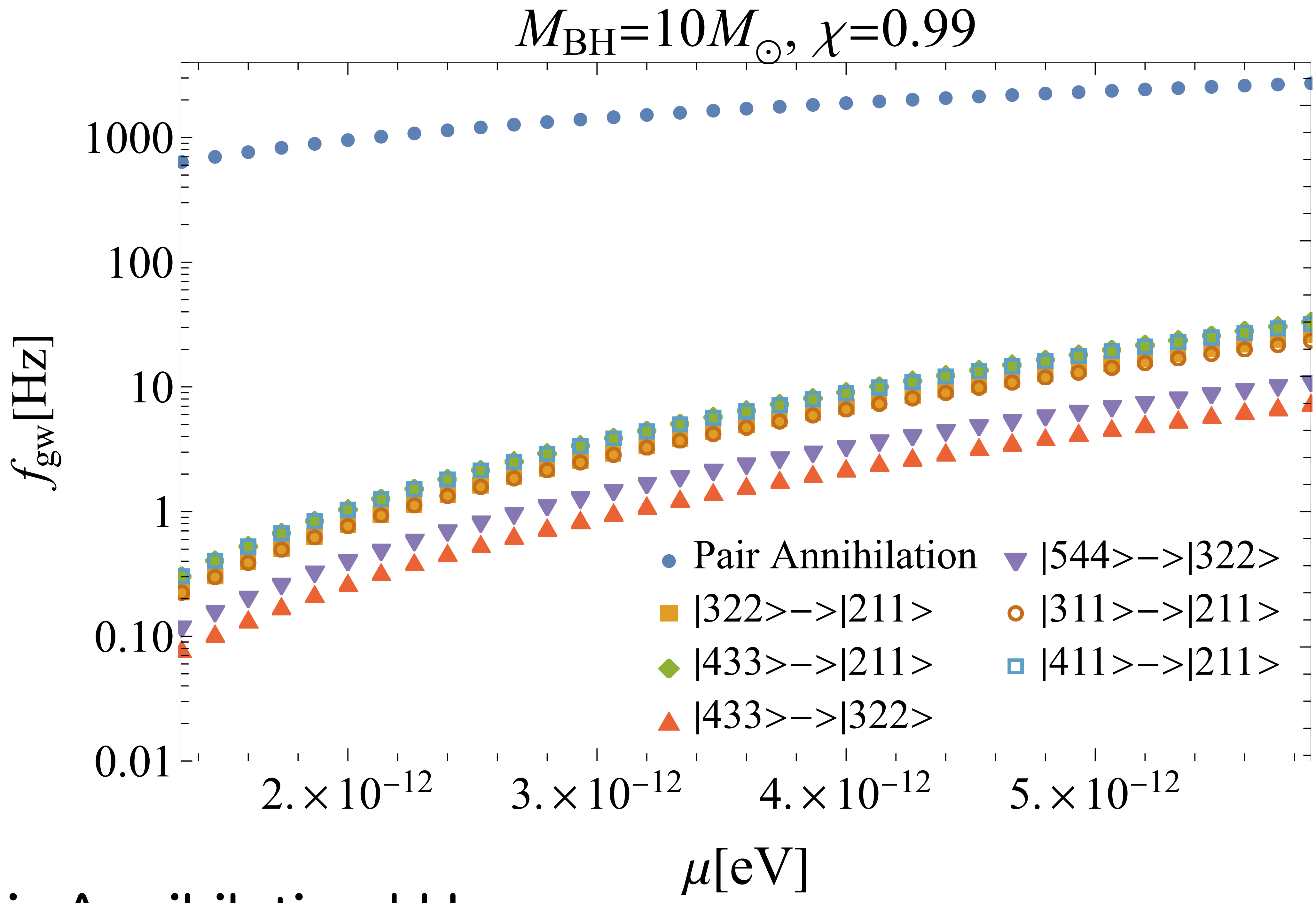
Level Transition

$$\omega_{\rm trans} = \omega_2 - \omega_1$$

Co-existence of many modes

→Rich level transition signal

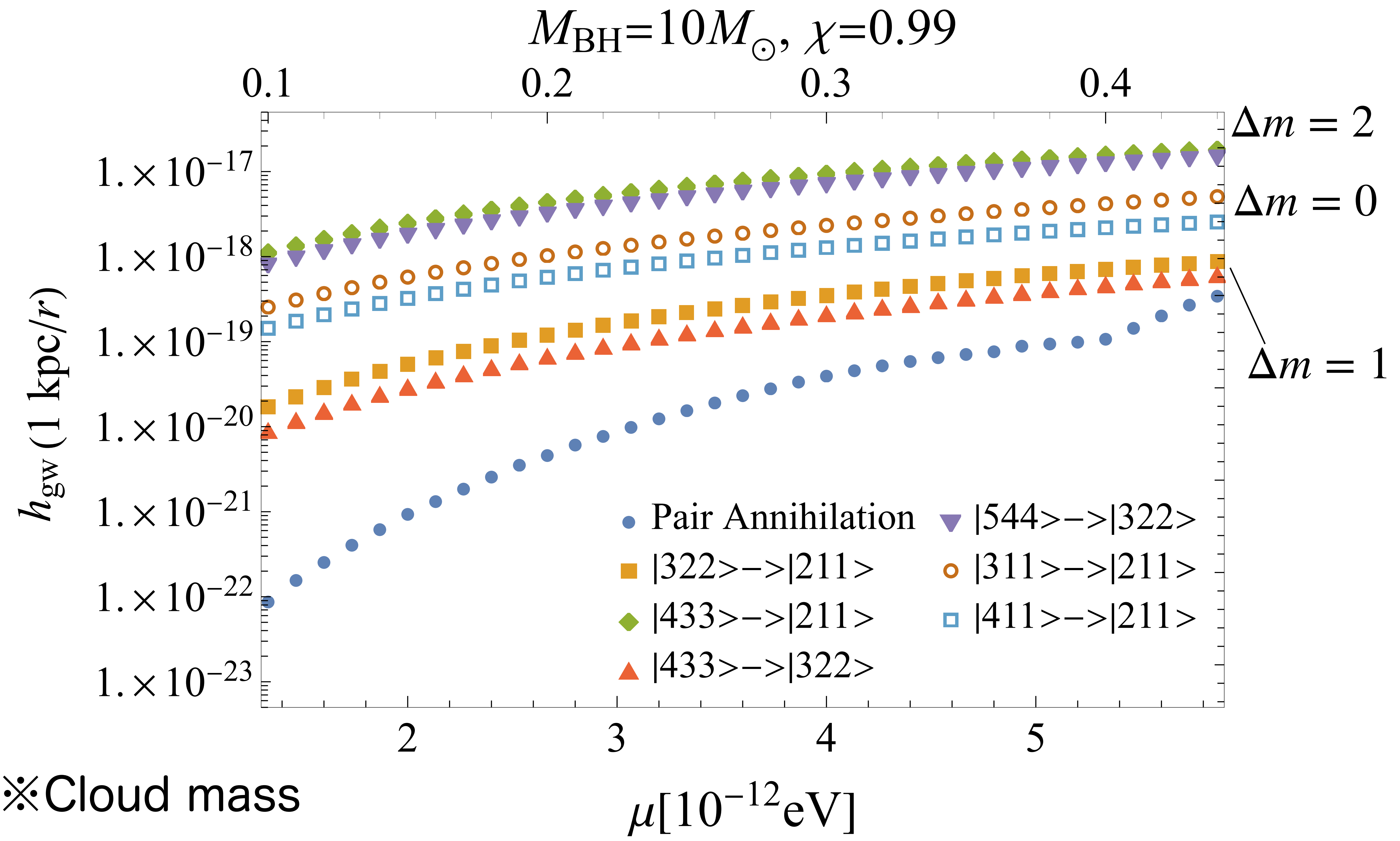
Gravitational wave frequency



Pair Annihilation:kHz

Level Transition:Hz

Gravitational wave amplitude

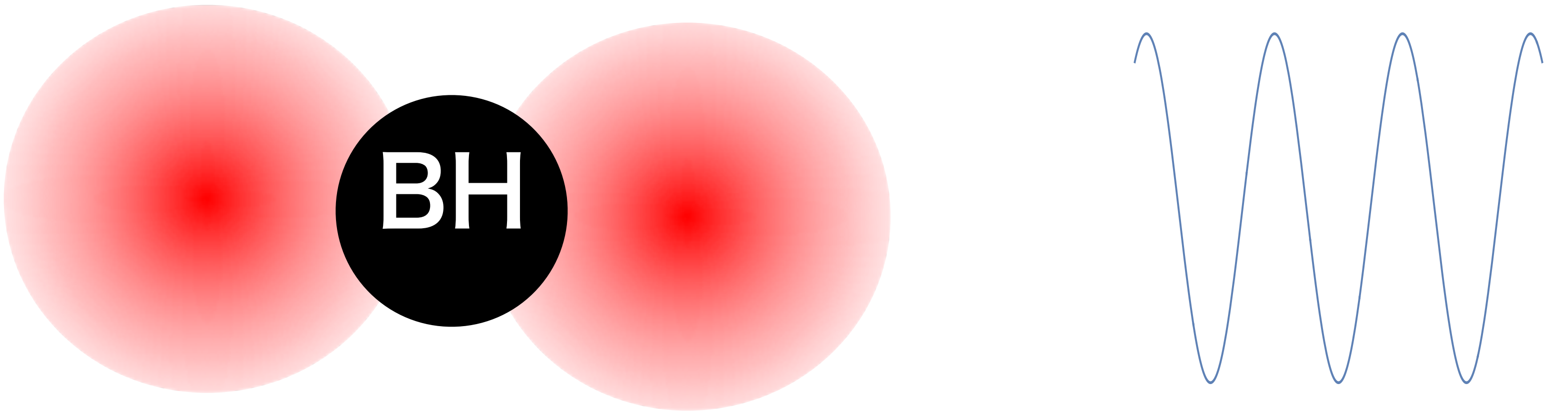


※Cloud mass

=Black hole mass

Gravitational wave amplitude

Suppression of the Pair annihilation signal



$$\lambda_{\mathrm{cloud}} \sim 1/\mu/(G\mu M_{\mathrm{BH}}) \quad \gg \quad \lambda_{\mathrm{gw}} \sim 1/\mu$$

Difference between the Level transition signal

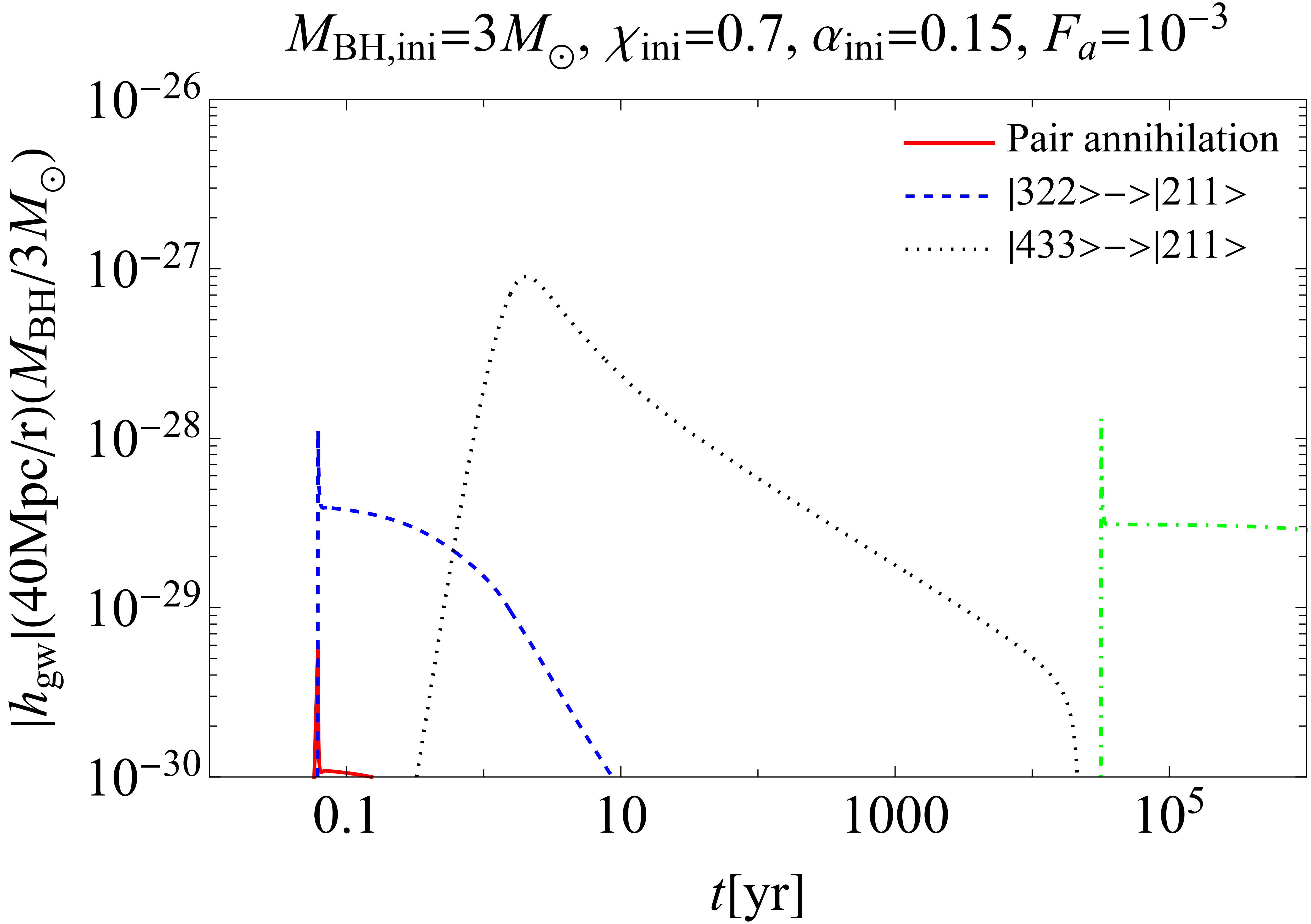
Quadrpole transition or not

$$< 211|Q_{ij}|433 > \neq 0$$

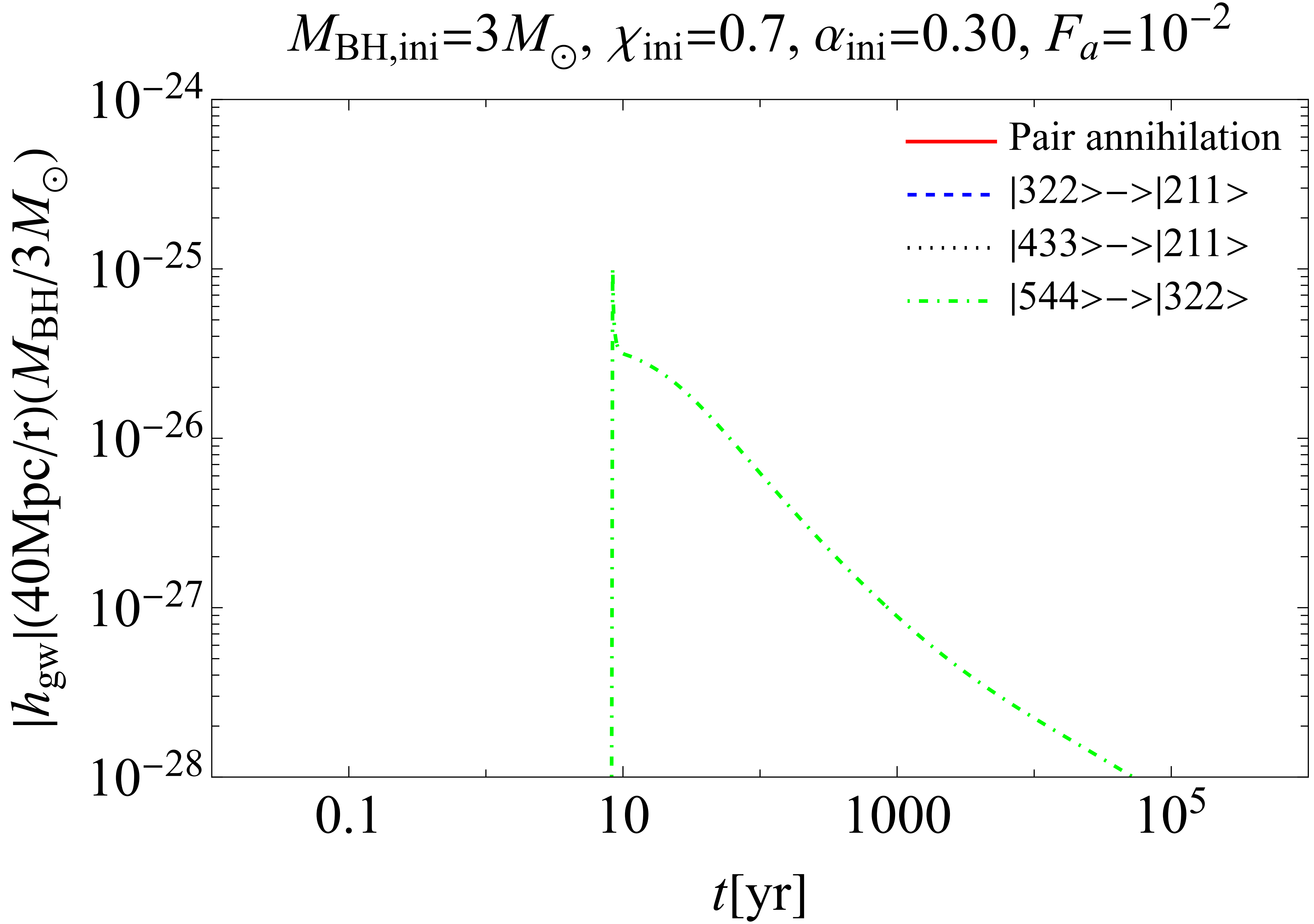
$$< 211|Q_{ij}|322 > = 0$$

Q_{ij}:mass quadrupole

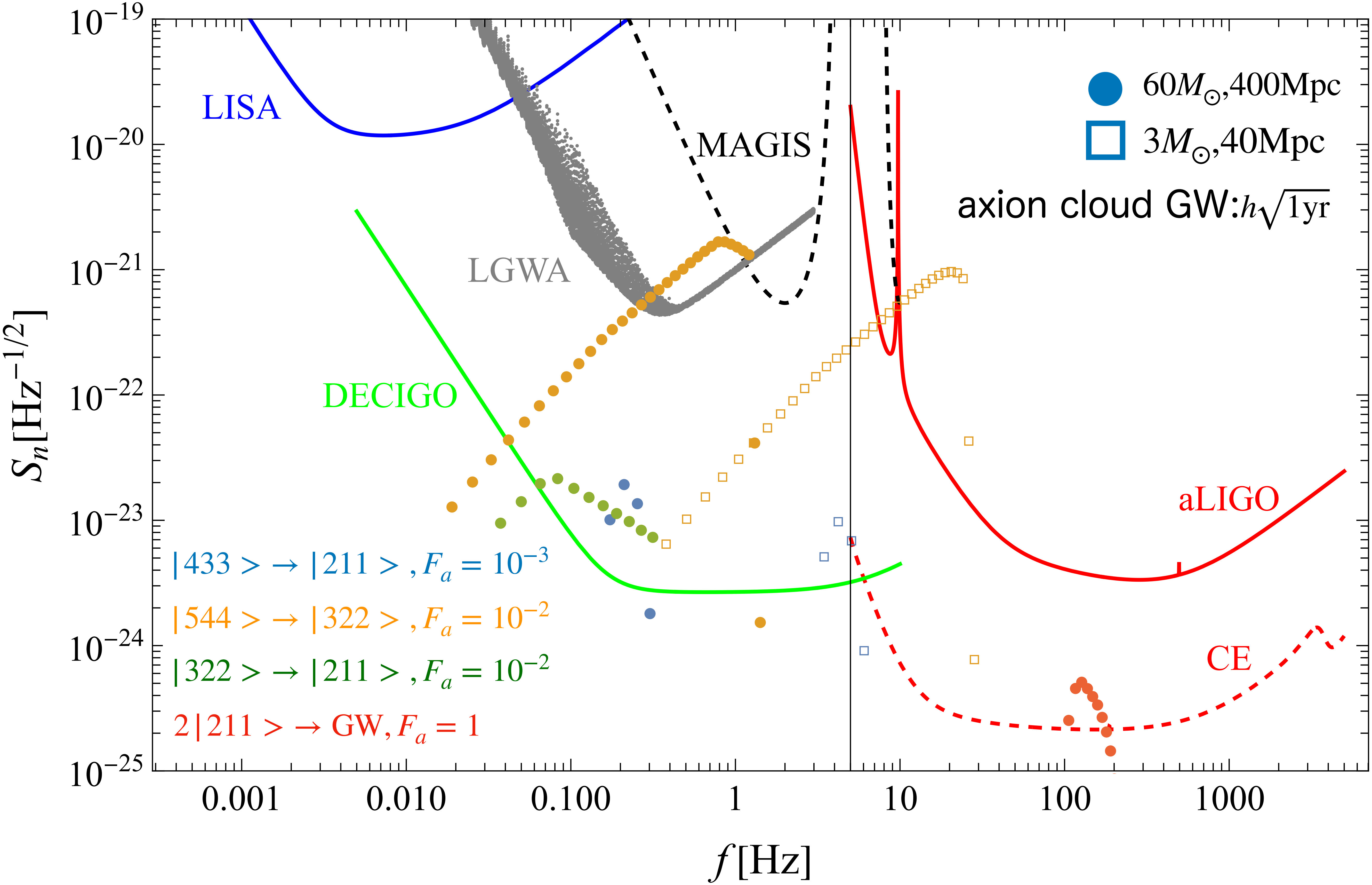
Gravitational wave amplitude



Gravitational wave amplitude



Can we observe?



Summary

- Investigated the evolution of the axion around the black hole, taking into account the relativistic correction, higher multipoles, and spin down.

- Self-interaction excites higher multipole moments, leading to a rich gravitational wave signal.
- Level transition gravitational wave with $\Delta m = 2$ can provide larger signal around GUT scale decay constant.
- For stellar mass black holes, frequency is around Hz.

- Ignored tidal effect from the environment, coupling to the photon,……. Maybe spoiled by these effects.

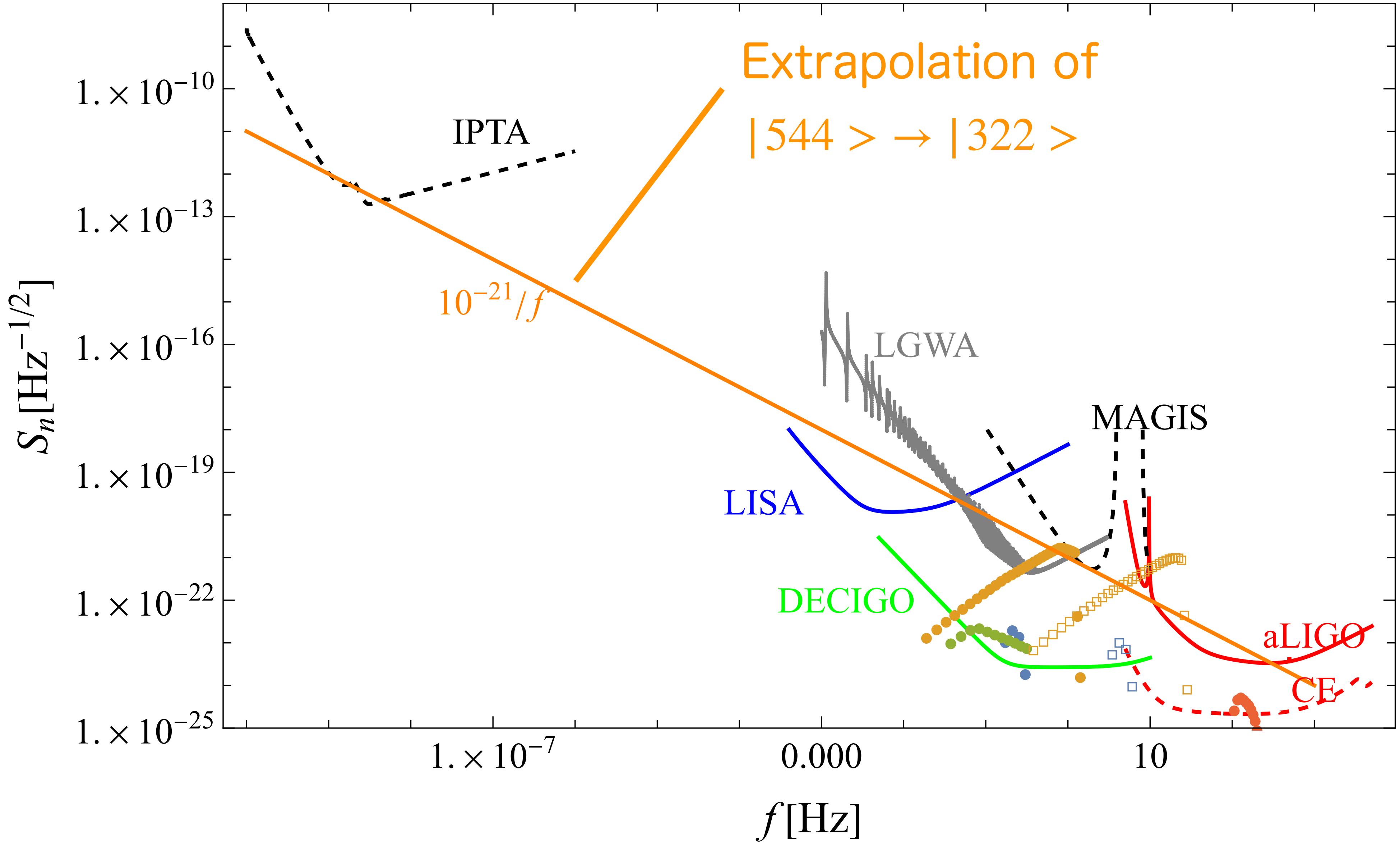
Back up

Lower Frequency?

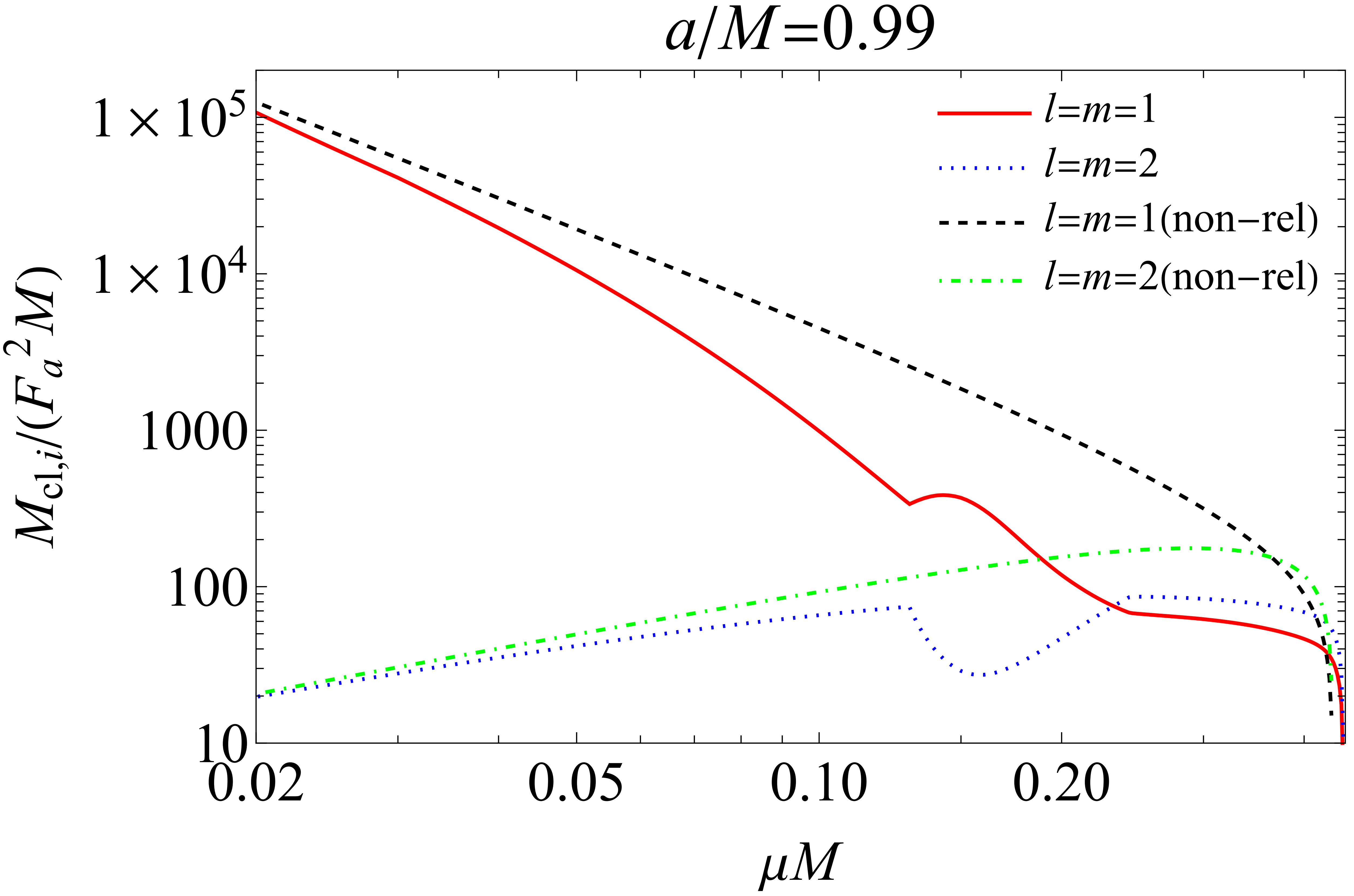
Can we observe with Pulsar Timing Array?

→Seems to be No. Higher multipole modes do not excite within the age of the universe.

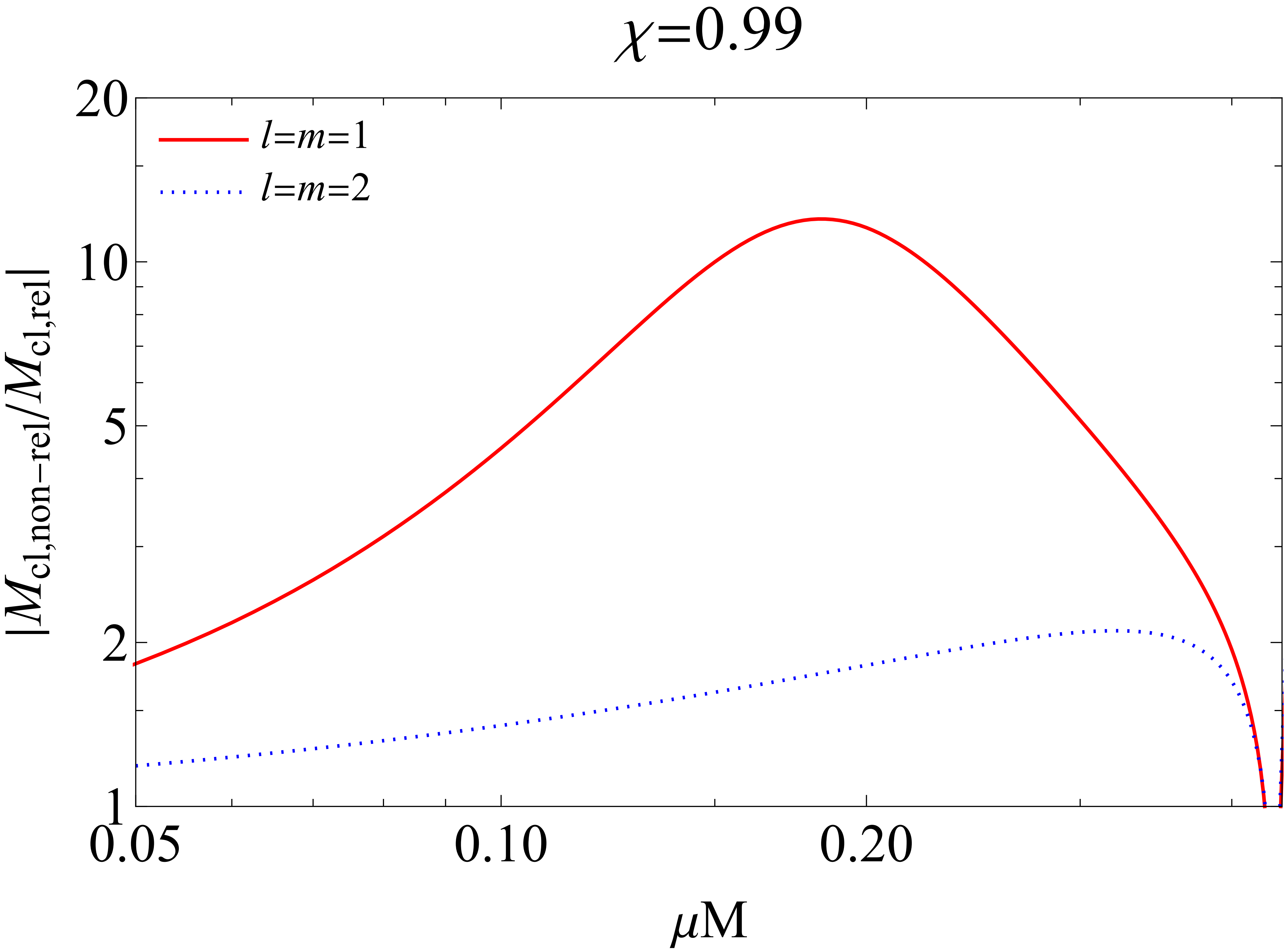
(Frequency$\propto M_{\rm BH}^{-1}$, Excitation time$\propto M_{\rm BH}$)



Non-relativistic VS relativistic



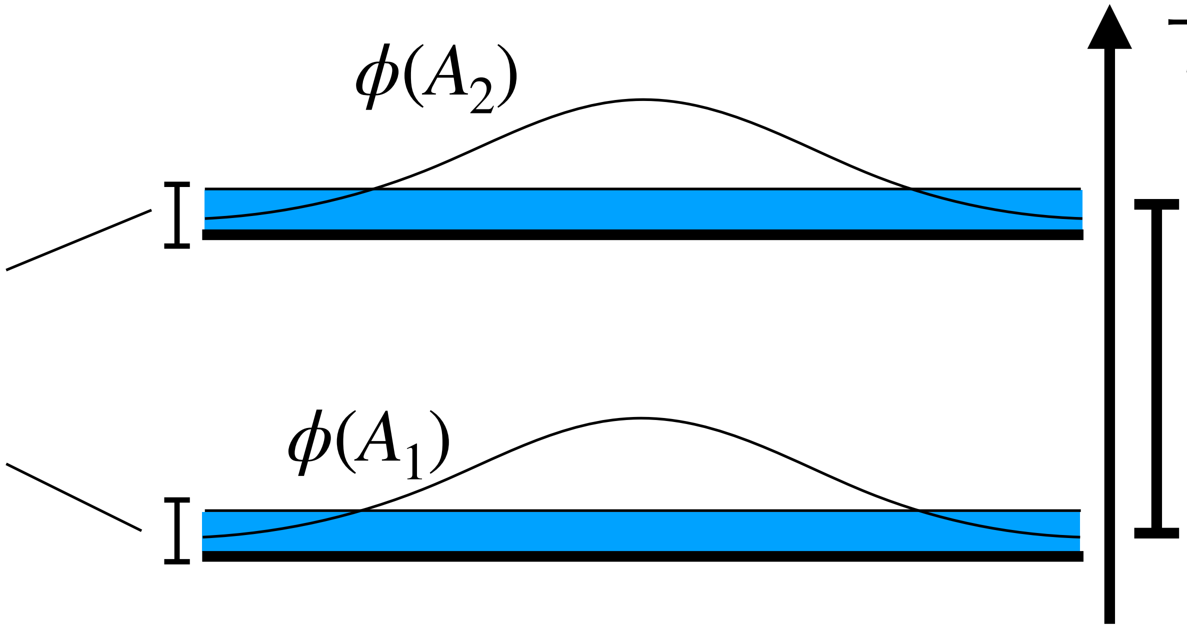
Non-relativistic VS relativistic



Non-linear calculation scheme

(HO et. al.,2022)

- Evolution is adiabatic($\omega_R \gg \omega_I$). For short time($\Delta t \ll \omega_I^{-1}$), configuration is approximately stationary.



Time

Approximately stationary

Growth by instability

Determine consistent long term evolution by energy conservation

$$\frac{\mathrm{d}E(A_0)}{\mathrm{d}A_0}\frac{\mathrm{d}A_0}{\mathrm{d}t} = -F_{tot}(A_0)$$

$E(A_0)$:Energy of$\phi(A_0)$

$F_{tot}(A_0)$:total energy flux

Main Idea

Find one-parameter family of stationary configuration $\{\phi(A_0)\}_{A_0}$, and join them by conservation law.

Non-linear calculation scheme

$$
\begin{aligned}
\phi(A_0) = &\left(\tilde{R}_{11}(r;A_0)Y_{11}(\theta) + \tilde{R}_{31}(r;A_0)Y_{31}(\theta) + \tilde{R}_{51}(r;A_0)Y_{51}(\theta)\right) e^{-i(\omega_0 t-\varphi)} \\
&+\left(\tilde{R}_{33}(r;A_0)Y_{33}(\theta) + \tilde{R}_{53}(r;A_0)Y_{53}(\theta)\right) e^{-3i(\omega_0 t-\varphi)} \\
&+\tilde{R}_{55}(r;A_0)Y_{55}(\theta)e^{-5i(\omega_0 t-\varphi)} + \mathrm{c.c.}
\end{aligned}
$$

- Helical symmetric
- A_0:Amplitude of $\tilde{R}_{11}$ at infinity

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta\frac{\mathrm{d}\tilde{R}_{nl}}{\mathrm{d}r}\right) + \Bigg[\frac{n^2(\omega_0(r^2+a^2)-am_0)^2}{\Delta} - \mu^2 r^2 + 2an^2\omega_0 m_0 - a^2n^2\omega_0^2 - l(l+1) \\
&\qquad +a^2(n^2\omega_0^2-\mu^2)\frac{1-2l(l+1)+2n^2m_0^2}{3-4l(l+1)}\Bigg]\tilde{R}_{nl} \\
&\qquad + a^2(n^2\omega_0^2-\mu^2)\left(\frac{(l-1-nm_0)(l-nm_0)}{(2l-3)(2l-1)}\frac{N^{nm_0}_{l-2}}{N^{nm_0}_{l}}\tilde{R}_{nl-2}\right. \\
&\qquad \left. + \frac{(l+2+nm_0)(l+1+nm_0)}{(2l+3)(2l+5)}\frac{N^{nm_0}_{l+2}}{N^{nm_0}_{l}}\tilde{R}_{nl+2}\right) \\
&\qquad + \int_0^{2\pi} d\varphi \int_{-1}^{1} dx\; Y_{lnm_0}(x)e^{-inm_0\varphi}(r^2+a^2x^2)V'(\phi) = 0\ ,
\end{aligned}
$$

Axion from string theory

Compactify ten dimensional space-time

$$S_4 = \frac{M_{pl}^2}{2}\int_{M_4} d^4x\sqrt{-g}\; R - \frac{1}{4g_{YM}^2}\int_{M_4} d^4x\sqrt{-g}\;\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{4\kappa_{10}^2}\int_{M_4\times Z}\tilde{H}_3\wedge *\tilde{H}_3$$

Constraint from supersymmetry

$$d\tilde{H}_3 = \frac{\kappa_{10}^2}{g_{10}^2}\left(\mathrm{tr}\left(R\wedge R\right) - \frac{1}{30}\mathrm{tr}\left(F\wedge F\right)\right)$$

“Lagrange multiplier” a (This becomes axion)

$$-\frac{V_Z}{4\kappa_{10}^2}\int\tilde{H}_3\wedge *\tilde{H}_3 + \int a\left[d\tilde{H}_3 - \frac{\kappa_{10}^2}{g_{10}^2}\left(\mathrm{tr}\left(R\wedge R\right) - \mathrm{tr}\left(F\wedge F\right)\right)\right]$$

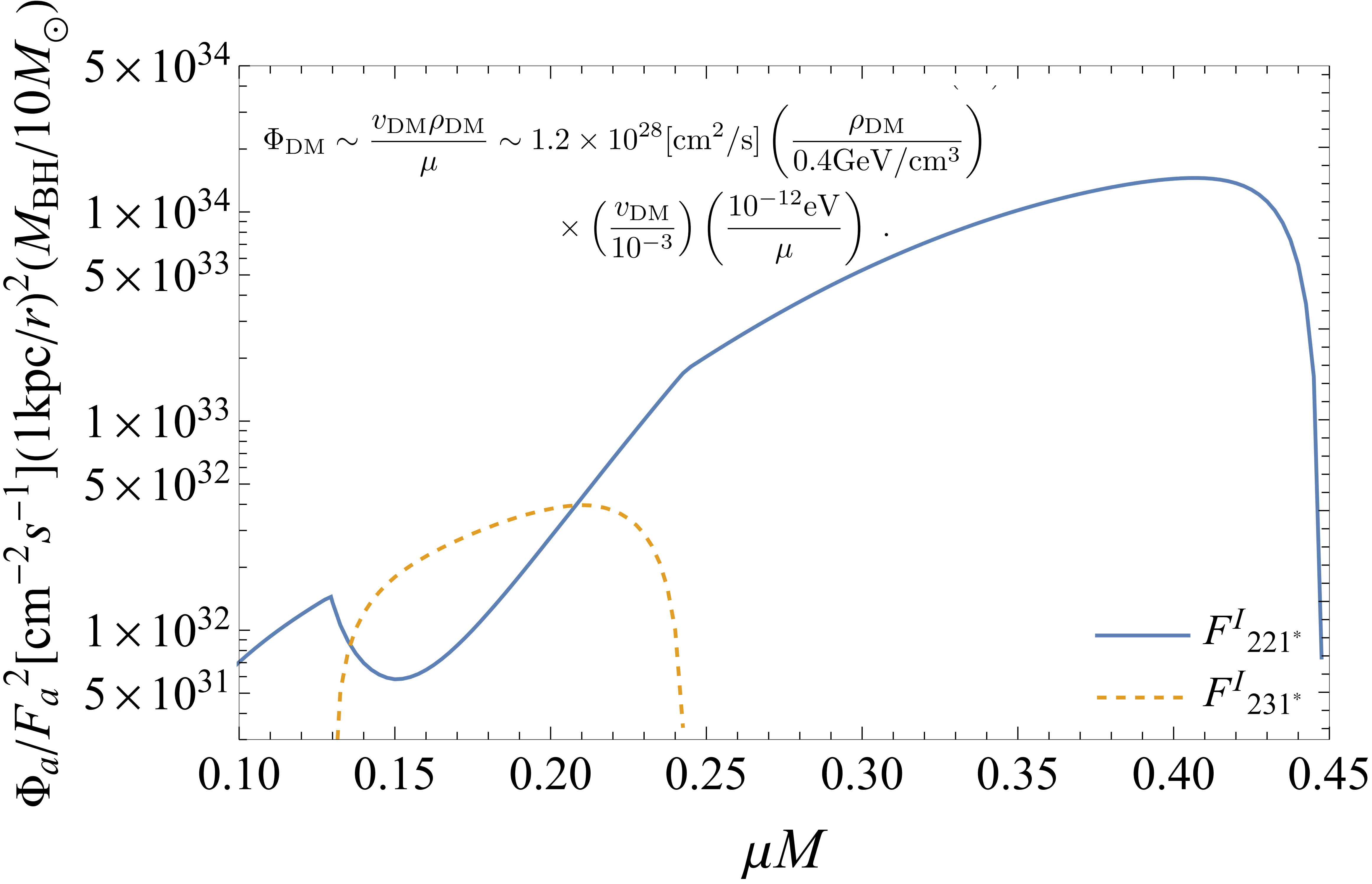
Integrate out H_3 and canonically normalize a

$$\int\sqrt{-g}\left[-\frac{1}{2}(\nabla_\mu\tilde{a})^2 + \frac{1}{64\pi^2}\frac{16\pi^2\kappa_{10}\sqrt{V_Z}}{\sqrt{2}g_{10}^2}\tilde{a}\left(\epsilon^{\rho\sigma\kappa\lambda}R_{\mu\nu\rho\sigma}R^{\mu\nu}_{\kappa\lambda} - \epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right)\right]$$

Axion wave emission

(HO+, 2024)

$M_{\rm BH}$=10$M_\odot$, χ=0.99

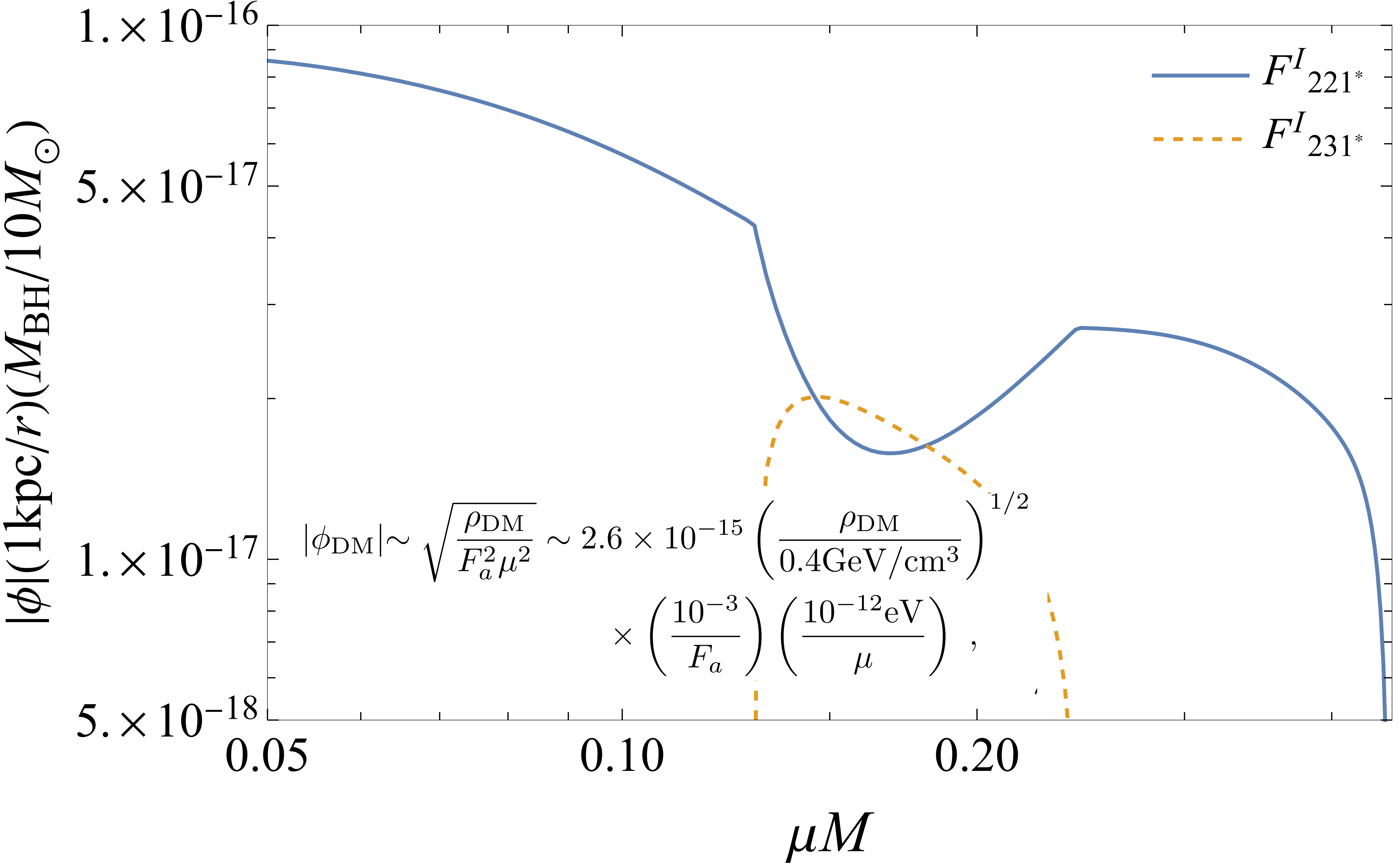


$$\Phi_{\rm DM} \sim \frac{v_{\rm DM}\rho_{\rm DM}}{\mu} \sim 1.2\times 10^{28}[\mathrm{cm}^2/\mathrm{s}]\left(\frac{\rho_{\rm DM}}{0.4\mathrm{GeV}/\mathrm{cm}^3}\right) \times \left(\frac{v_{\rm DM}}{10^{-3}}\right)\left(\frac{10^{-12}\mathrm{eV}}{\mu}\right).$$

Axion wave emission

$M_{\rm BH}=10M_\odot,\ \chi=0.99$

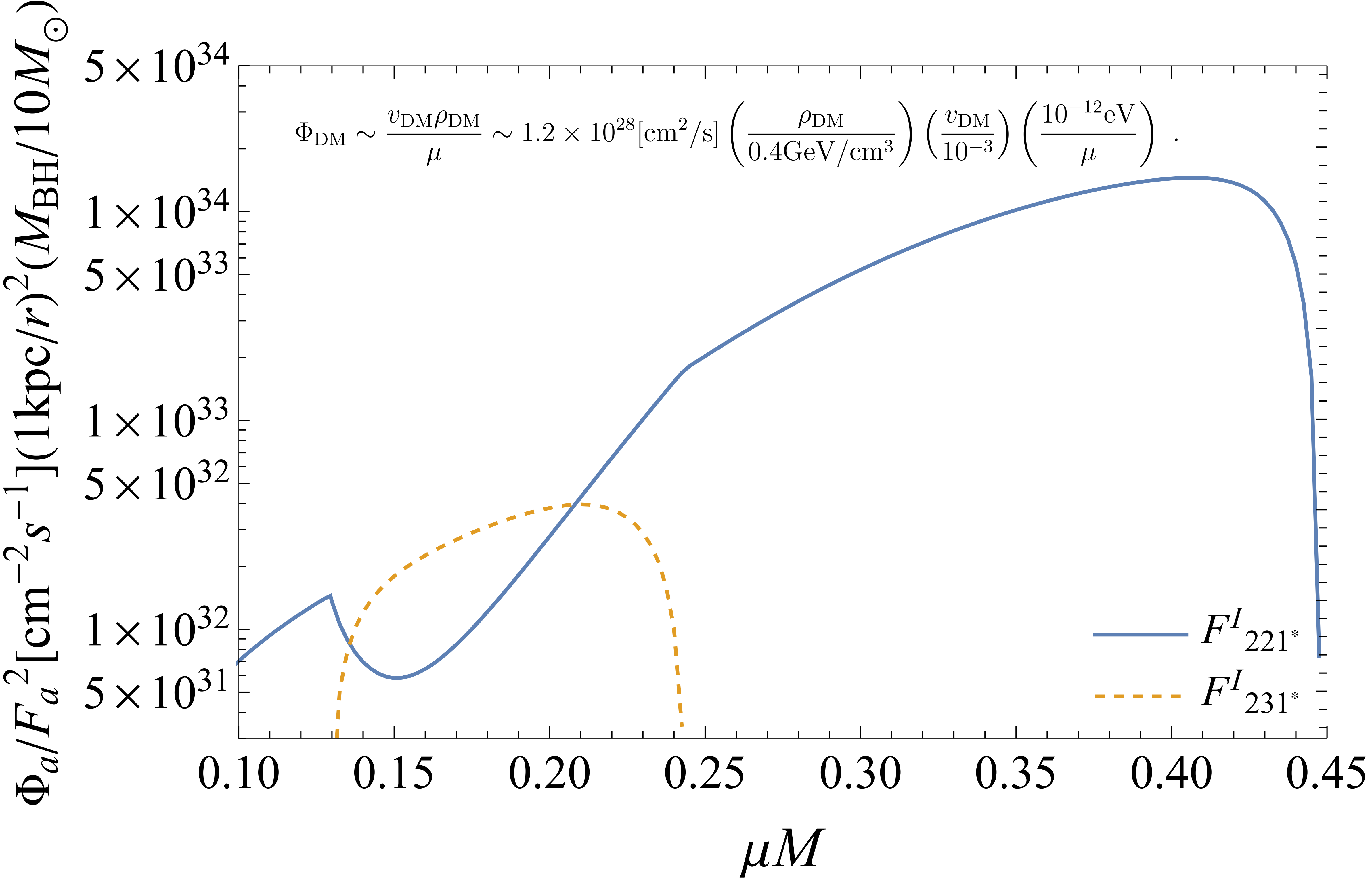
(HO+, 2024)



$$|\phi_{\rm DM}| \sim \sqrt{\frac{\rho_{\rm DM}}{F_a^2\mu^2}} \sim 2.6\times10^{-15}\left(\frac{\rho_{\rm DM}}{0.4{\rm GeV/cm^3}}\right)^{1/2} \times\left(\frac{10^{-3}}{F_a}\right)\left(\frac{10^{-12}{\rm eV}}{\mu}\right),$$

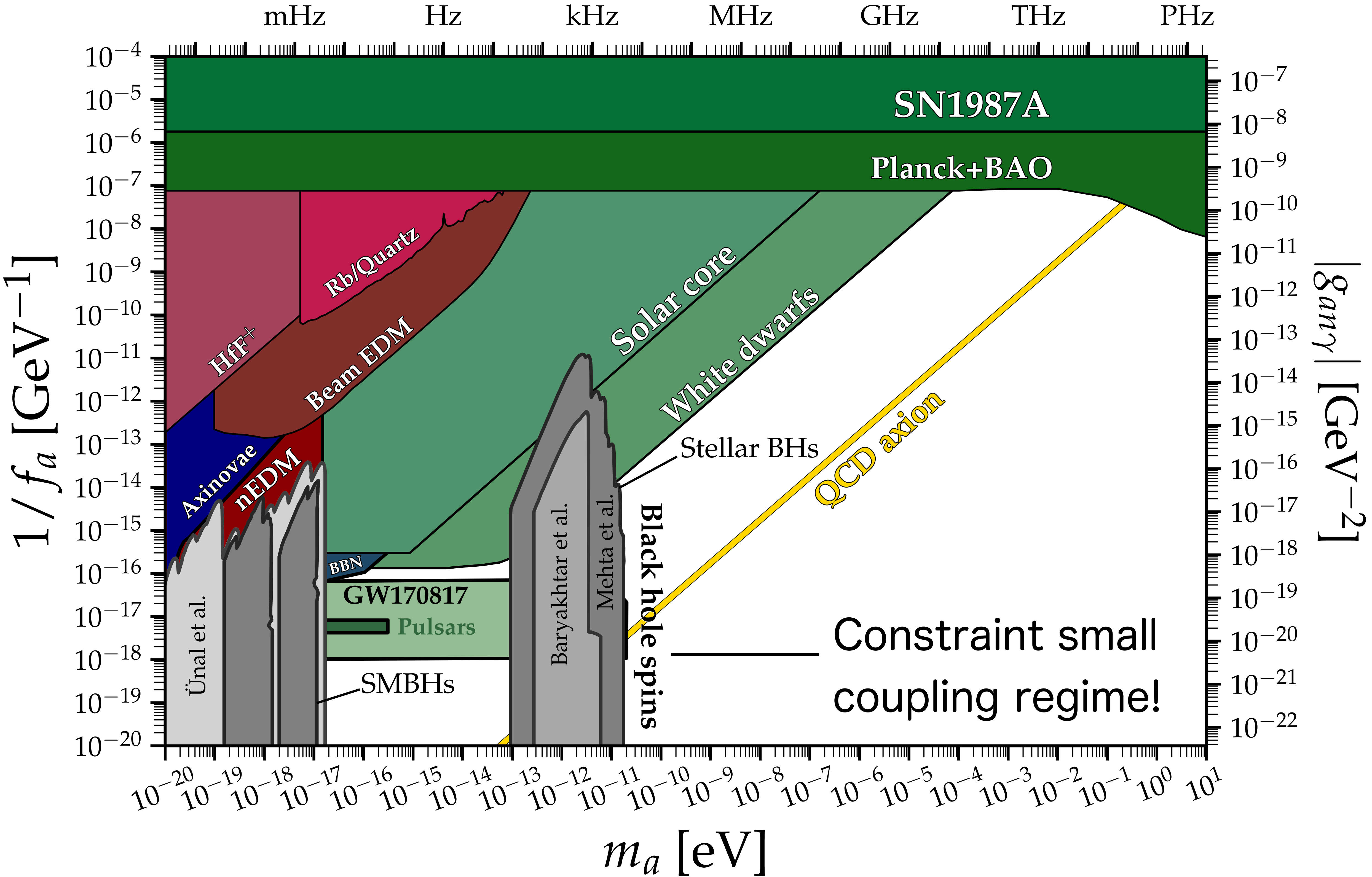
Axion wave

$M_{\mathrm{BH}}=10M_{\odot}$, $\chi=0.99$



$$\Phi_{\mathrm{DM}} \sim \frac{v_{\mathrm{DM}}\rho_{\mathrm{DM}}}{\mu} \sim 1.2\times 10^{28}[\mathrm{cm}^2/\mathrm{s}]\left(\frac{\rho_{\mathrm{DM}}}{0.4\mathrm{GeV/cm}^3}\right)\left(\frac{v_{\mathrm{DM}}}{10^{-3}}\right)\left(\frac{10^{-12}\mathrm{eV}}{\mu}\right) .$$

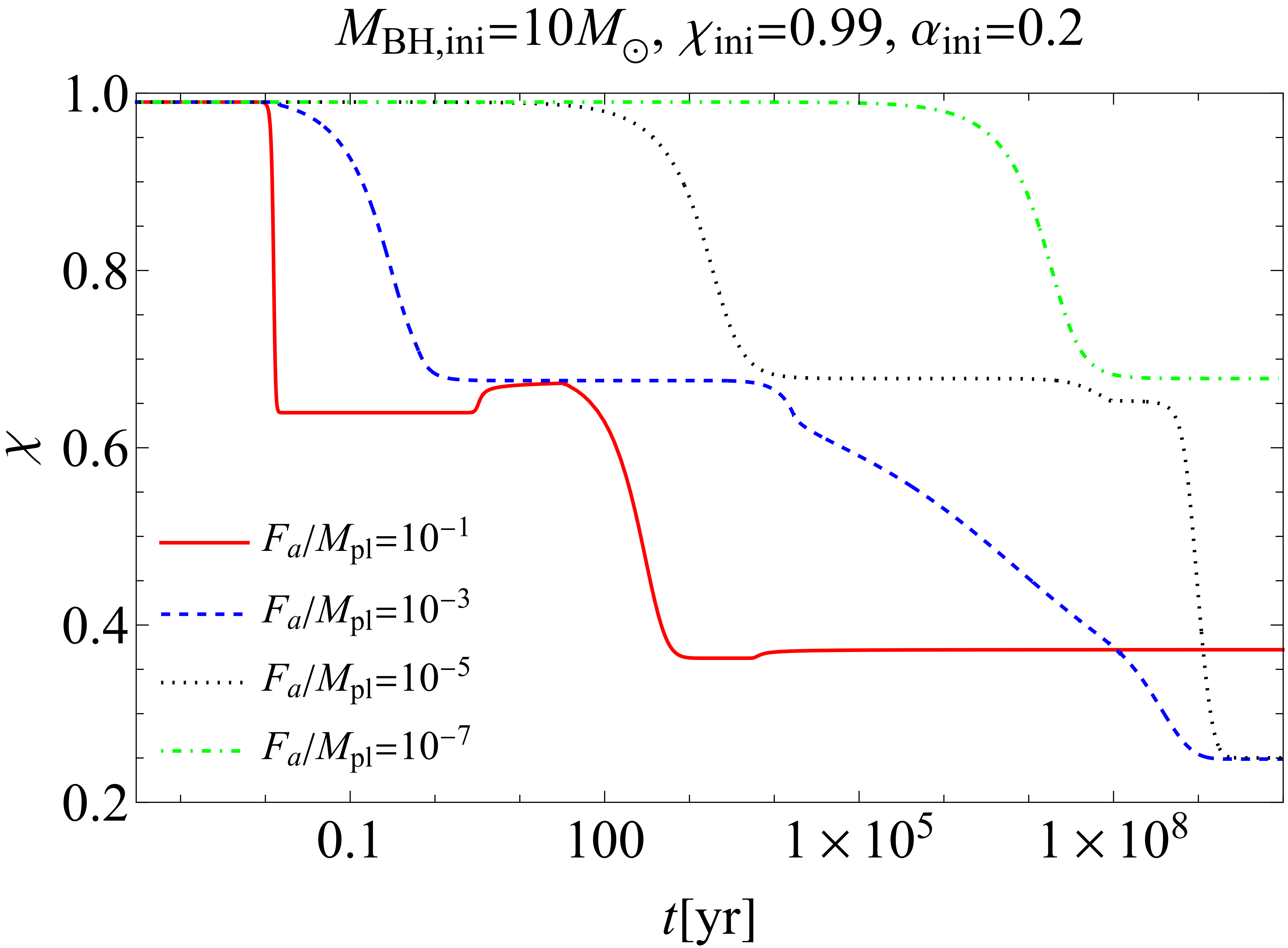
Limit on axion



Constraint small coupling regime!

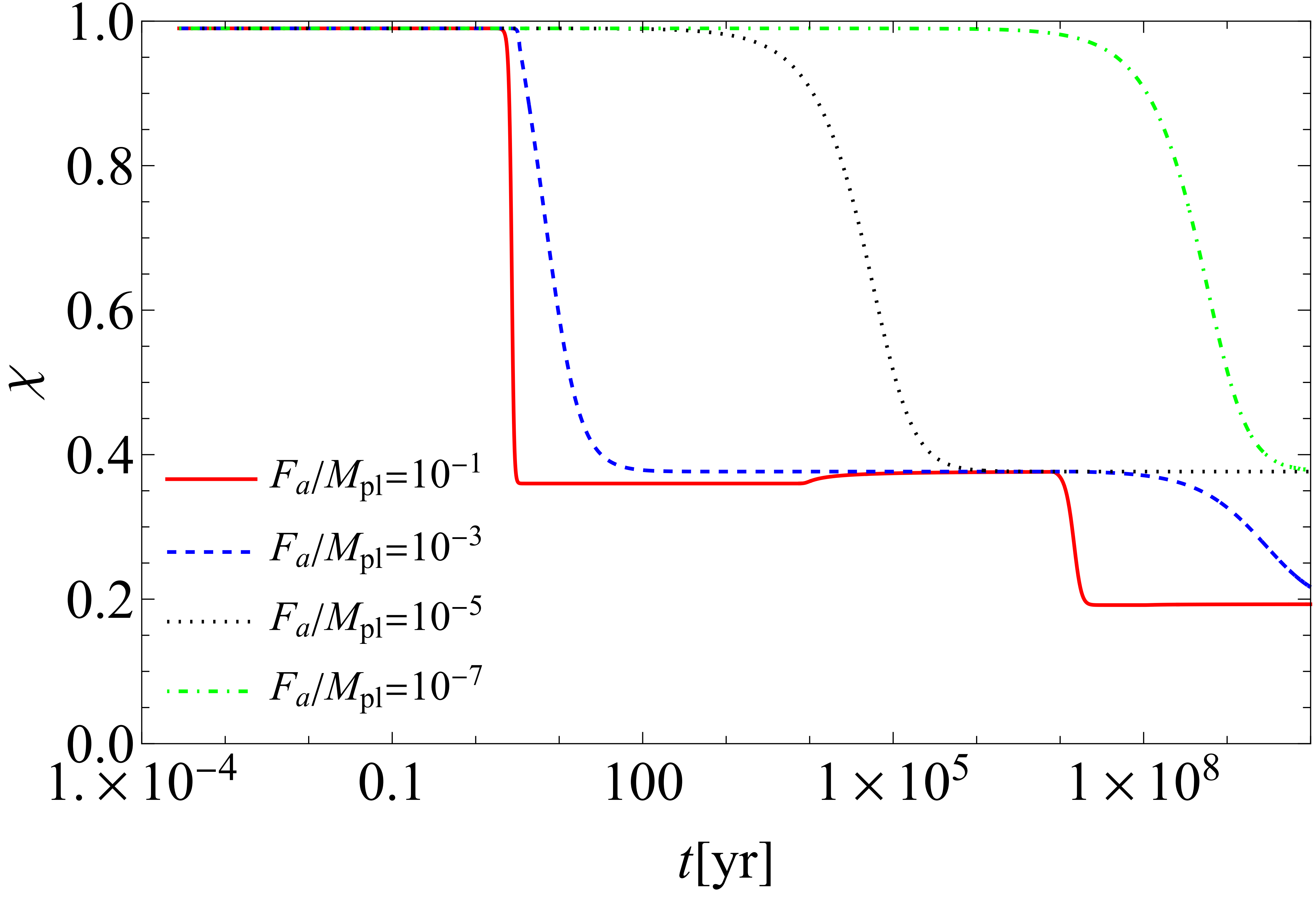
(https://cajohare.github.io/AxionLimits/)

Spin evolution



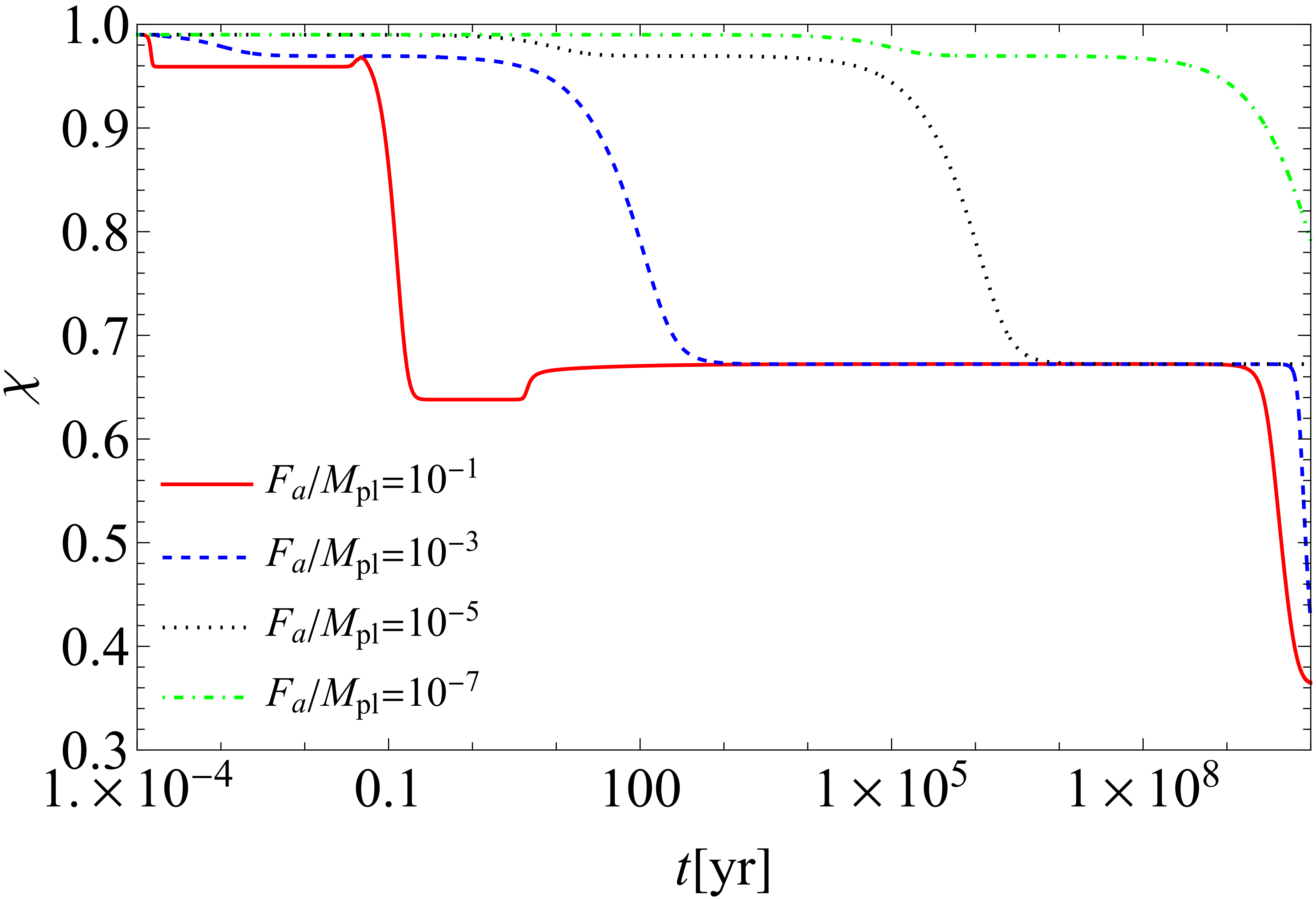
Spin evolution

$M_{\rm BH,ini}=10M_\odot$, $\chi_{\rm ini}=0.99$, $\alpha_{\rm ini}=0.1$



Spin evolution

$M_{\mathrm{BH,ini}}=10M_{\odot}$, $\chi_{\mathrm{ini}}=0.99$, $\alpha_{\mathrm{ini}}=0.4$



Superradiance

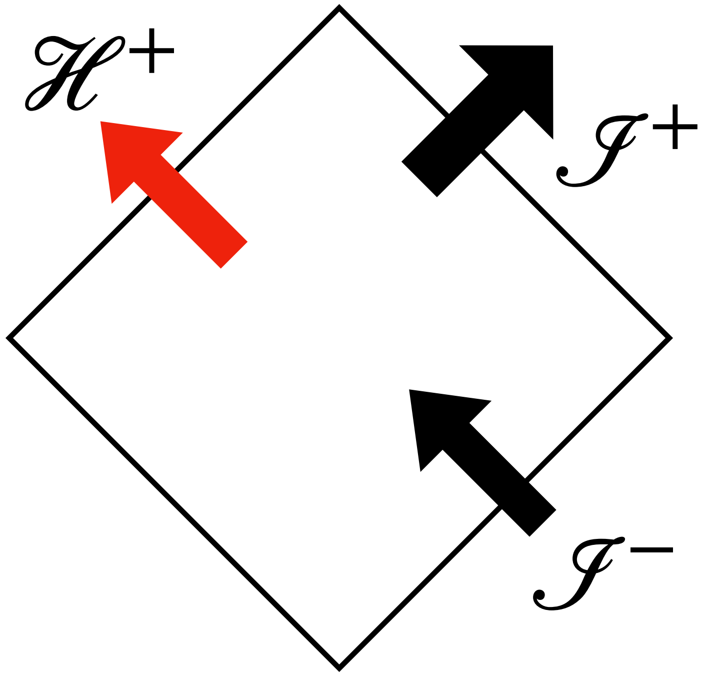
アクシオンのようなボゾンはブラックホールからエネルギーを引き抜ける.

$$J_{\mathscr{H}^+} = -\int_{r=r_+} J^\mu n_\mu \; 2Mr_+ d\theta d\varphi \;, \qquad J_\mu = -\frac{i}{2}(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)$$

$J_{\mathscr{H}^+}$:event horizonを横切る粒子数フラックス

$n^\mu \propto (\partial_t)^\mu + \Omega_H(\partial_\varphi)^\mu$:event horizonの生成子

Ω_H:event horizonの角速度



$\phi = R_{lm\omega}(r)S_{lm\omega}(\theta)e^{-i(\omega t - m\varphi)}$, $\omega > 0$を仮定

➡ $J_{\mathscr{H}^+} = 4\pi M r_+(\omega - m\Omega_H)\,|\,R_{lm\omega}(r_+)\,|^2$

$J_{\mathscr{H}^+} < 0$ for $\omega - m\Omega_H < 0$

負の粒子数フラックス=ブラックホールの外で粒子数増える

Superradiance!

Possible effects on axion cloud

アクシオン雲が進化するとき，以下のような様々な効果が効く

(Arvanitaki et. al.,2011,……)

1. 降着によるBHスピンの上昇 (Brito et. al.,2014)
2. 連星による潮汐力 (Baumann et. al.,2019)
3. 電磁場との相互作用
4. 重力波放射
5. 自己相互作用

BHのスピン対質量プロットから制限をつける際に重要

共鳴が起き，雲が消えてしまう.

3.はstring axionを考えると小さい. (Arvanitaki et. al.,2011)

4.は進化の初期段階では効かない. (Kodama&Yoshino,2014)

Coupling with EM

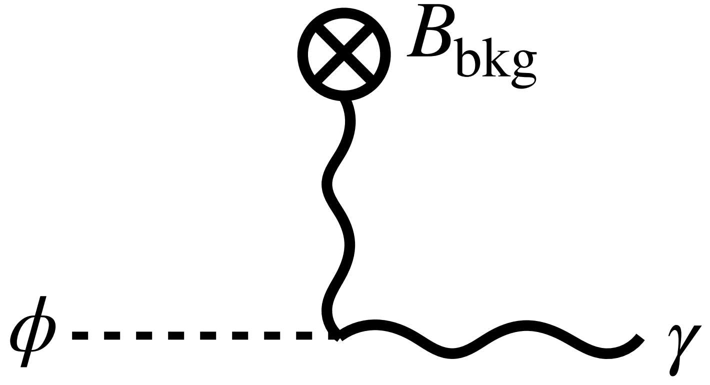
$$S_{int} = \frac{1}{32\pi^2 F_a}\frac{4\alpha}{3}\int d^4x\sqrt{-g}\,\phi\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

ϕ:アクシオン　F_a:崩壊定数

$F_{\mu\nu}$:電磁場　α:微細構造定数

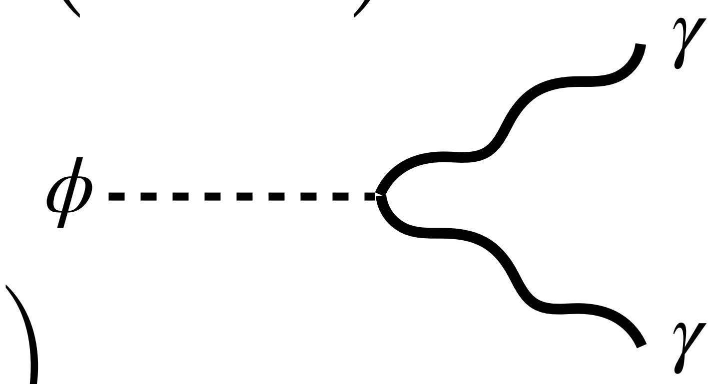
1. 背景の磁場と相互作用して光子に変換

(Arvanitaki et. al.,2011)



$$\Gamma_{\phi\gamma} \sim 7\times 10^{-11}{\rm yr}^{-1}\left(\frac{10^{16}{\rm GeV}}{F_a}\right)^2\left(\frac{\mu}{6\times 10^{-10}{\rm eV}}\right)\left(\frac{B_{\rm bkg}}{4\times 10^{8}{\rm G}}\right)^2$$

2. アクシオンが２つの光子に崩壊



$$\Gamma_{\phi\gamma\gamma} \sim 8\times 10^{-65}{\rm yr}^{-1}\left(\frac{10^{16}{\rm GeV}}{F_a}\right)^2\left(\frac{\mu}{6\times 10^{-10}{\rm eV}}\right)$$

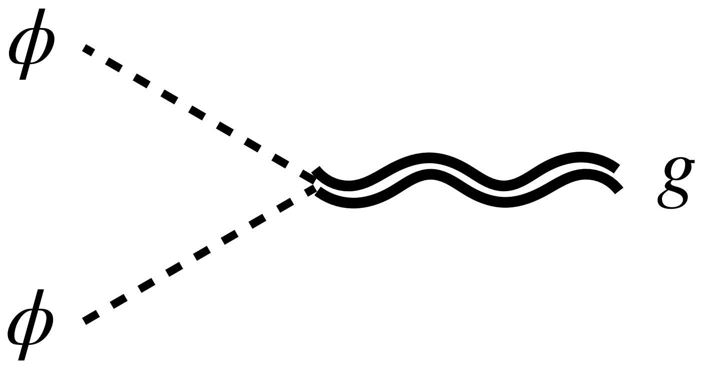
いずれもsuperradiant instabilityに比べて十分長いので無視する

Gravitational wave emission

重力場との結合から単色重力波を放射する.
背景重力波や連続重力波として観測可能(?)

1. Axionが対消滅し，gravitonを生成

(Arvanitaki et. al.,2011,
Kodama&Yoshino, 2014)



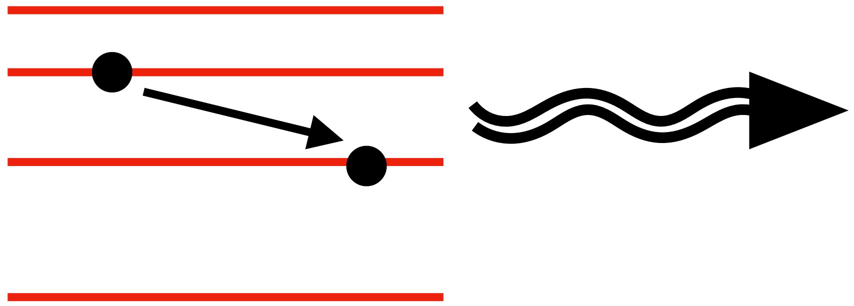
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} \sim N_{110}^2 \frac{9\pi(\mu M)^{18}}{2^{26} M^4}(35 + 28\cos\theta + \cos 4\theta)$$

N_{110}:$l = m = 1, n = 0$の雲の粒子数

$\omega_I \sim (\mu M)^9$なので，かなり成長しないと効かない

2. 束縛状態間の遷移に伴う重力波放射

(Arvanitaki et. al.,2011)



3. String axionなら自然に

$$S_{int} \propto \int d^4x \sqrt{-g}\,\phi\,\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta}$$

が入るが，相互作用が弱すぎて効かない.

Self-interaction vs GW emission

自己相互作用と重力波によるエネルギーロスのうち
どちらが効くか？

単位時間あたりのエネルギー放射で比較

$$\frac{P(2\times(1,1,0)\rightarrow \mathrm{graviton})}{P(3\times(1,1,0)\rightarrow \mathrm{axion})}\sim 10^{-6}\frac{M}{M_{\mathrm{cl}}}\left(\frac{F_a/M_{\mathrm{pl}}}{10^{-2}}\right)^4\frac{1}{(\mu M)^4}$$

M_{cl}が中心BHに対して小さすぎない限り自己相互作用が効く

M_{cl}が小さすぎるときはそもそも重力波放射が弱すぎて,

superradiant growthが卓越

重力波放射は成長に大きな影響を与えないため，無視する