Bridging the μHz gap in the GWs spectrum

Diego Blas Temiño

Jenkins, Bourguin, Foster, Hees, Herrero-Valea, Xue;
Zwick, Souyer, O'Neill, Derdzinski, Saha, D'Orazio, Kelley

ICREA

IFAE

BIST Barcelona Institute of Science and Technology

2107.04063
2107.04601
2406.02306

AGENCIA ESTATAL DE INVESTIGACIÓN

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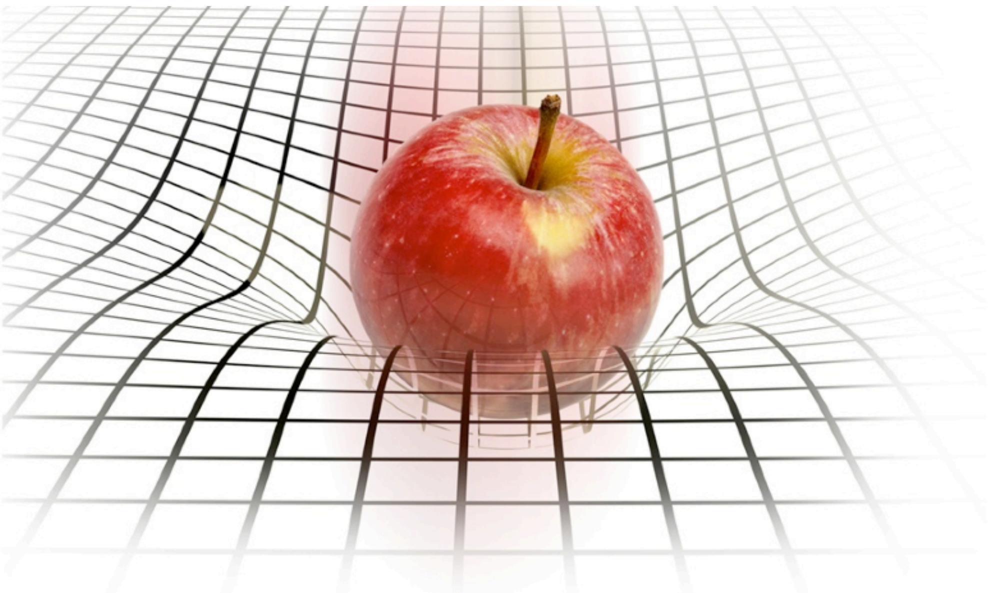
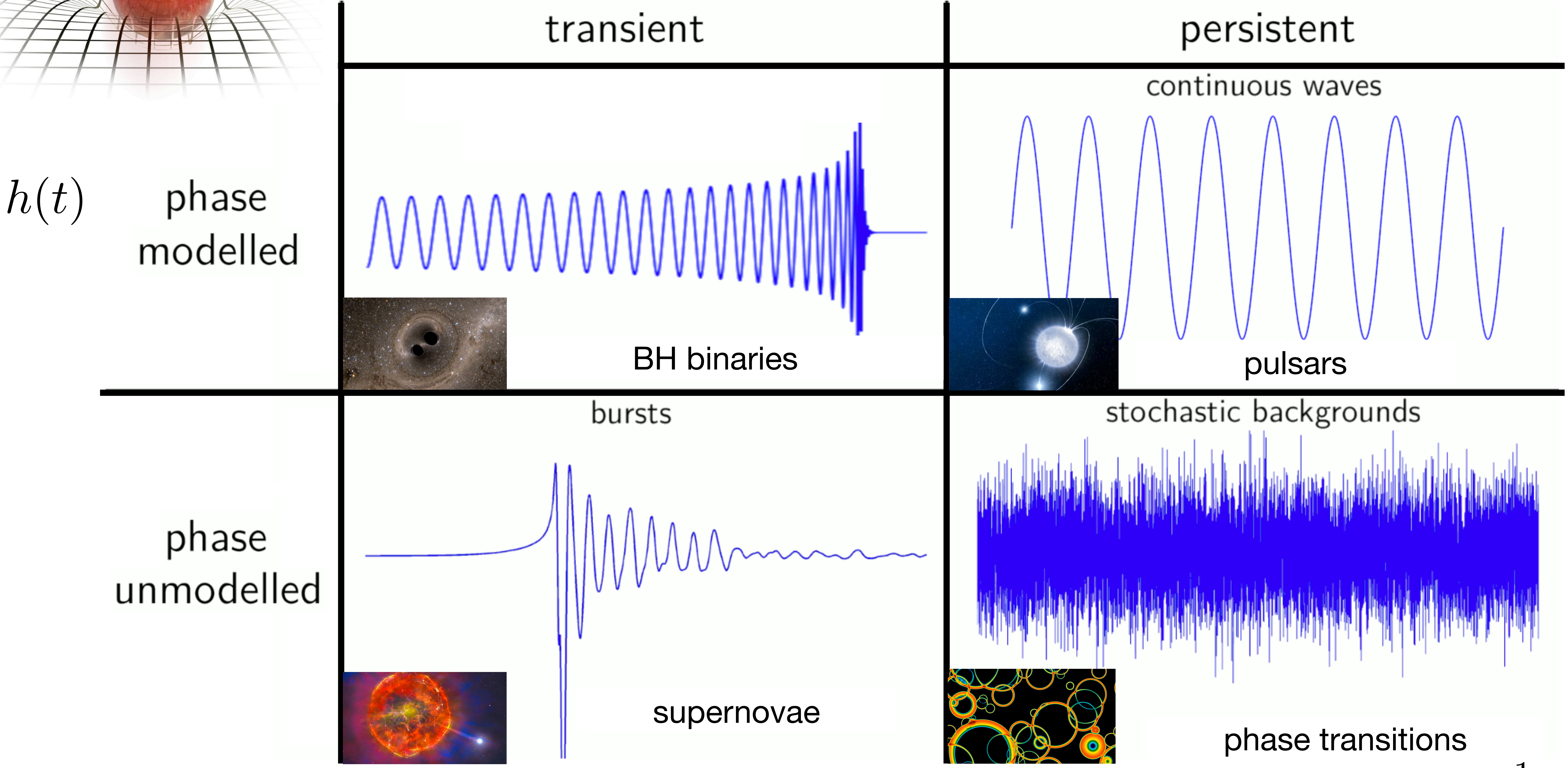
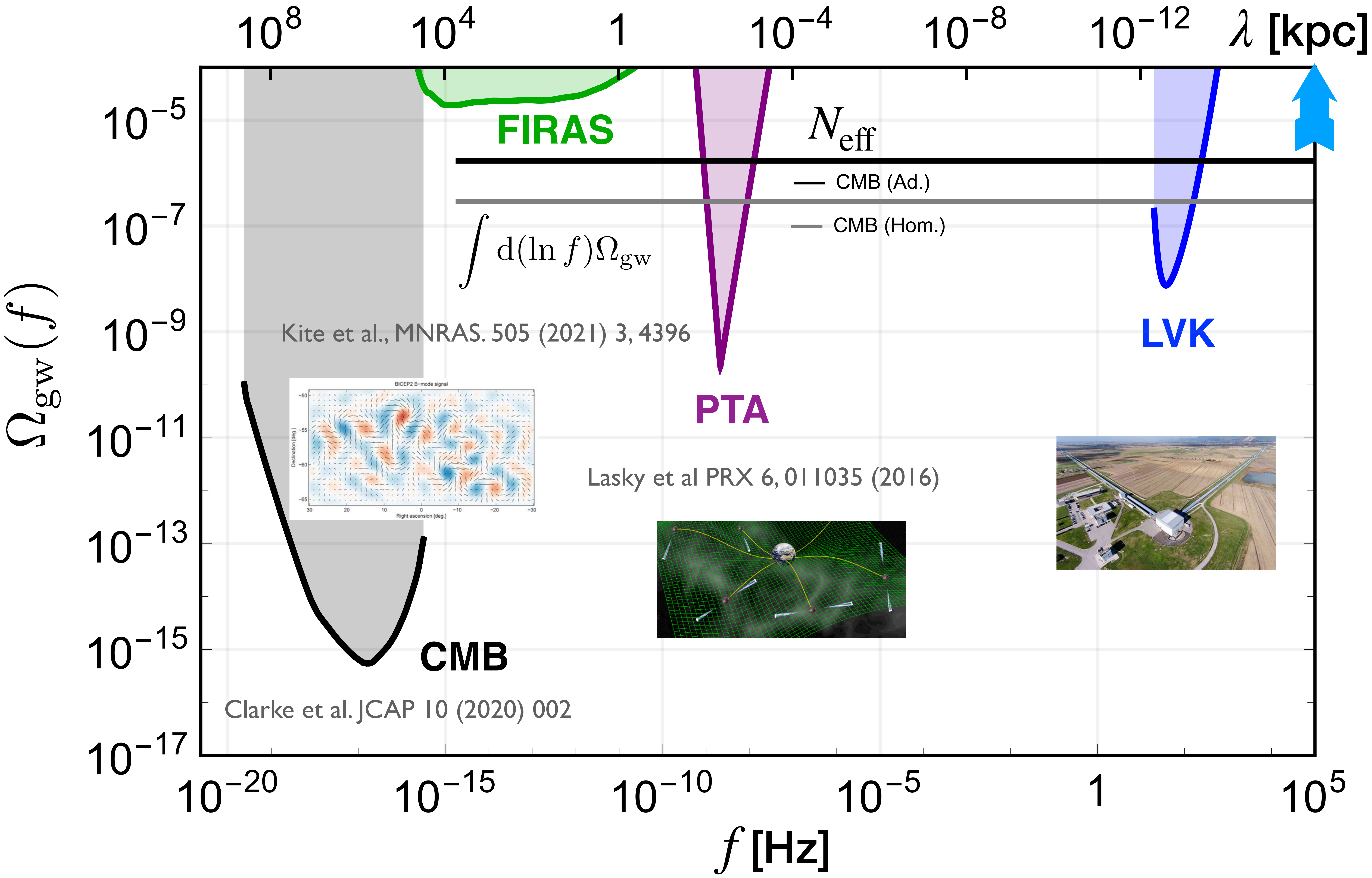


Taxonomy of GWs

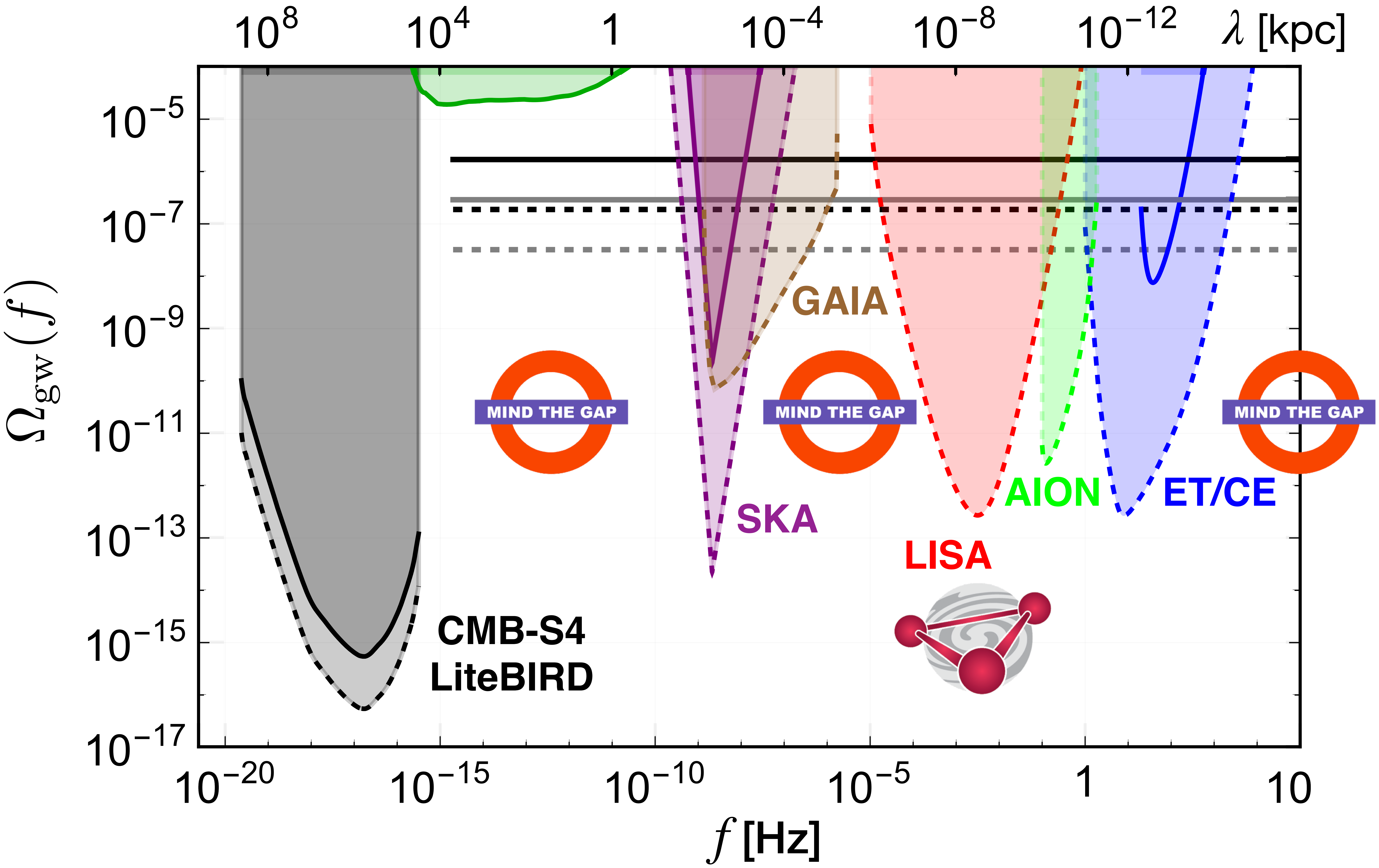


$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}(\ln f)}$$

GWs soundscape today

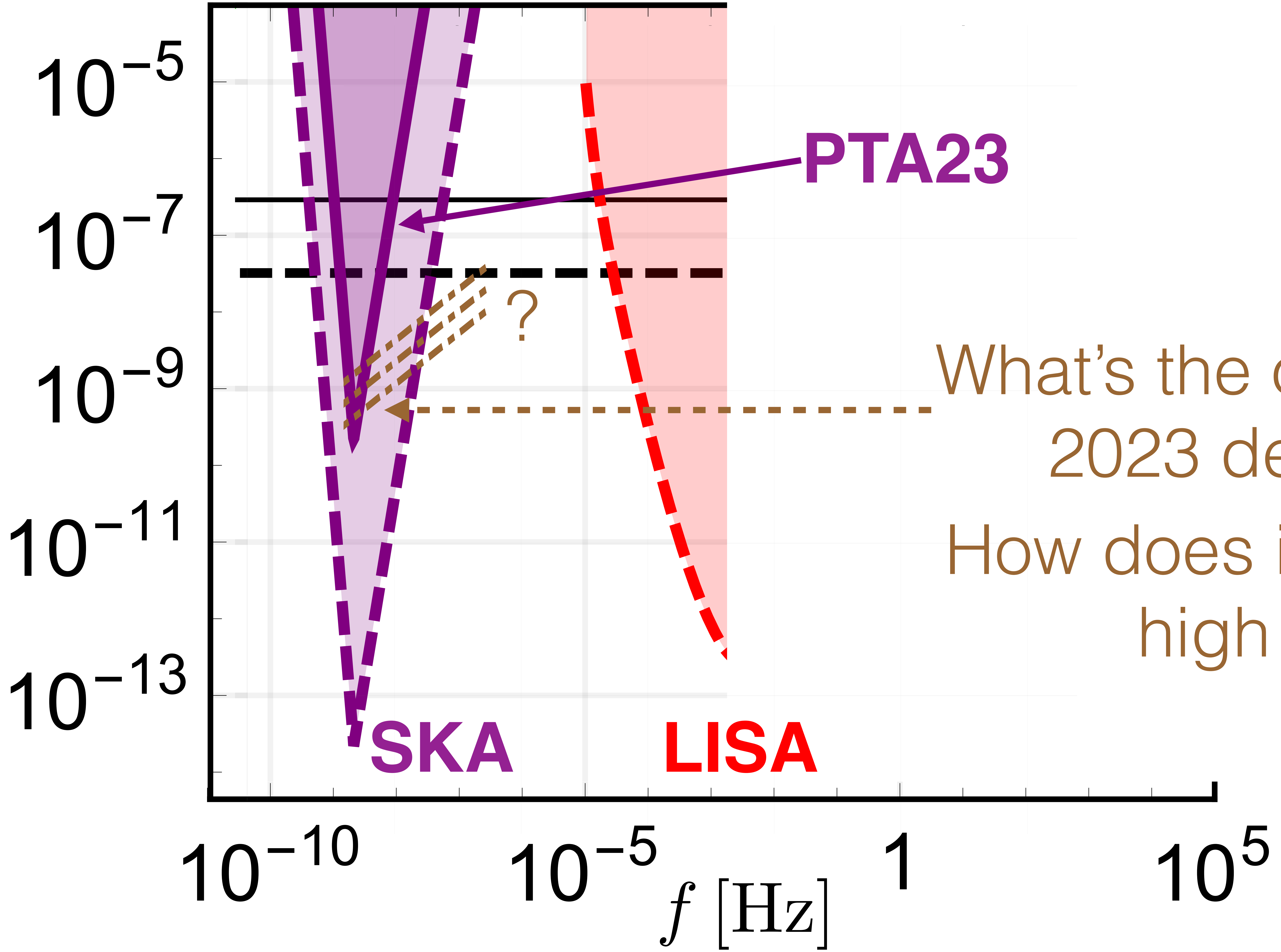


GWs soundscape ca. 2040



Possible **backgrounds** & ideas at μHz: a rich band

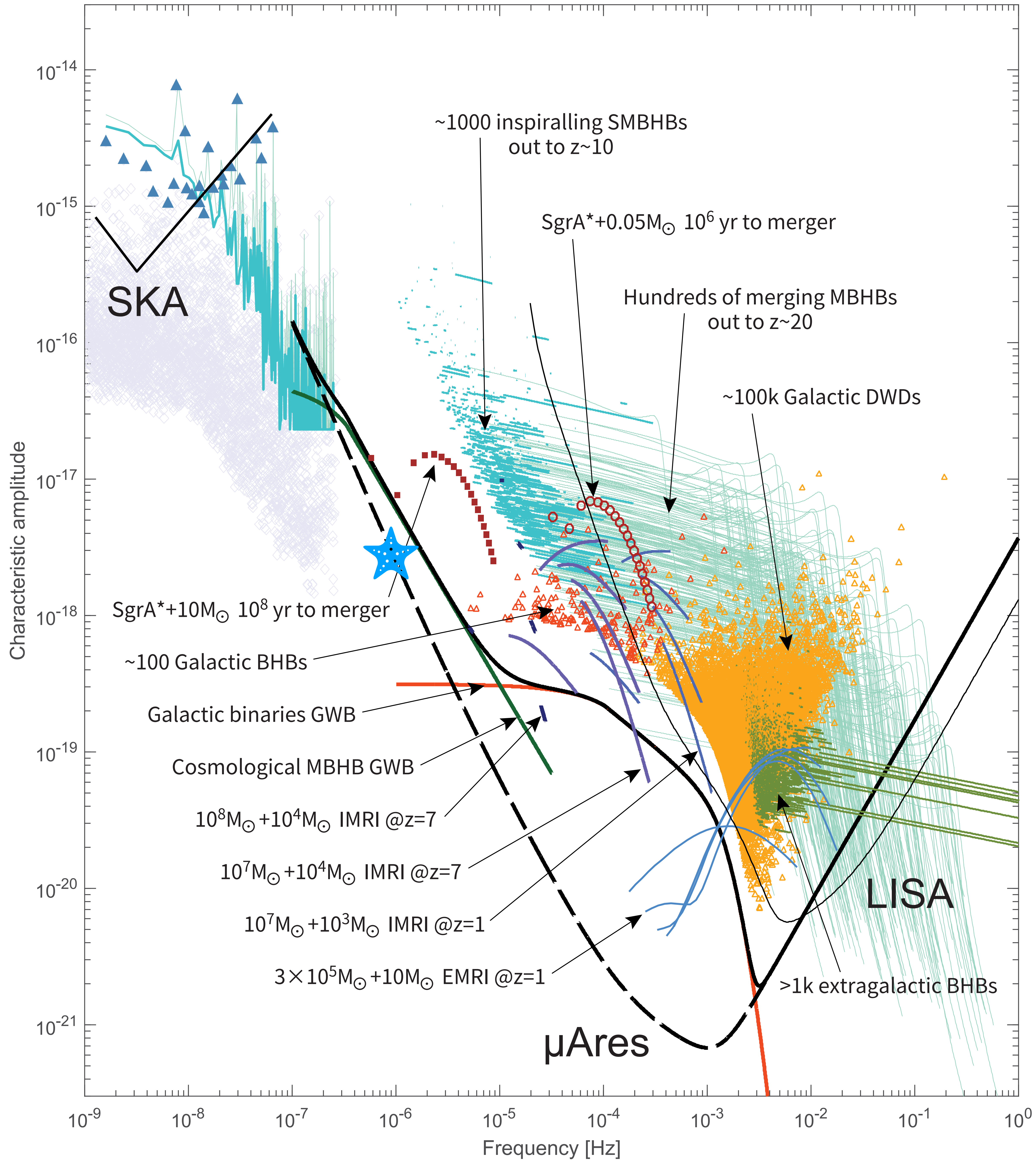
Possible **backgrounds** & ideas at μHz: a rich band



What's the origin of the 2023 detection?

How does it change at high freq?

Other astro signals



Sesana et al. 1908.11391

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d(\ln f)}$$

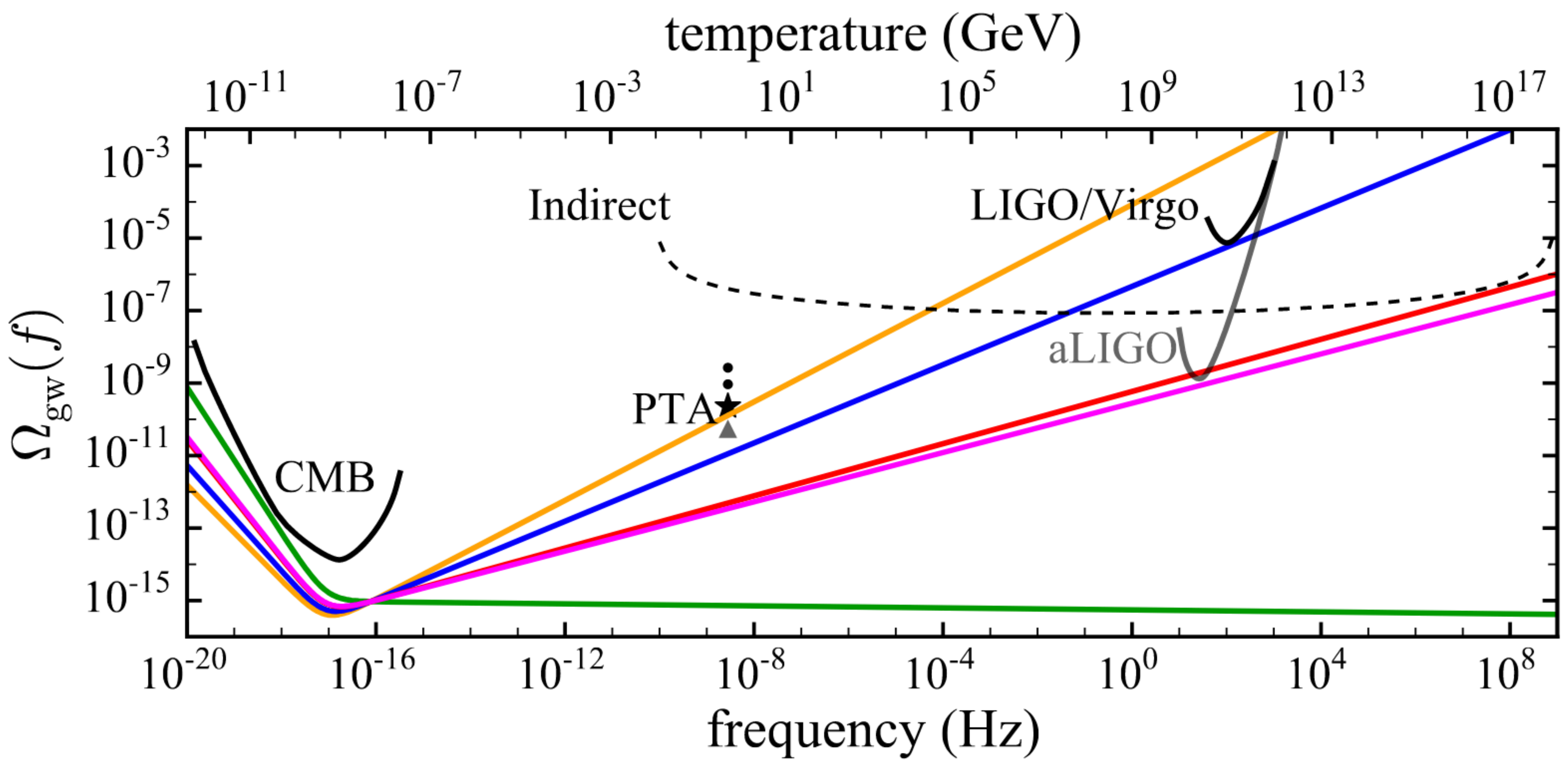
$$\rho_{\rm GW} \sim M_P^2 \omega^2 h_{GW}^2$$

$$\rho_c = 1.2 \times 10^{11} \frac{M_\odot}{{\rm Mpc}^3}$$

Backgrounds from fundamental physics

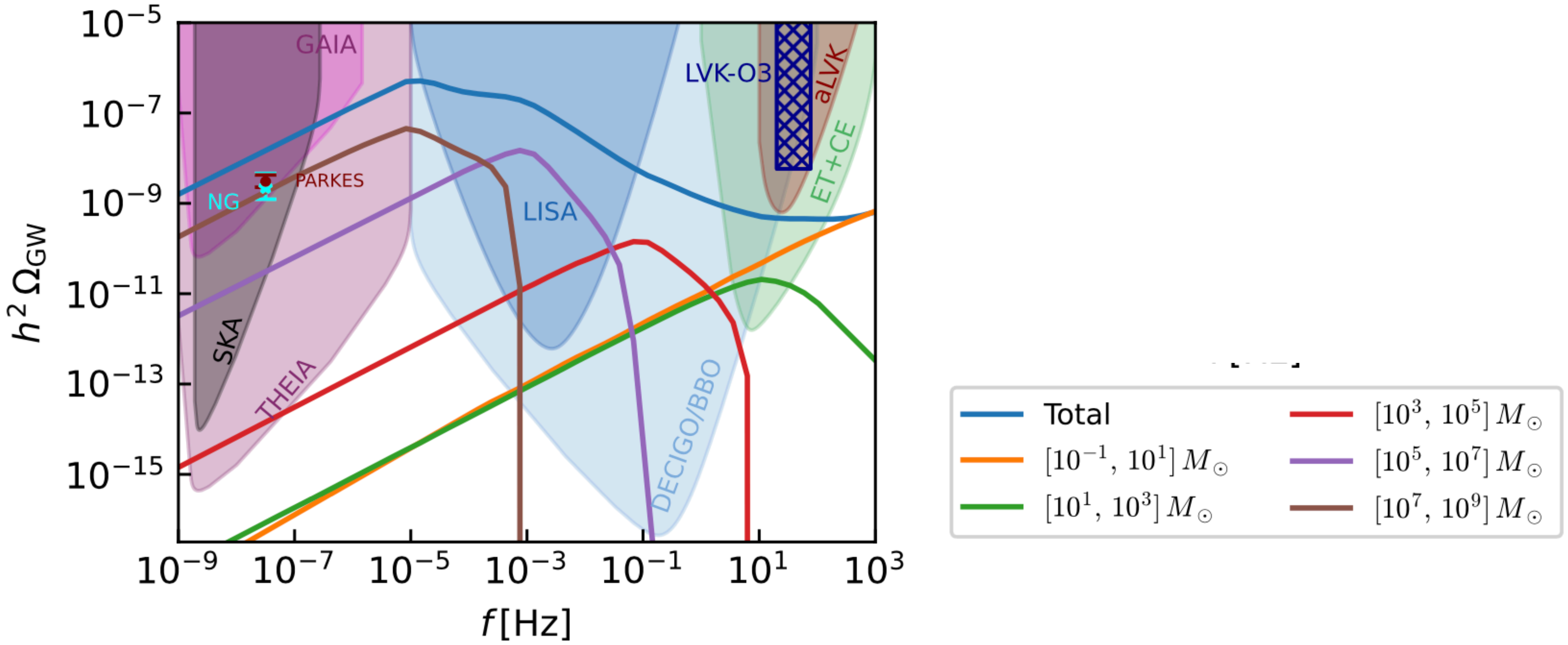
Inflation

Lasky et al PRX 6, 011035 (2016)



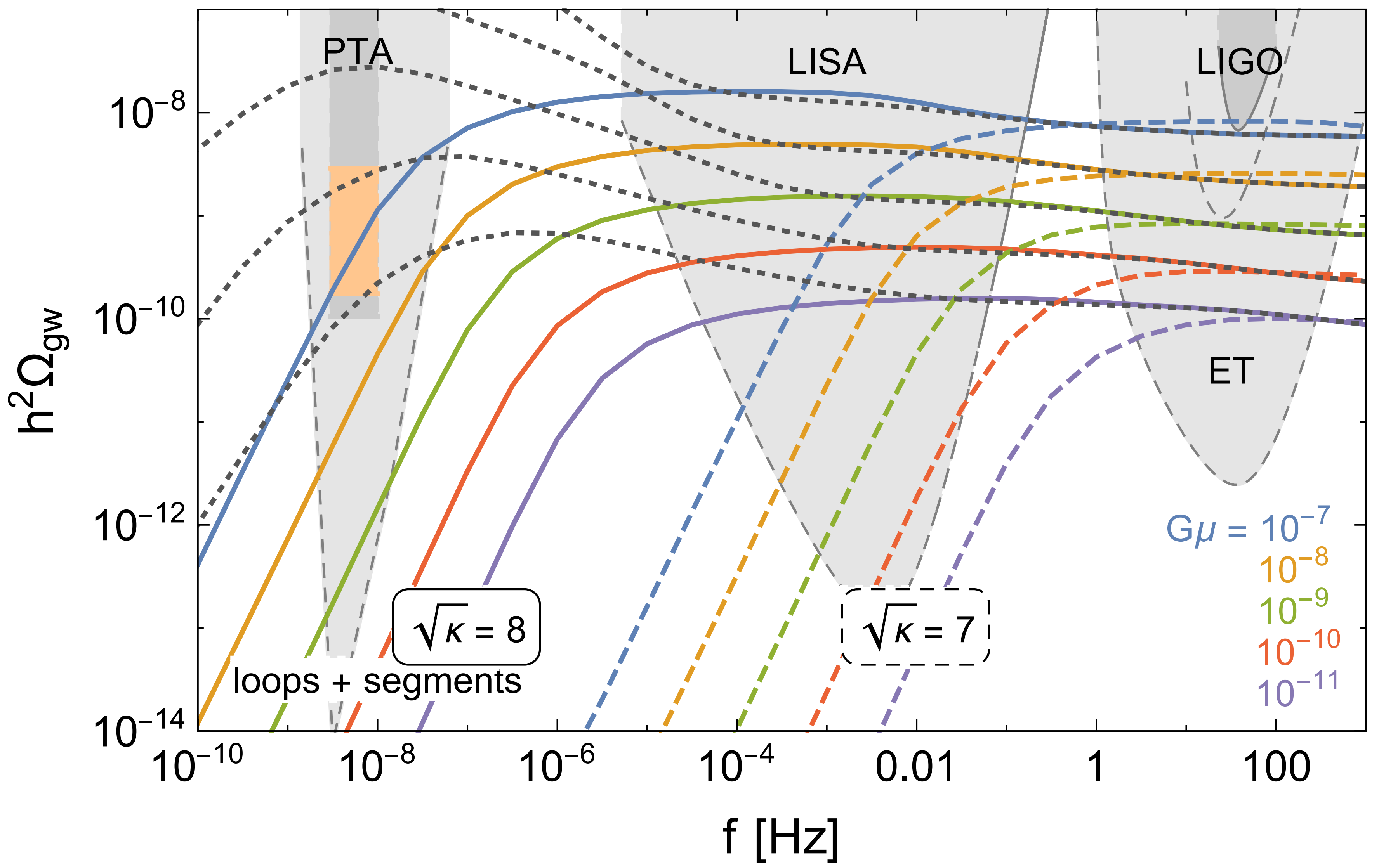
PBH

Braglia et al. JCAP 12 (2021) 12



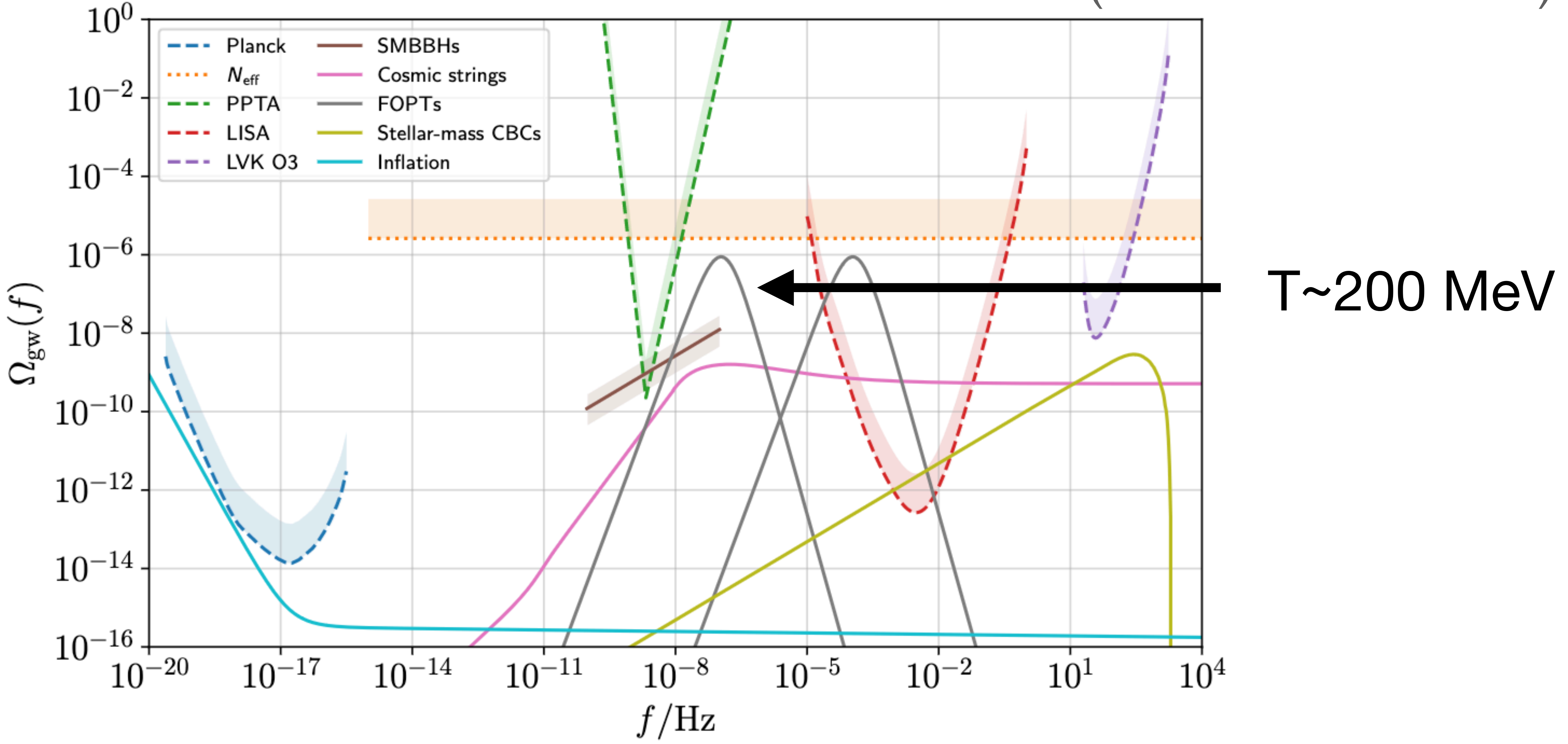
Cosmic Strings

Buchmuller et al. 2107.04578



FOPT

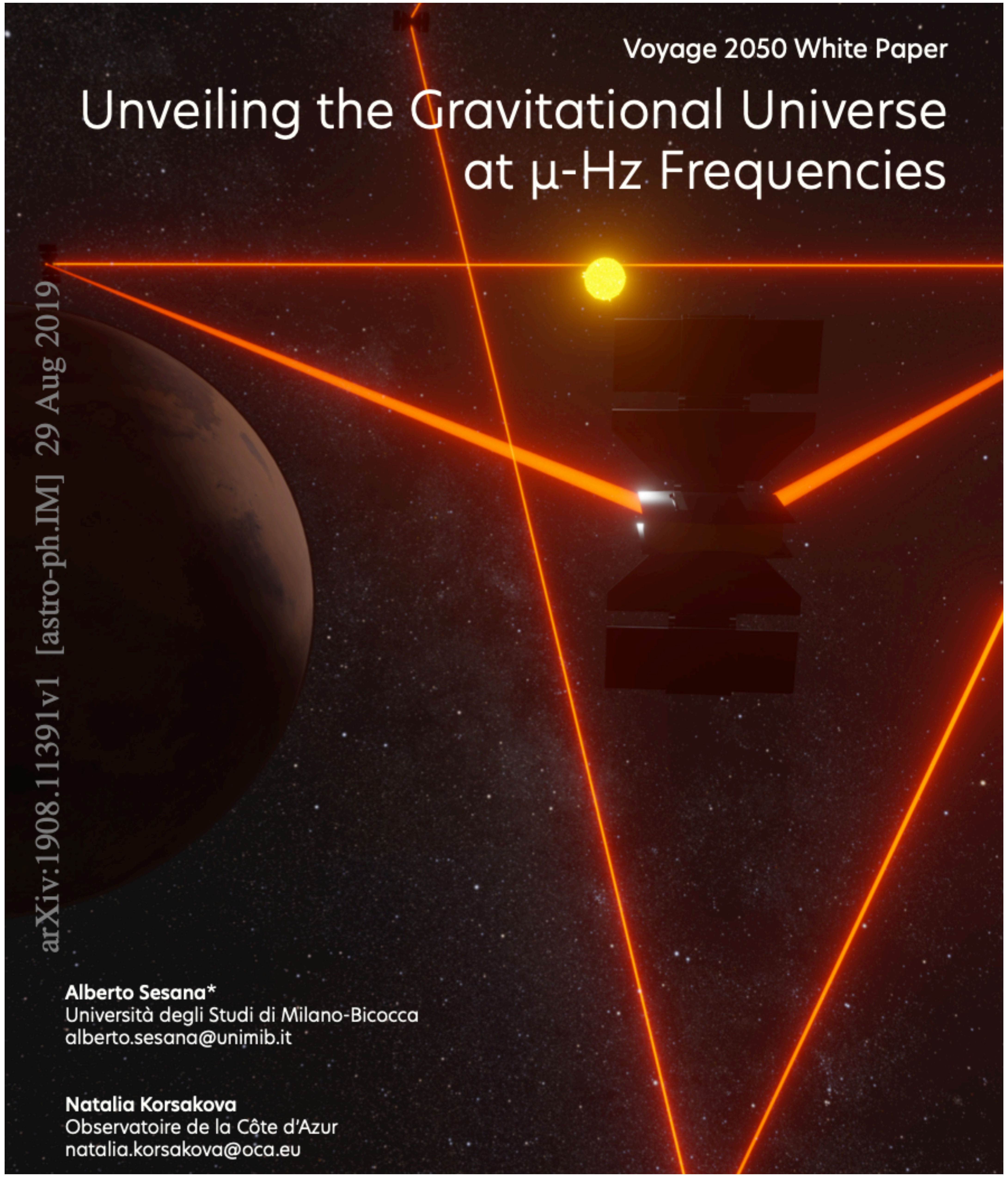
Renzini et al 2202.00178
(based on 1512.06239)



Possible backgrounds & **ideas** at μHz: a rich band

What can we do?

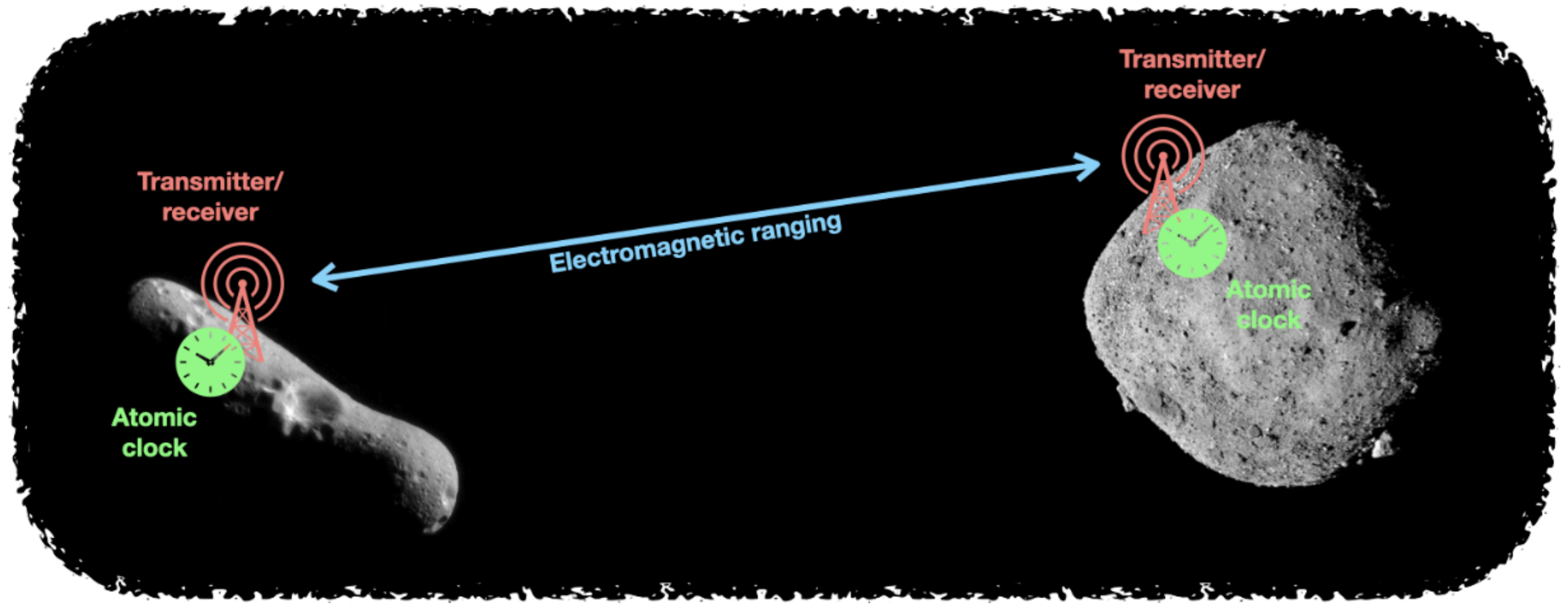
i) μAres: LISA-like concept

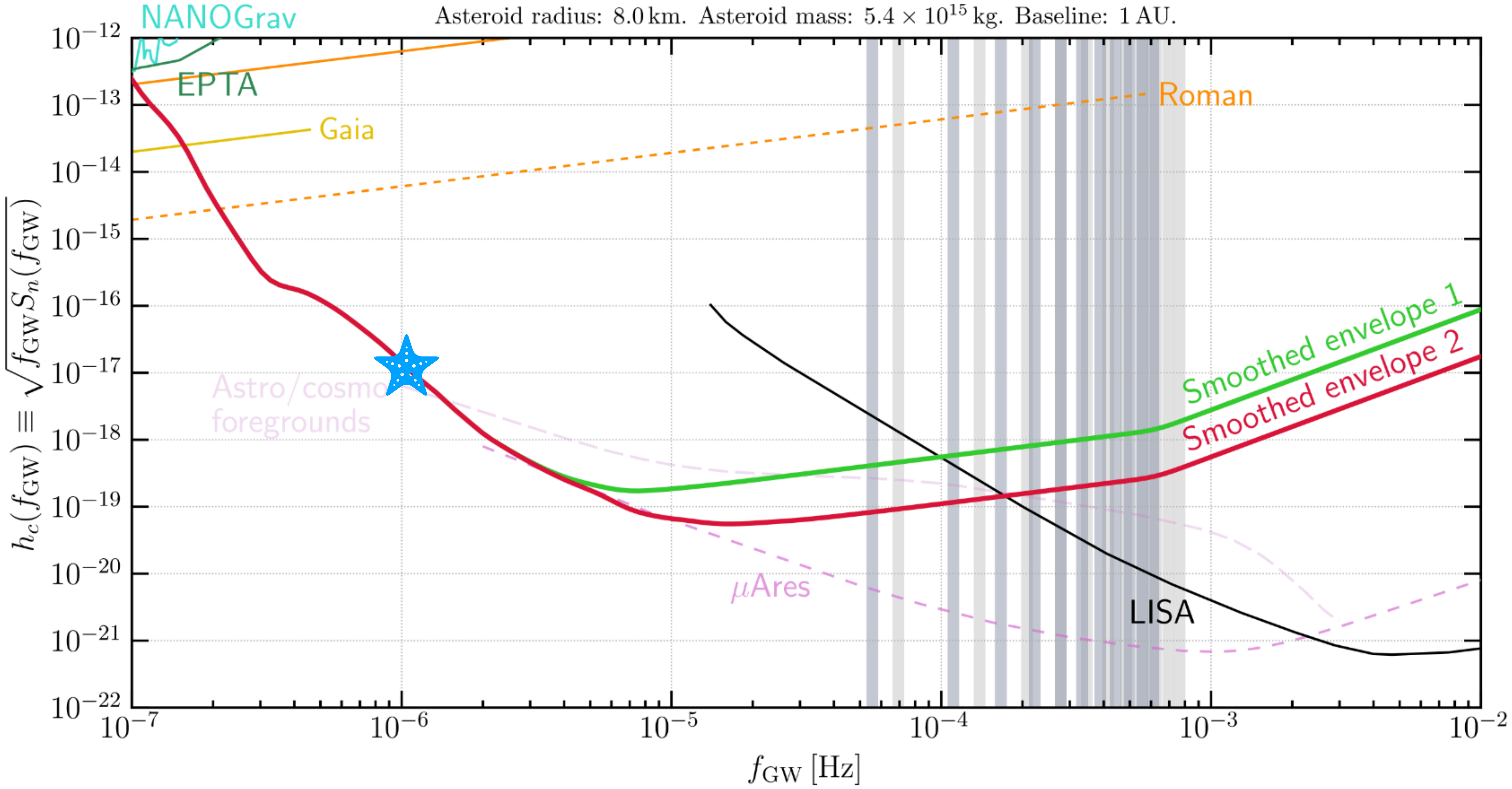


The μAres detection landscape
z [AU]
y [AU]
x [AU]
LISA
μAres (in plane)
μAres (out of plane)
Characteristic amplitude
Frequency [Hz]
SKA
LISA
μAres
~1000 inspiralling SMBHBs
out to z~10
SgrA*+0.05M⊙ 10^6 yr to merger
Hundreds of merging MBHBs
out to z~20
~100k Galactic DWDs
SgrA*+10M⊙ 10^8 yr to merger
~100 Galactic BHBs
Galactic binaries GWB
Cosmological MBHB GWB
10^8M⊙+10^4M⊙ IMRI @z=7
10^7M⊙+10^4M⊙ IMRI @z=7
10^7M⊙+10^3M⊙ IMRI @z=1
3×10^5M⊙+10M⊙ EMRI @z=1
>1k extragalactic BHBs

ii) Ranging of asteroids?

Fedderke et al 2112.11431





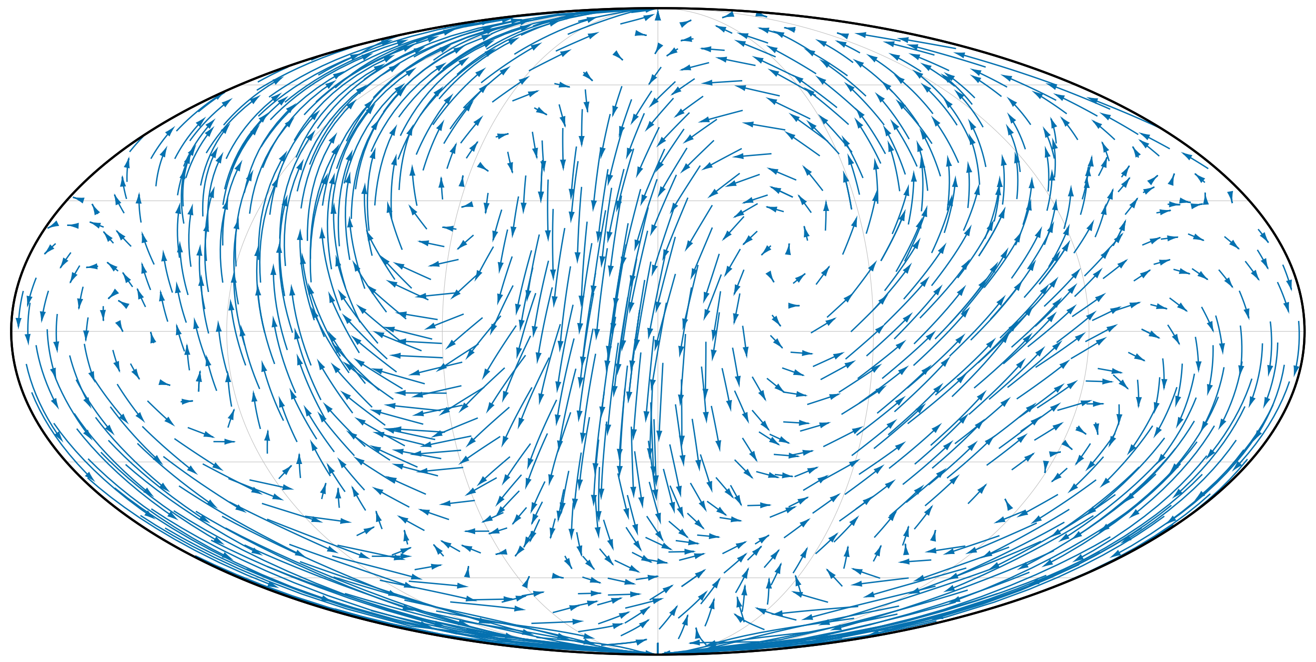
iii) Future astrometry?

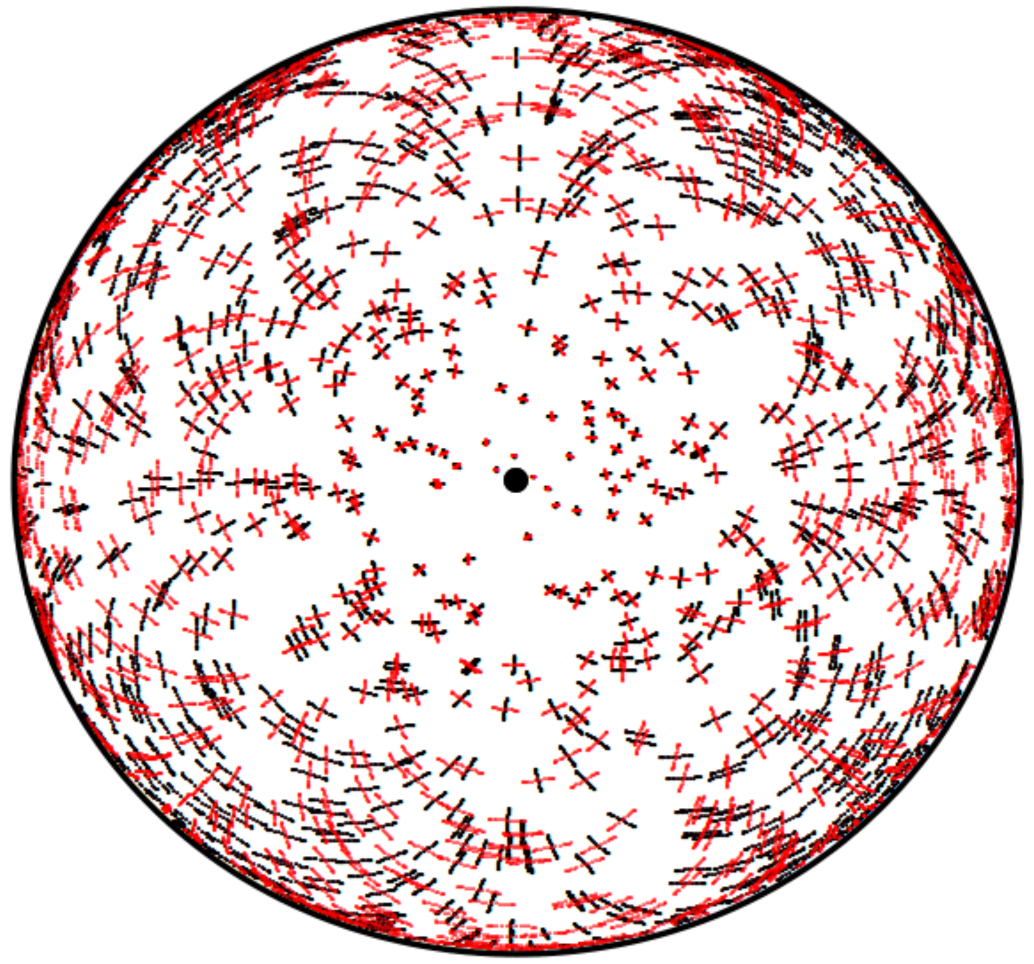
e.g. Moore et al 1707.06239
Mihaylov et al. 1804.00660

Klioner 1710.11474

Garcia-Bellido et al. 2104.04778

Monitoring many stars (GAIA or better)





Fedderke et al 2204.07677

Stellar interferometry

We evaluate the potential for gravitational-wave (GW) detection in the frequency band from 10 nHz to 1 μHz using extremely high-precision astrometry of a small number of stars

at characteristic strains around $h_c \sim 10^{-17} \times (\mu\text{Hz}/f_{\text{GW}})$. The astrometric angular precision required to see these sources is $\Delta\theta \sim h_c$ after integrating for a time $T \sim 1/f_{\text{GW}}$. We show that jitter in the photometric center of WD of this type due to starspots is bounded to be small enough to permit this high-precision, small-N approach. We discuss possible noise arising from stellar reflex motion induced by orbiting objects and show how it can be mitigated. The only plausible technology able to achieve the requisite astrometric precision is a space-based stellar interferometer. Such a future mission with few-meter-scale collecting dishes and baselines of $\mathcal{O}(100\,\text{km})$ is sufficient to achieve the target precision. This collector size is broadly in line with the collectors proposed for

Çalışkan et al 2312.03069

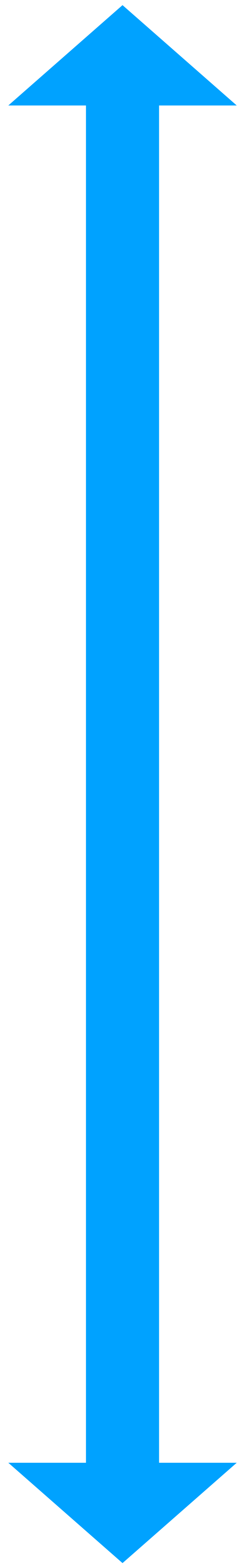
iv) Atomic interferometry in space: AEDGE

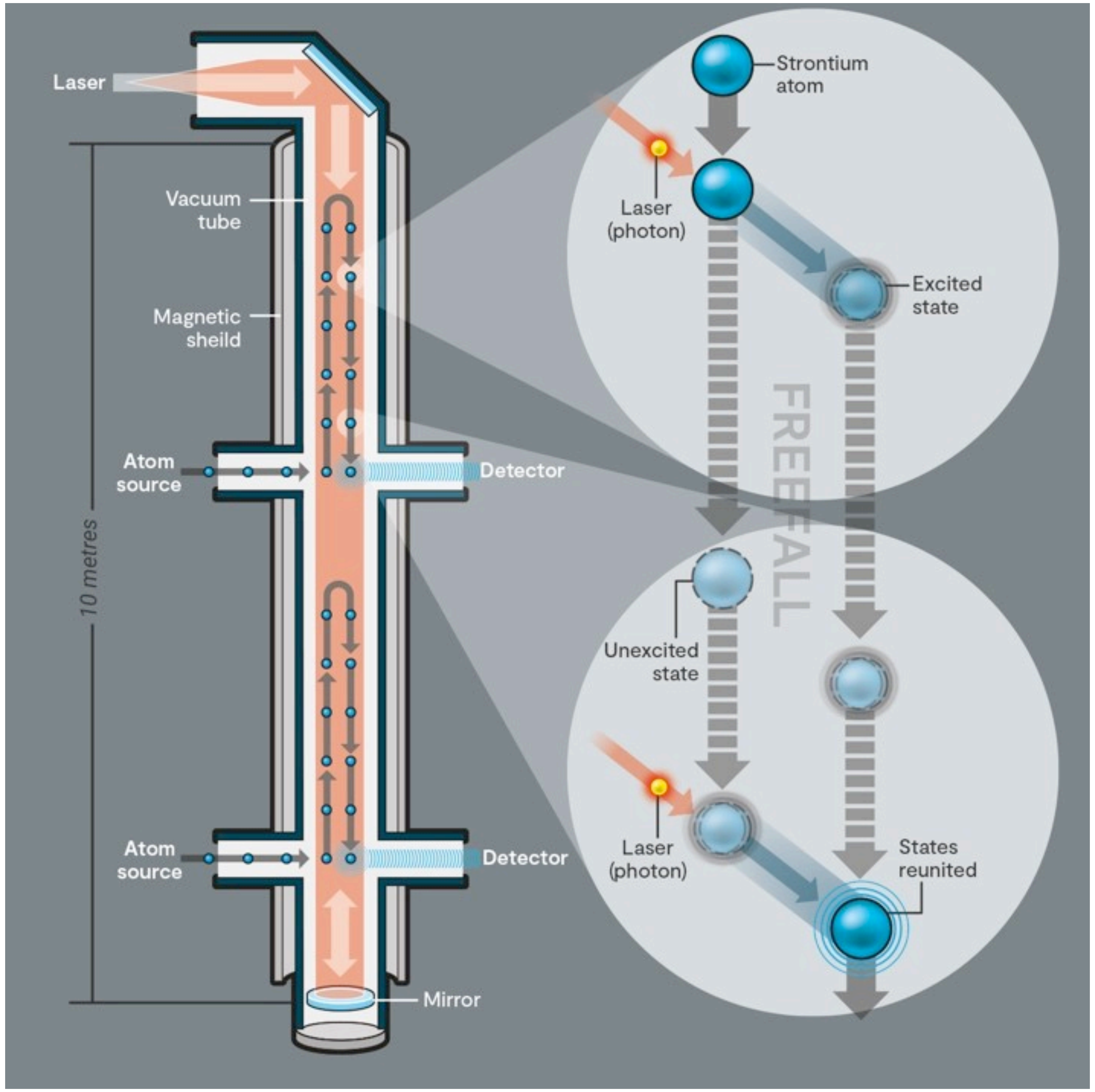
Abou El-Neaj et al 1908.00802

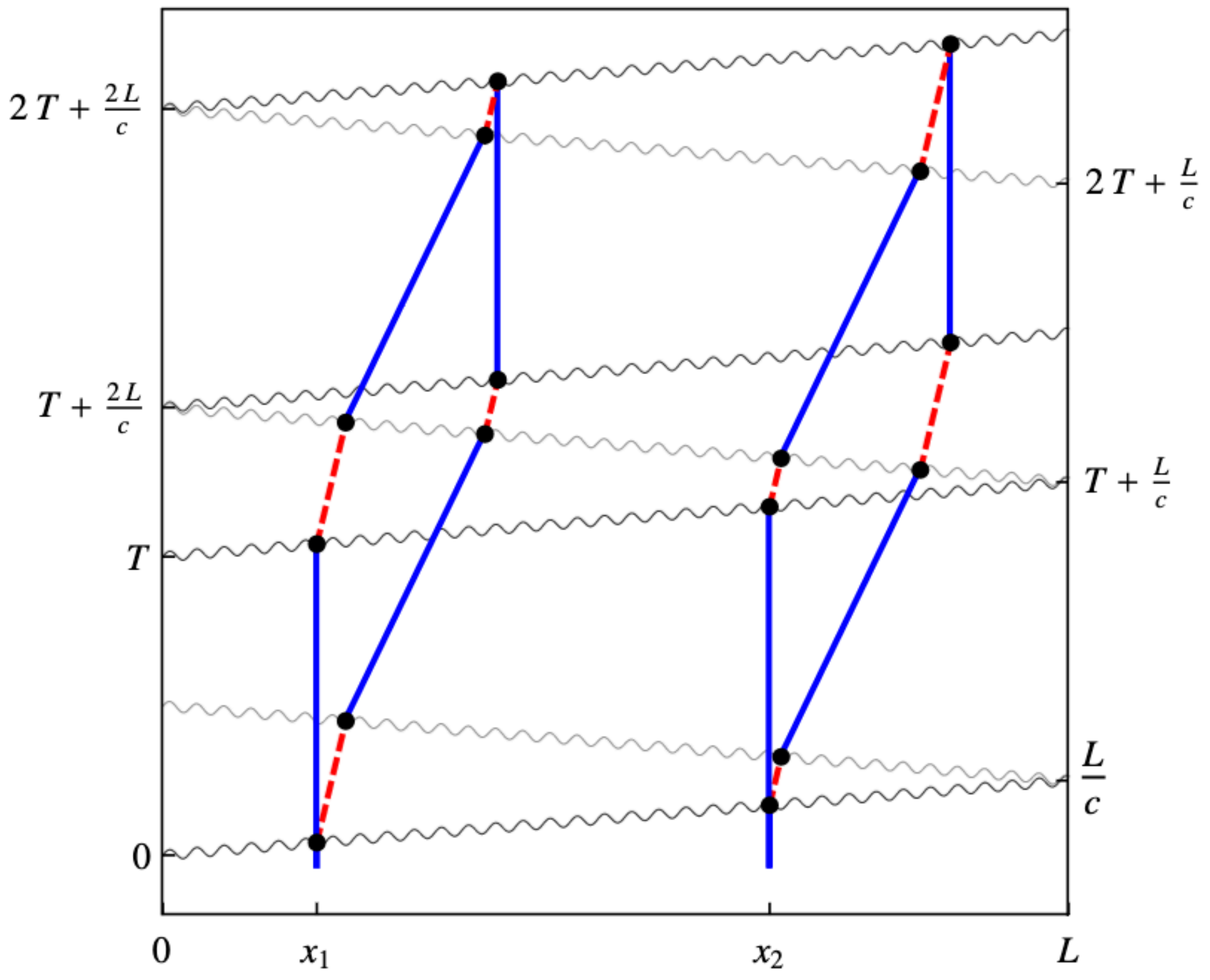
Graham et al 1206.0818 (MAGIS)

Badurina et al 2108.02468 (AION)

40000 km

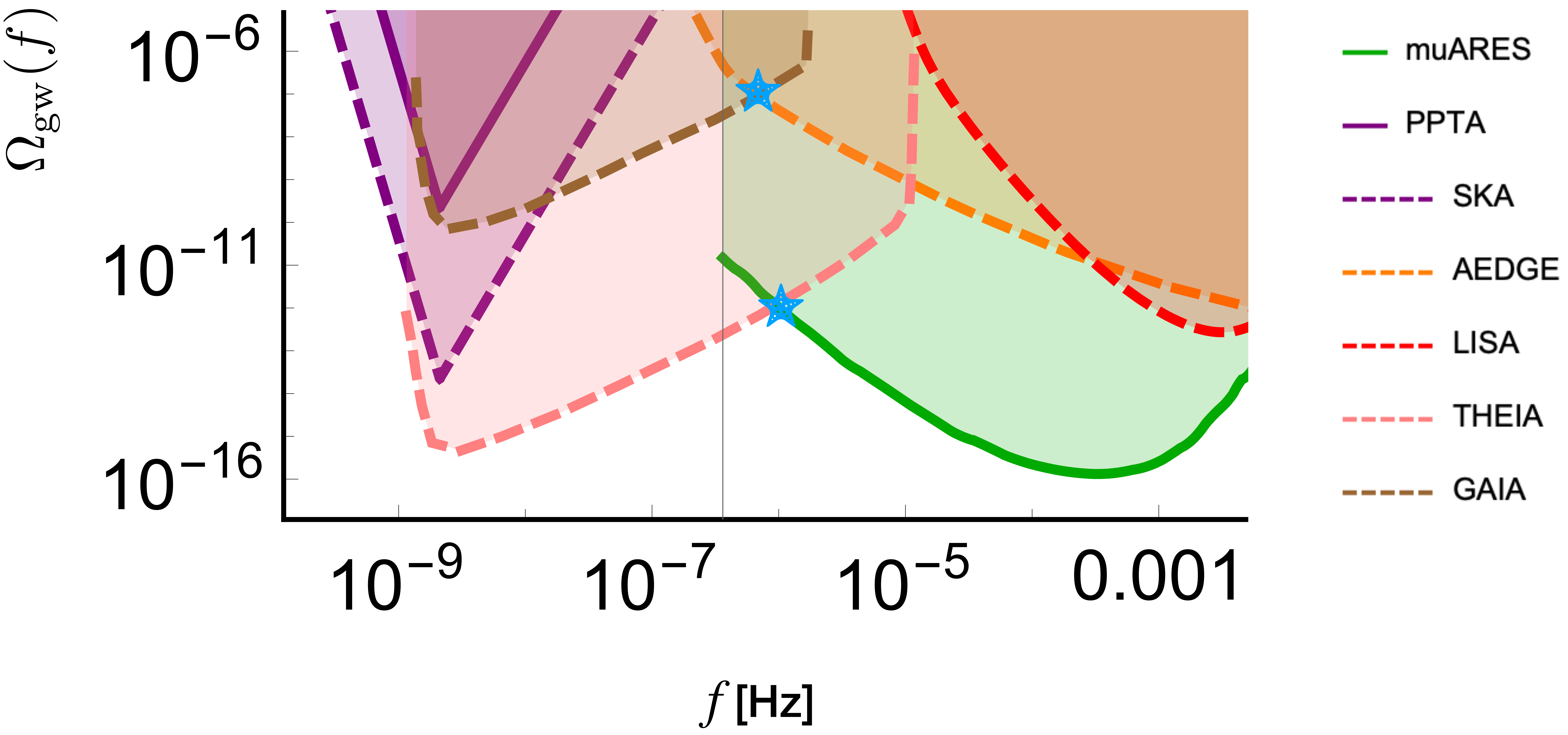




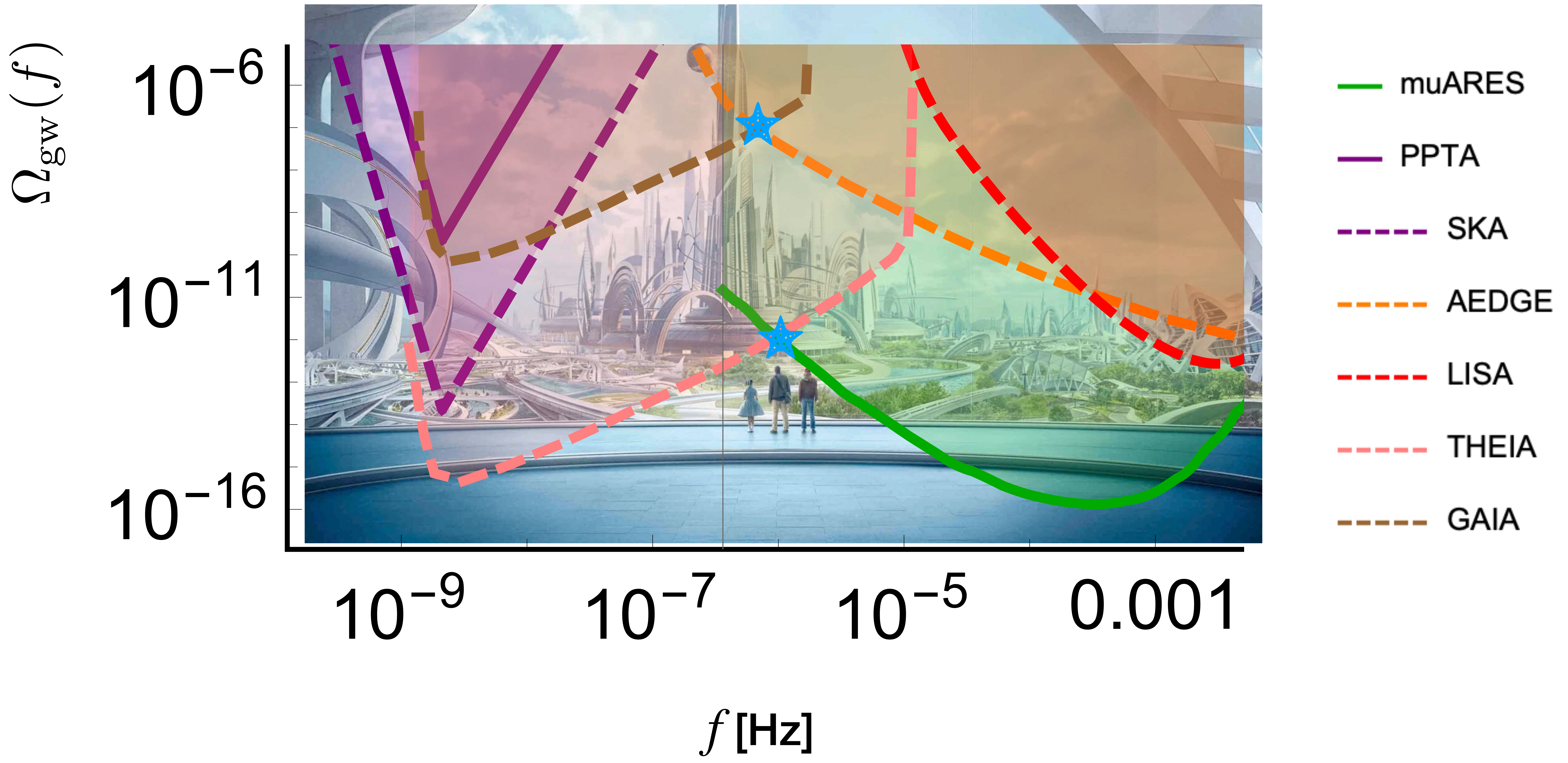


$$\Delta\phi \sim \omega L h$$

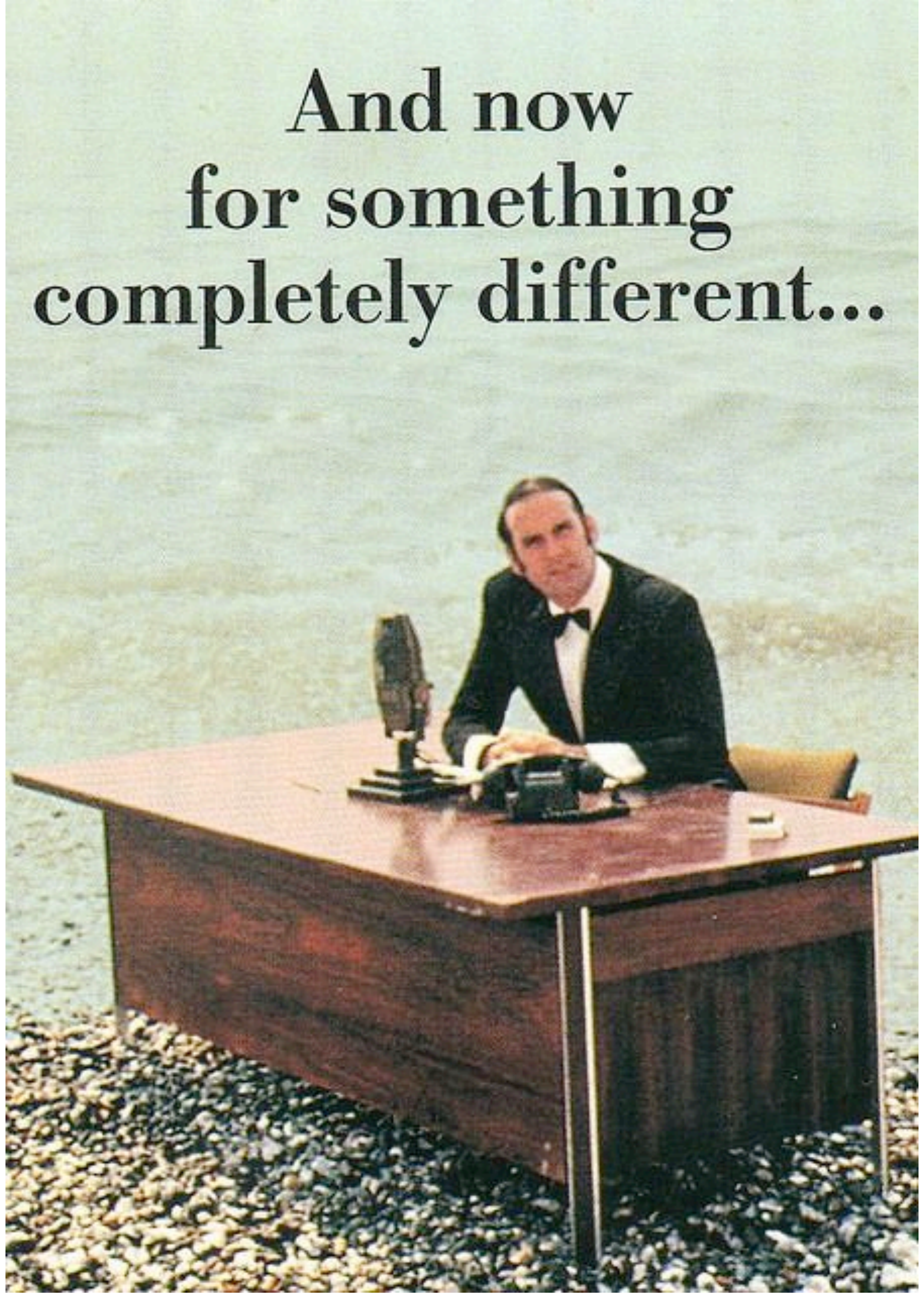
The most optimistic future...



The most optimistic future…



Is this all we can do in this band?



$$f \sim \mu\mathrm{Hz}$$

few days

$$\lambda \sim 10^{11}\,\mathrm{km} \sim 66\ \mathrm{AU}$$

↑

Natural units for Solar System!

Binary resonance: a brief history

discussed by Misner, Thorne, and Wheeler. . .

1. The Relative Motions of Two Freely Falling Bodies

As a gravitational wave passes two freely falling bodies, their proper separation oscillates (Figure 37.3). This produces corresponding oscillations in the redshift and round-trip travel times for electromagnetic signals propagating back and forth between the two bodies. Either effect, oscillating redshift or oscillating travel time, could be used in principle to detect the passage of the waves. Examples of such detectors are the Earth-Moon separation, as monitored by laser ranging [Fig. 37.2(a)]; Earth-spacecraft separations as monitored by radio ranging; and the separation between two test masses in an Earth-orbiting laboratory, as monitored by redshift measurements or by laser interferometry. Several features of such detectors are explored in exercises 37.6 and 37.7. As shown in exercise 37.7, such detectors have so low a sensitivity that they are of little experimental interest.

. . . but that was *50 years ago!*

investigated more recently by Lam Hui *et al*, PRD (2013),
similar ideas used to search for dark matter by Blas *et al*, PRL (2017)

time for a closer look?

Absorption of GWs by binaries

$f \sim \mu\mathrm{Hz}$

few days

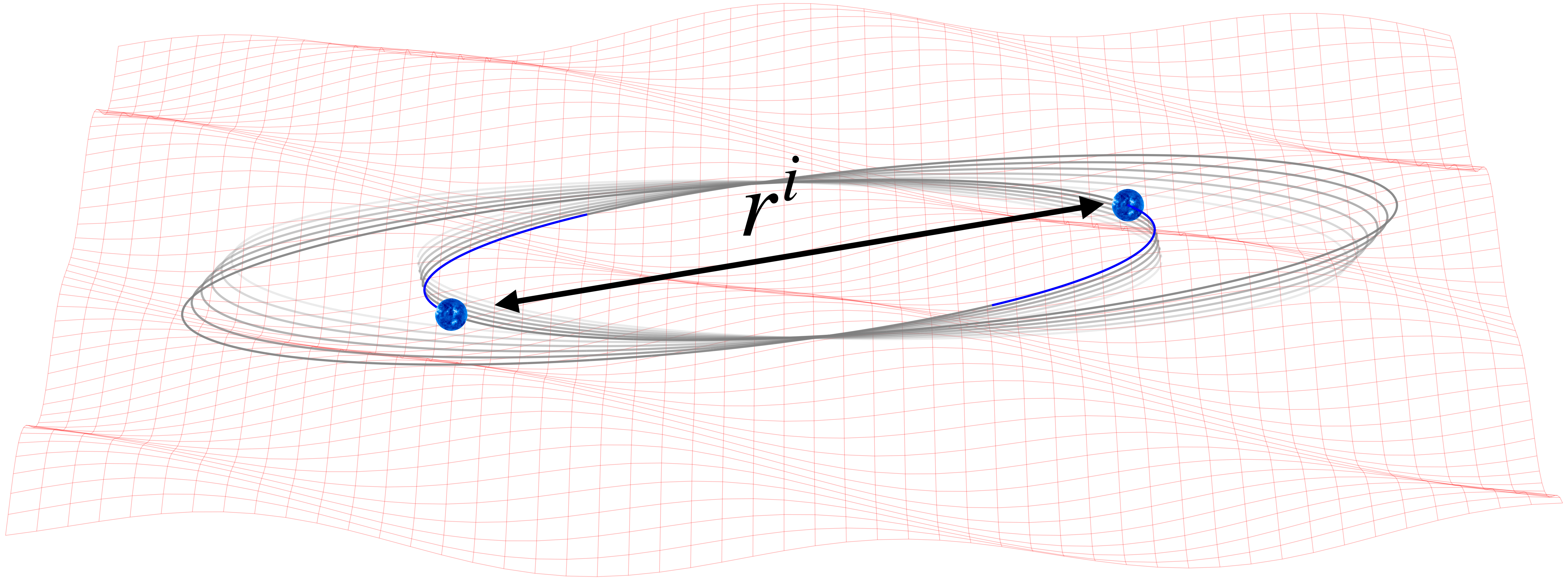
Intuitive idea (from '60s)

Influence of a GW on a binary system (e.g. non-relativistic)

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Newtonian potential

GW

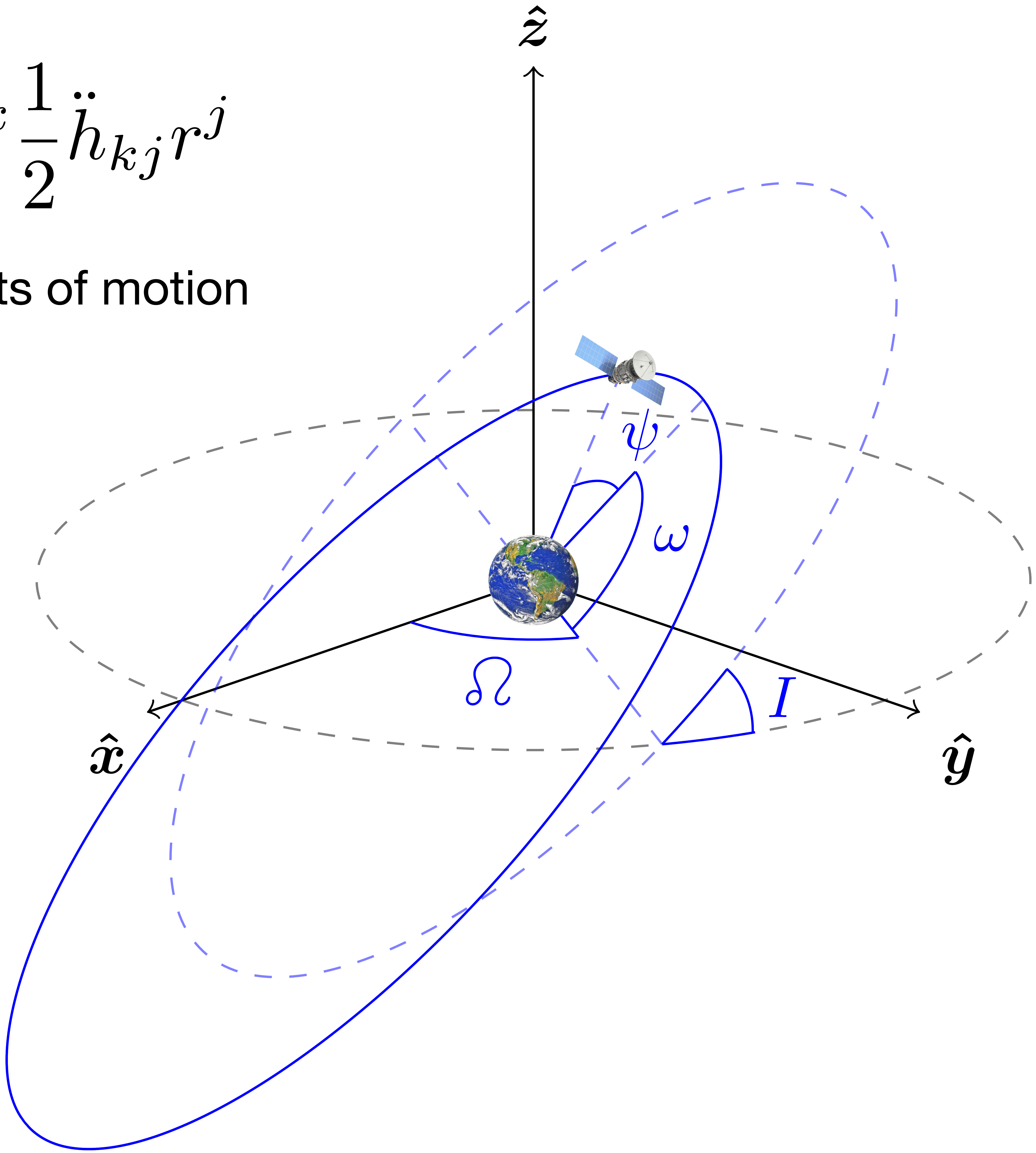


Absorption of GWs by binaries

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Better characterised for its 6 Newtonian constants of motion

- **period** P, **eccentricity** e:
  size and *shape* of orbit
- **inlination** I, **ascending node** Ω:
  orientation in space
- **pericentre** ω,
  mean anomaly at epoch ε:
  radial and angular *phases*



Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

- for generic perturbation:

$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi} + \mathcal{F}_\theta\right],$$

$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2},$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$

Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

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$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

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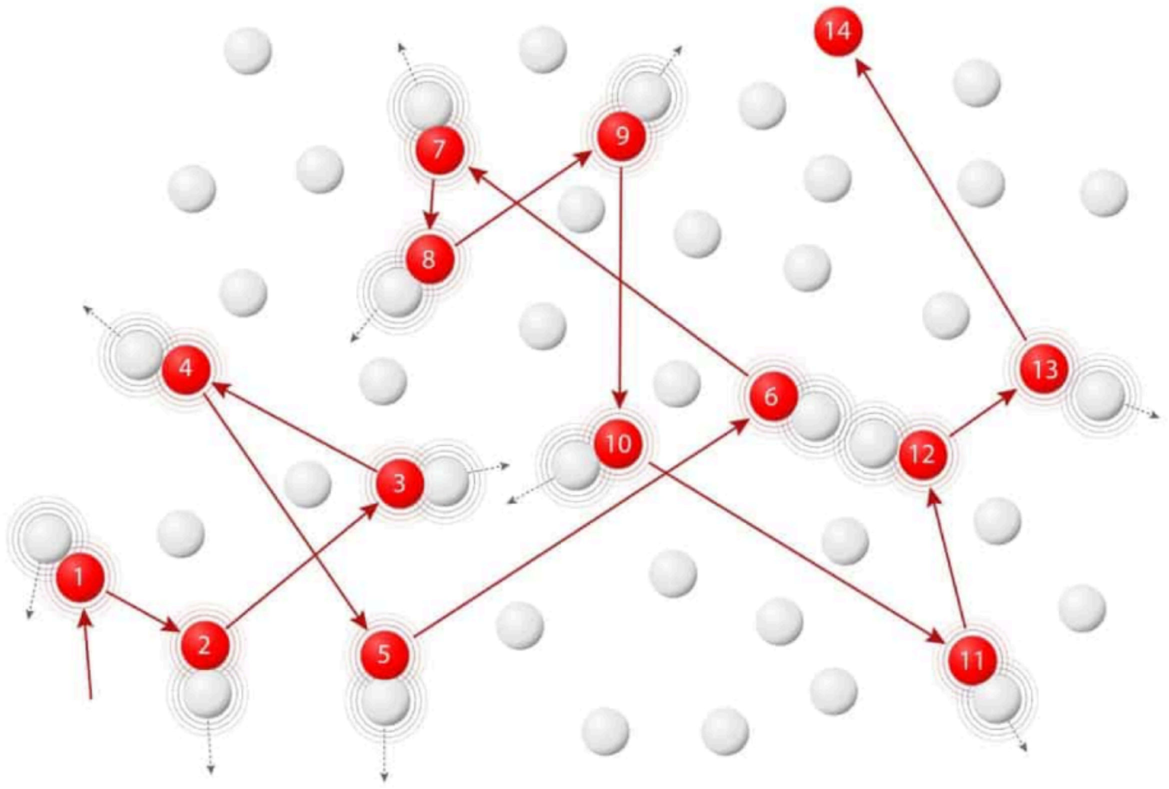
$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$



For the SGWB… Fokker-Planck approach

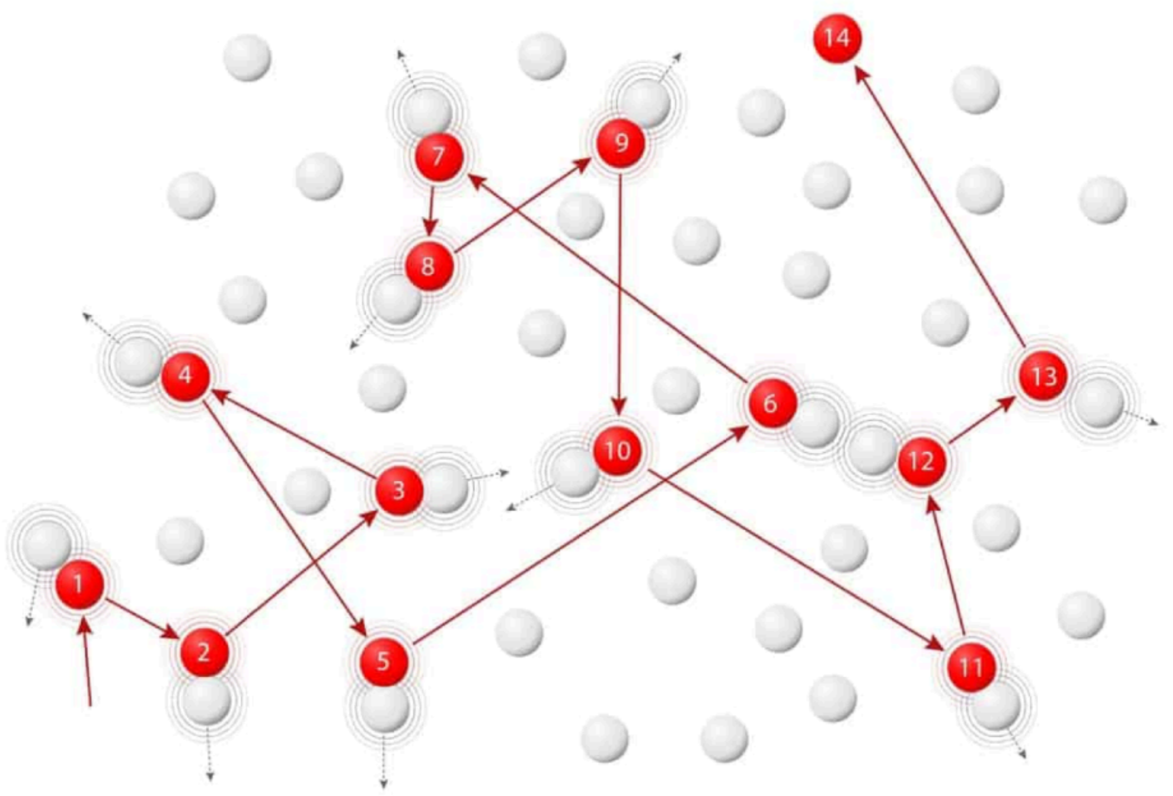
$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

deterministic

stochastic

$$\dot{X}_i(\boldsymbol{X}, t) = V_i(\boldsymbol{X}) + \Gamma_i(\boldsymbol{X}, t)$$

we move from dynamics of the variable to dynamics of the **distribution *W(X)***



$$\frac{\partial W}{\partial t} = -\partial_i \left(D_i^{(1)} W \right) + \partial_i \partial_j \left(D_{ij}^{(2)} W \right)$$

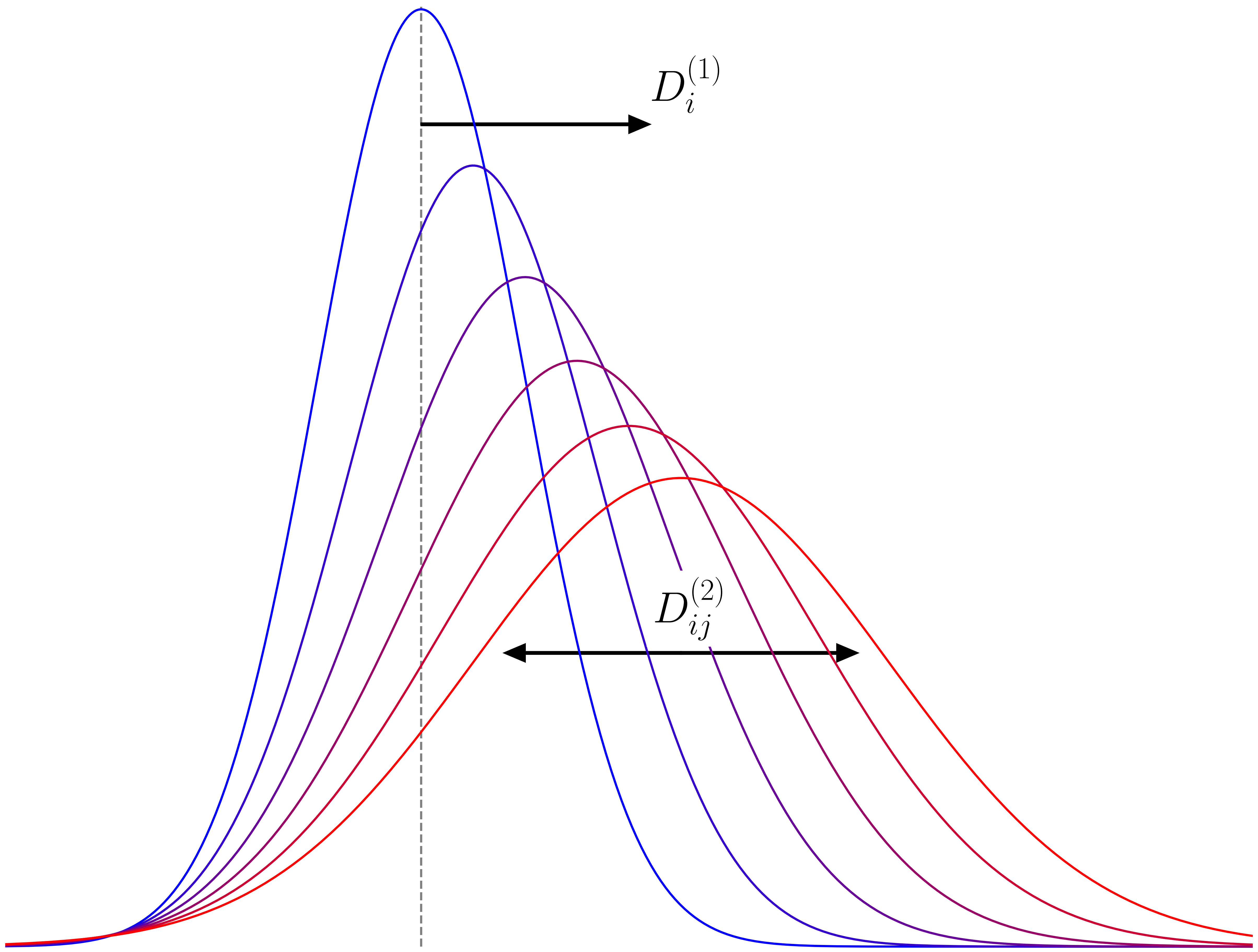
with $\partial_i \equiv \partial / \partial X_i$

$$D_i^{(1)} = V_i + \lim_{\tau \to 0} \frac{1}{\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t'} \mathrm{d}t'' \left\langle \Gamma_j \left(\boldsymbol{x}, t''\right) \partial_j \Gamma_i \left(\boldsymbol{x}, t'\right) \right\rangle .$$

$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{2\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t+\tau} \mathrm{d}t'' \left\langle \Gamma_i \left(\boldsymbol{x}, t'\right) \Gamma_j \left(\boldsymbol{x}, t''\right) \right\rangle .$$

Secular effects (accumulate with time)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\boldsymbol{X}, t)$ of orbital elements $\boldsymbol{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}\left(D_i^{(1)} W\right) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\left(D_{ij}^{(2)} W\right)$$

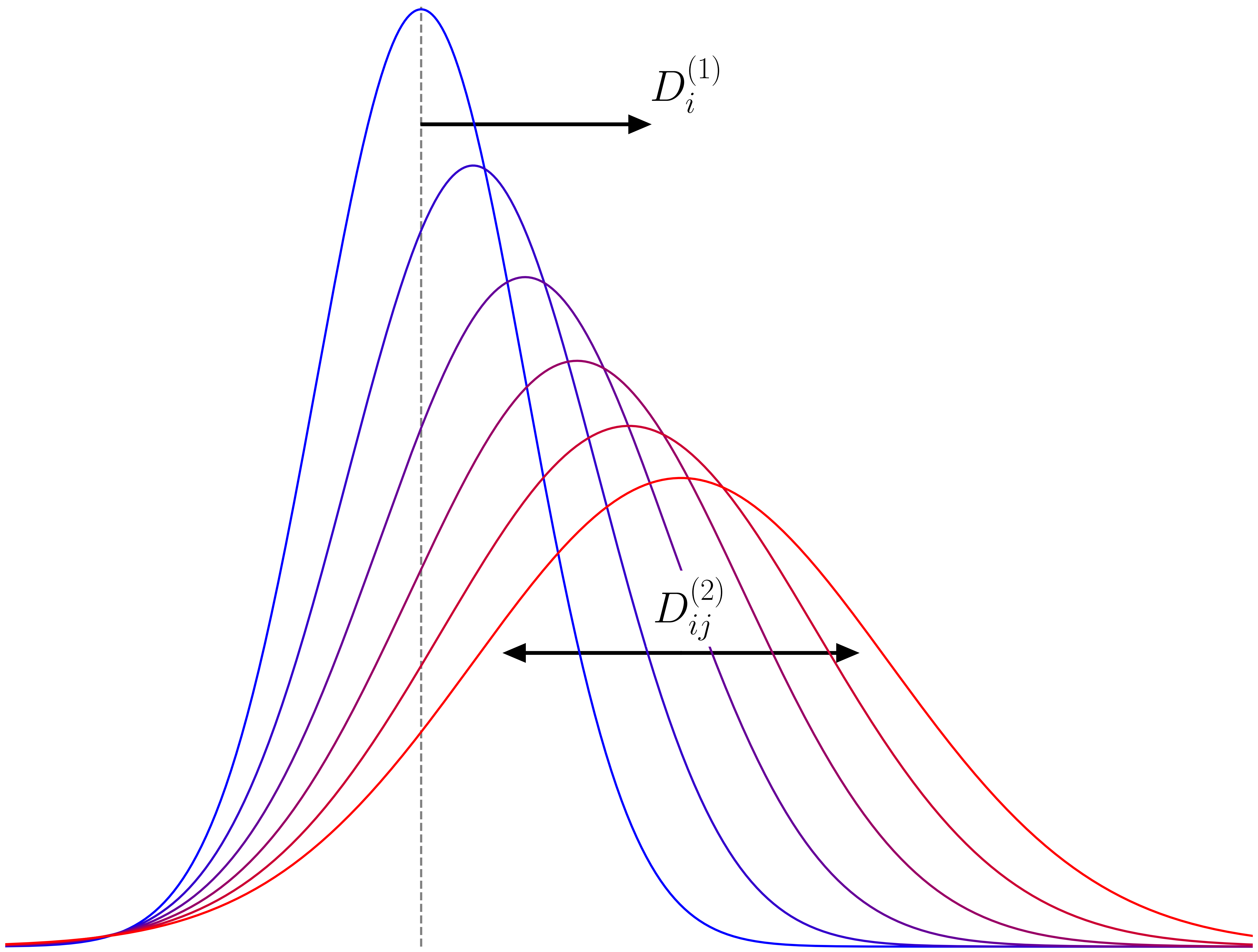
- *drift* and *diffusion* coefficients (averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X})\Omega_{\mathrm{gw}}(n/P)$$

$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X})\Omega_{\mathrm{gw}}(n/P)$$

Secular effects (accumulate with time)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



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(averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X})\,\Omega_{\mathrm{gw}}(n/P)$$

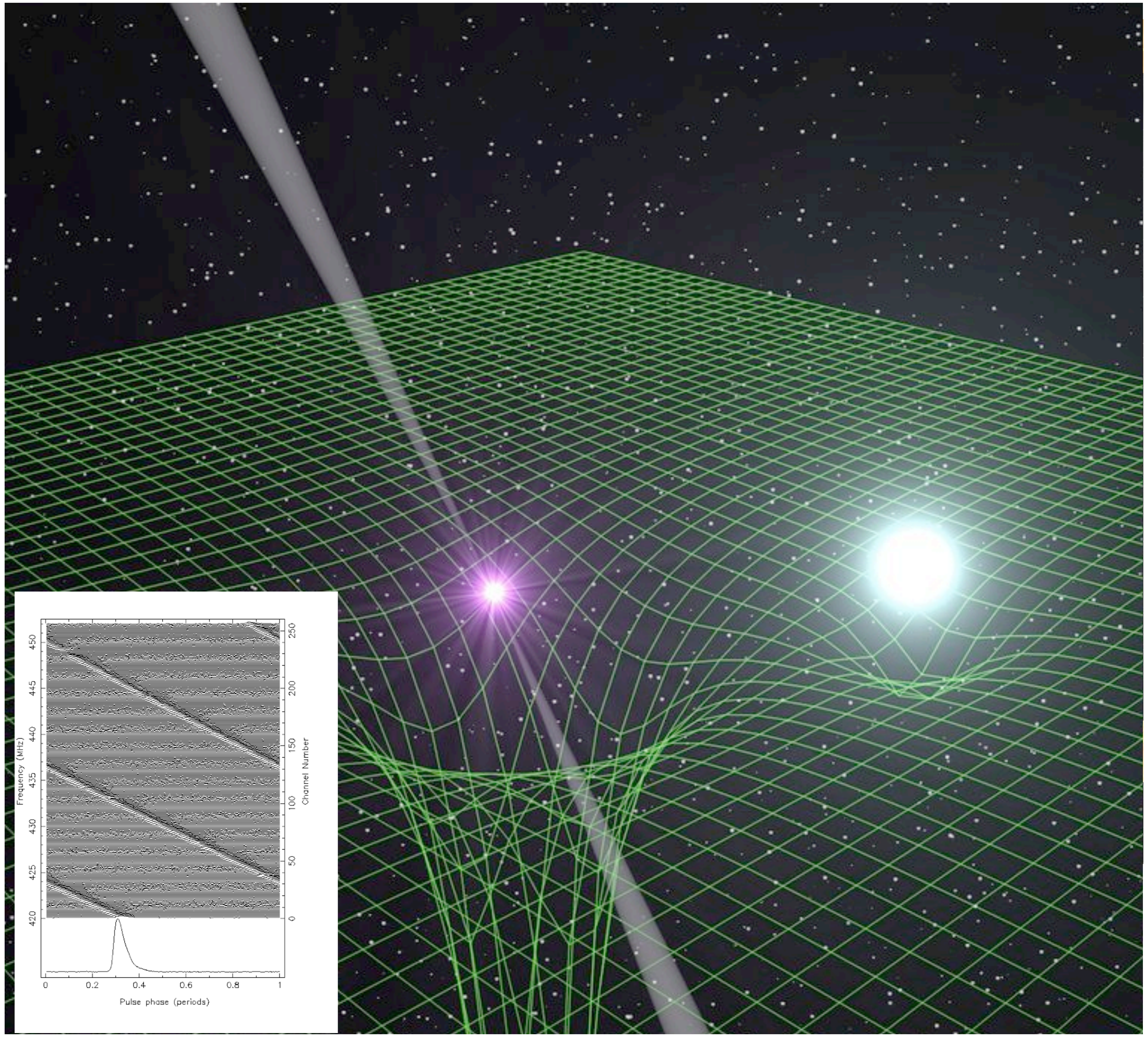
$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X})\,\Omega_{\mathrm{gw}}(n/P)$$

Two probes

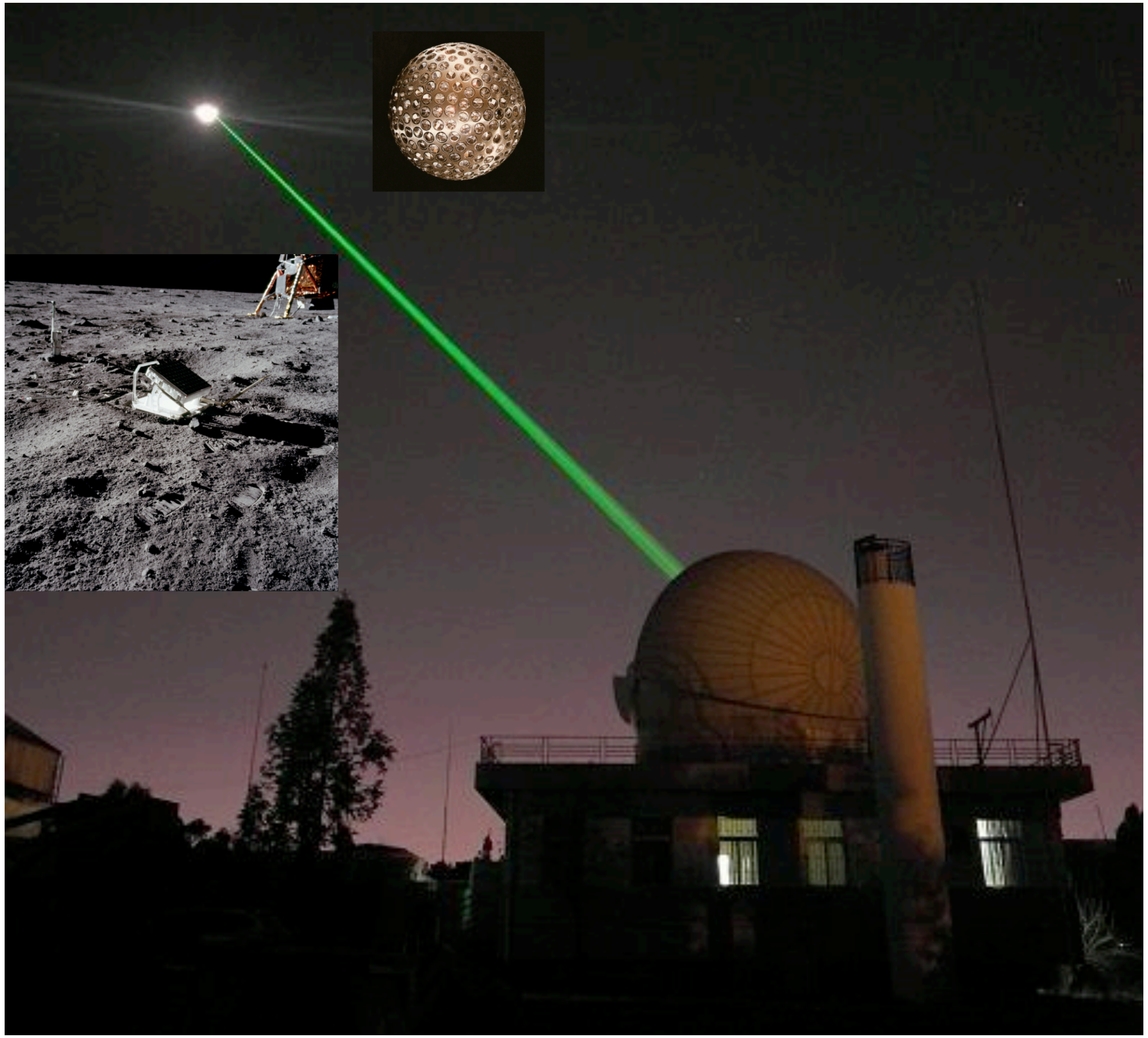
$f \sim \mu\text{Hz}$

few days

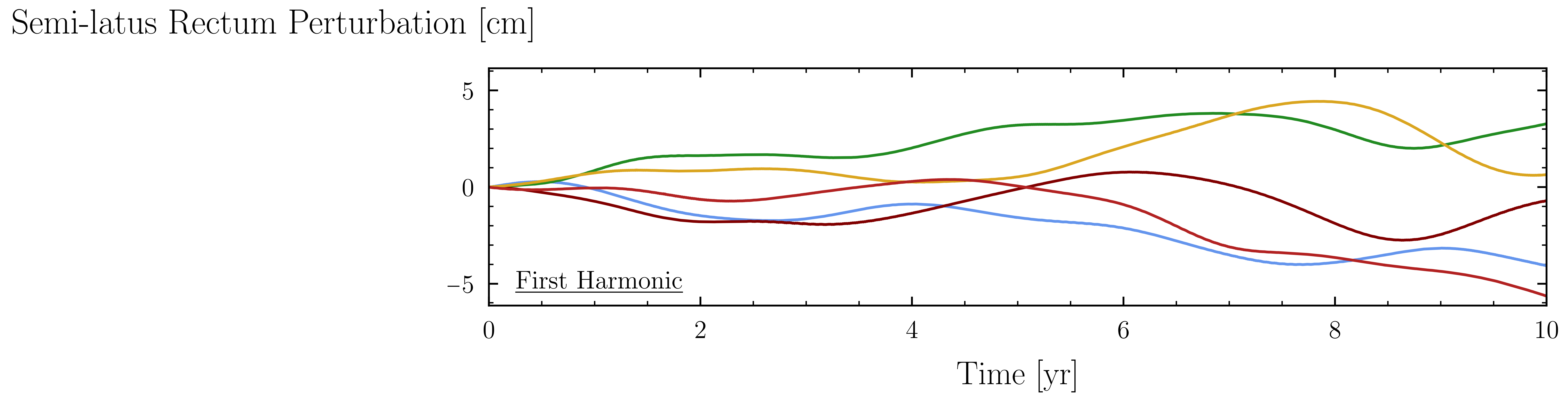
timing of binary pulsars

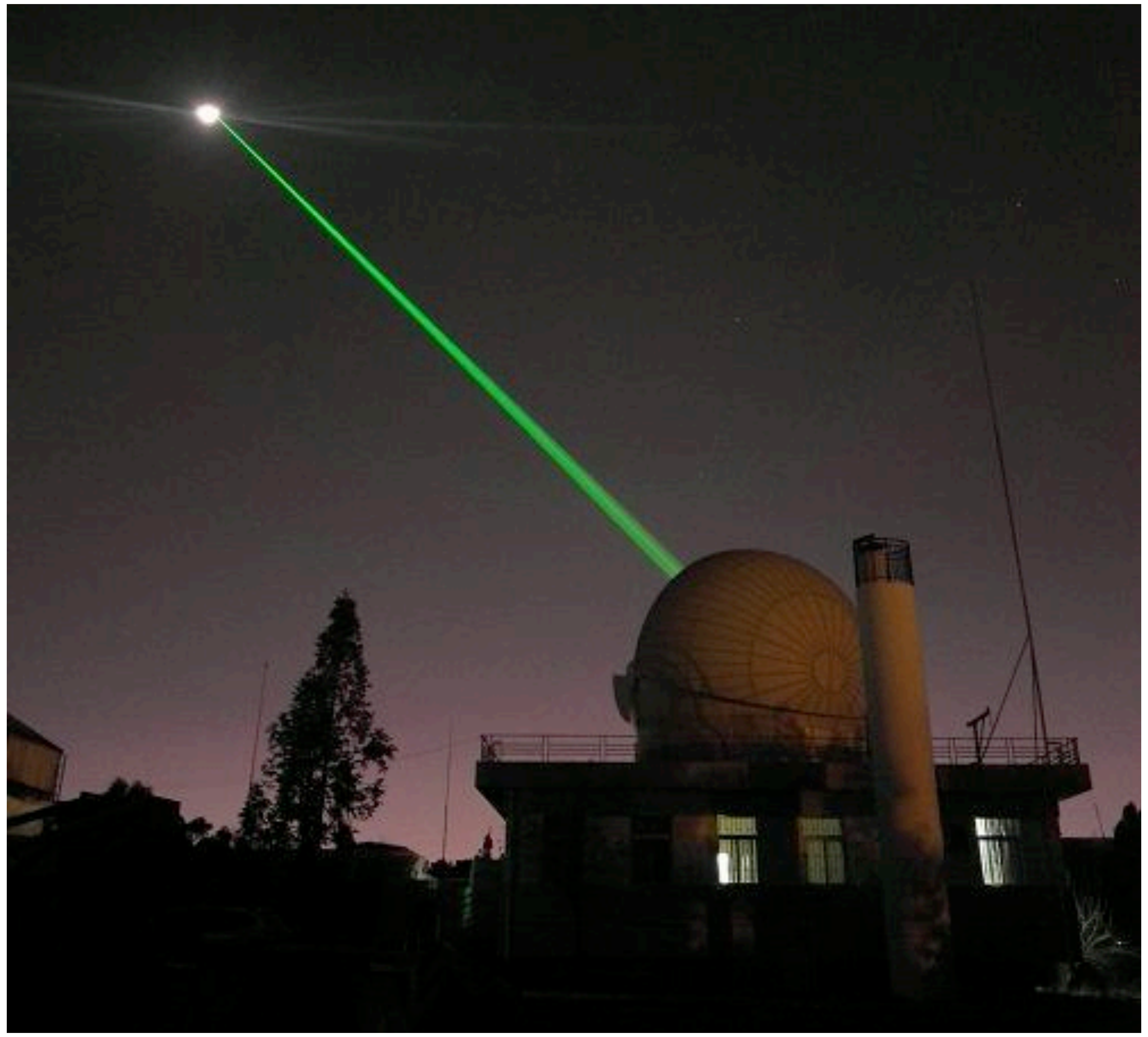


lunar and satellite laser ranging



Confirming with simulations

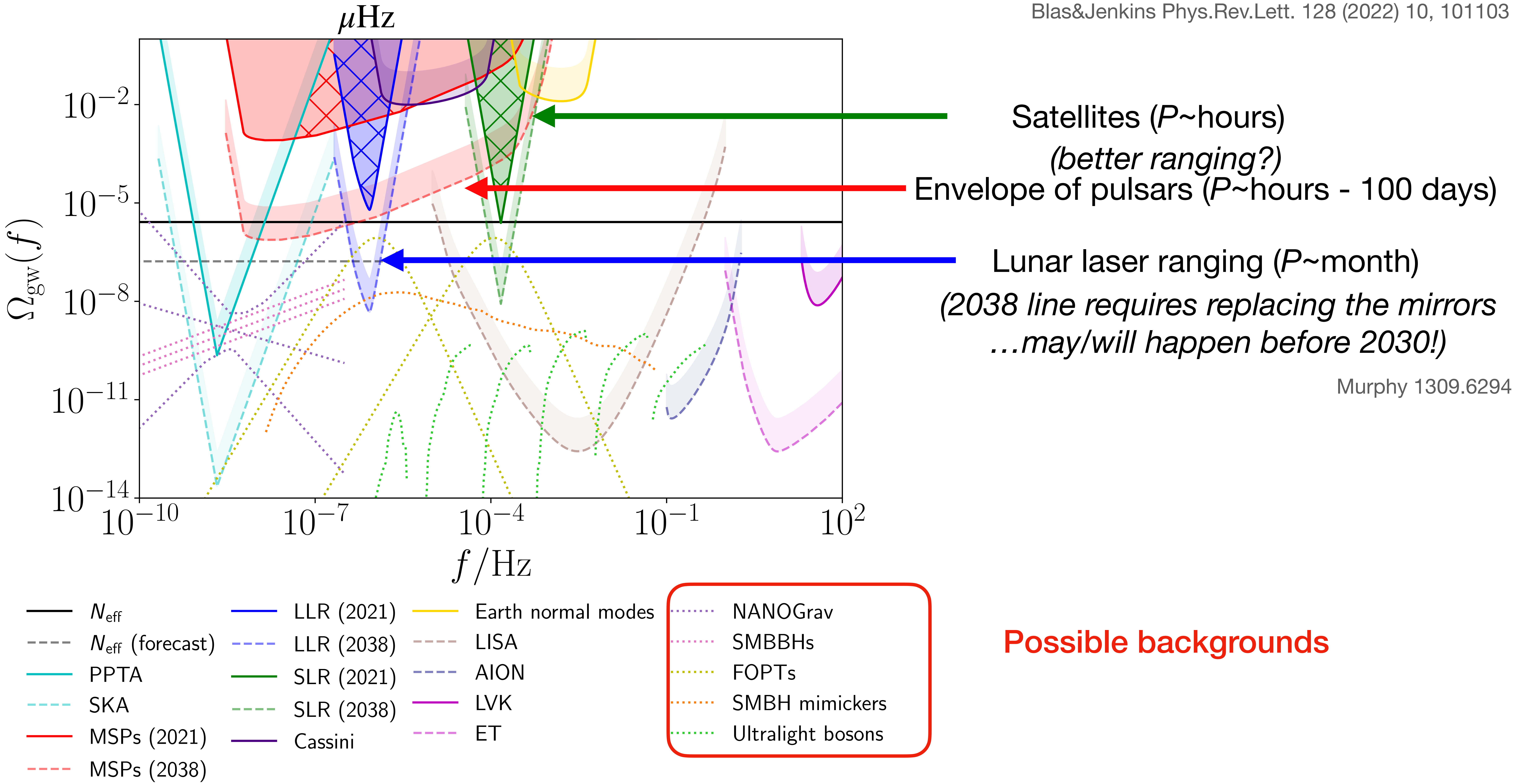




Credit: J. Foster
(work in progress:
Blas, Bourguin, Foster, Hees, Herrero-Valea, Jenkins, Xue)

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103

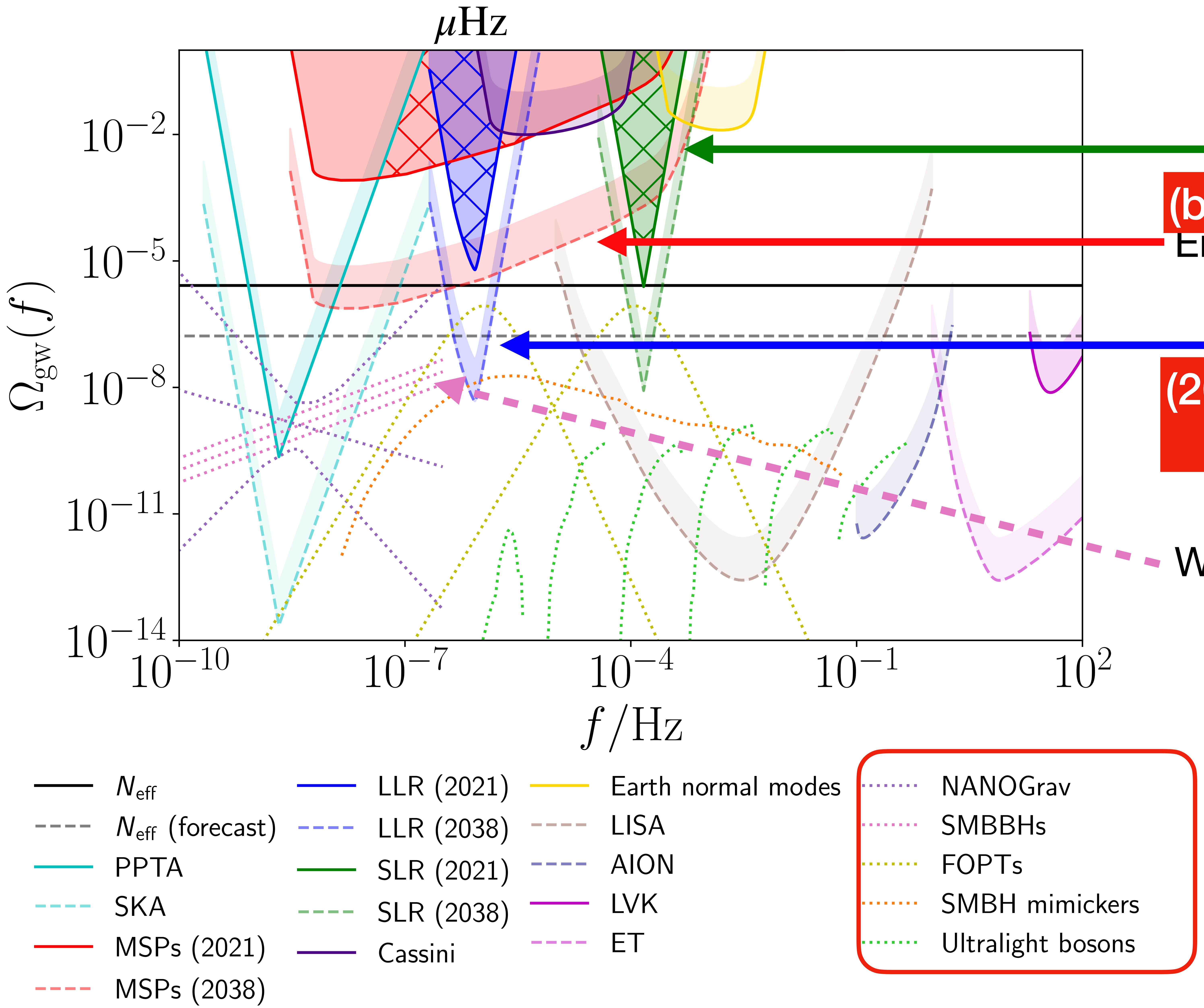


Murphy 1309.6294

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (P~hours)

(better ranging? new satellites?)

Envelope of pulsars (P~hours - 100 days)

Lunar laser ranging (P~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

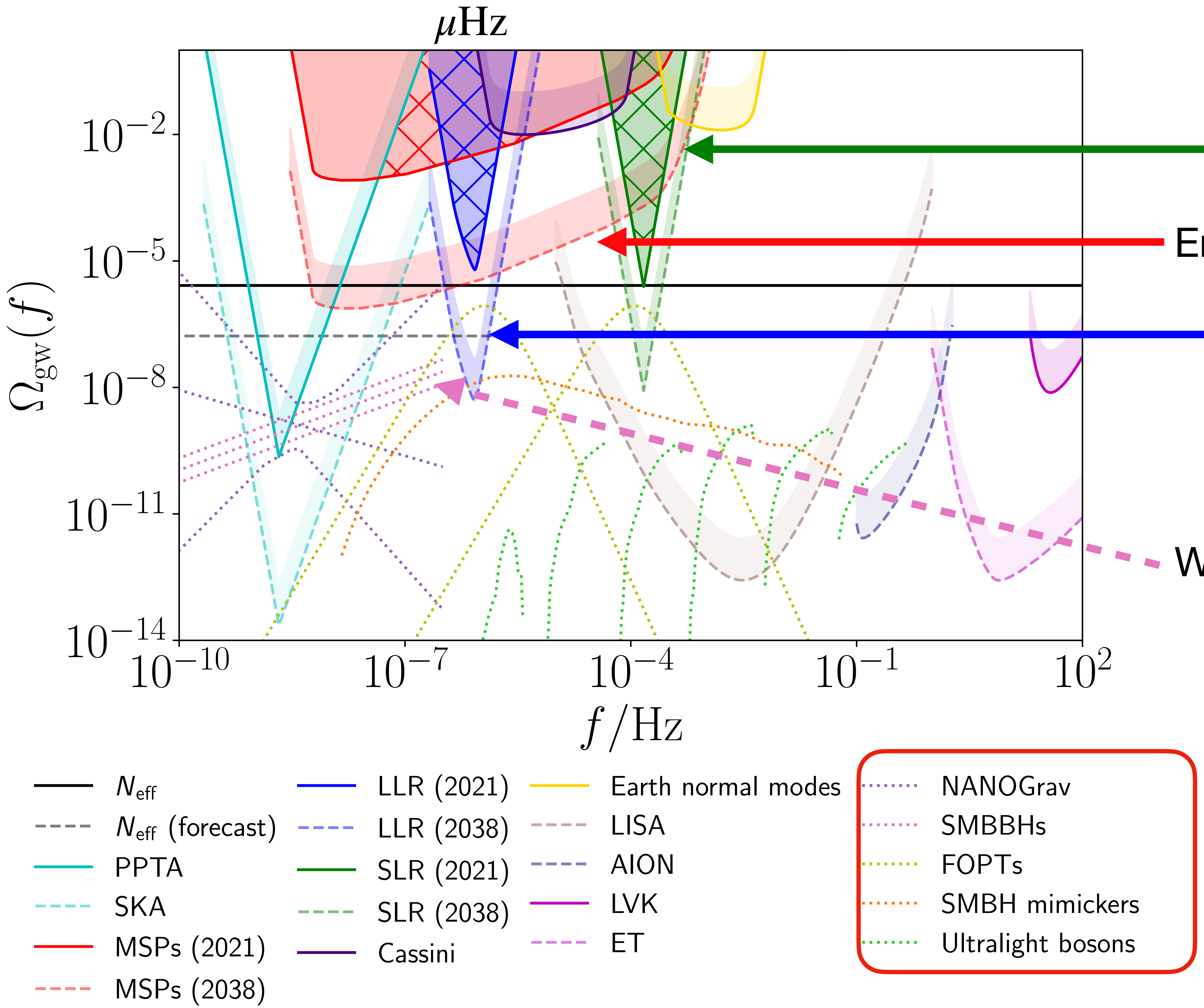
Murphy 1309.6294

We may see the signal of PTAs!!!

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Murphy 1309.6294

We may see the signal of PTAs!!!

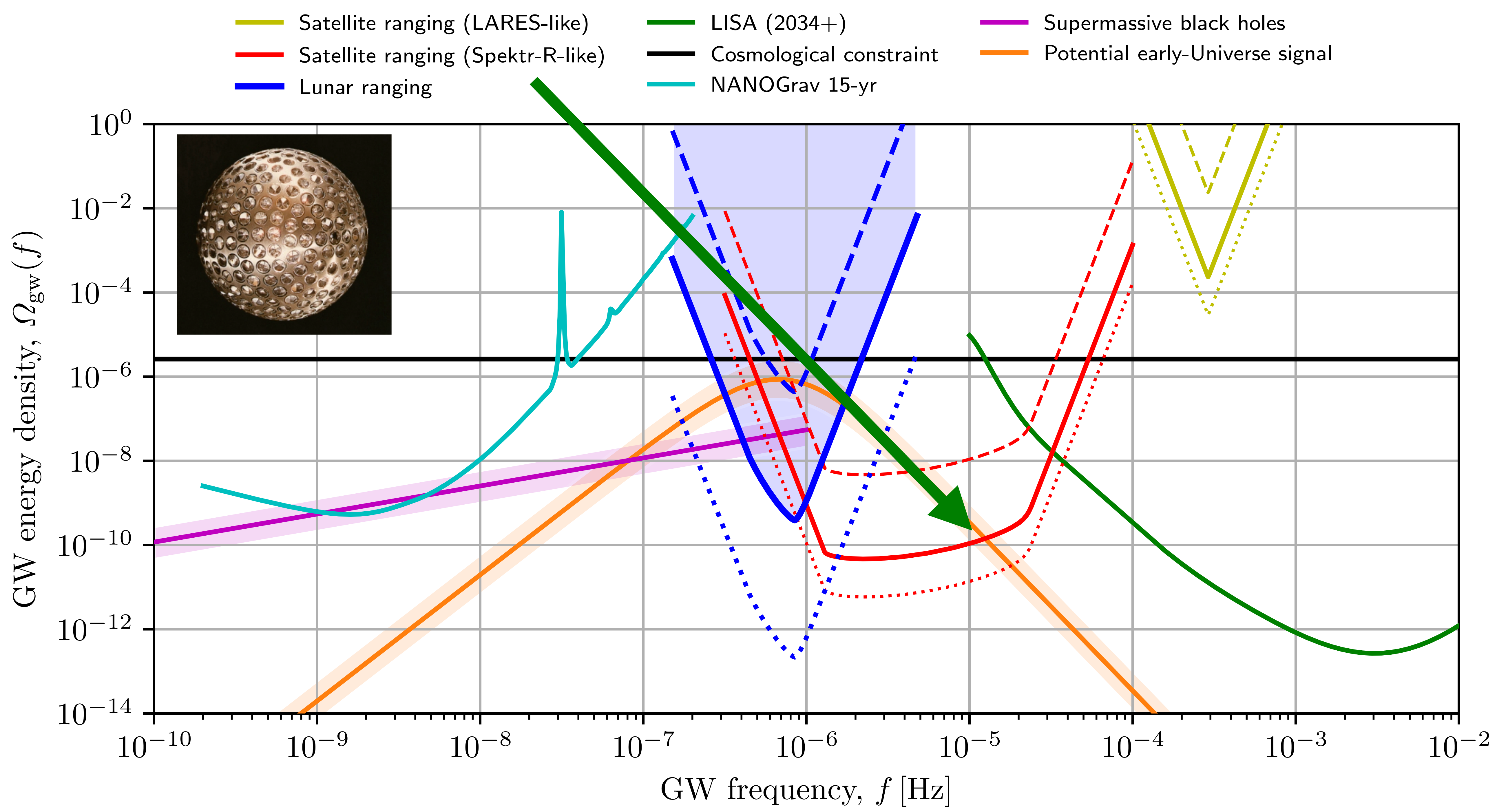
in 2050

$$\Omega \lesssim 3 \times 10^{-9} \quad \text{at} \quad f \sim \mu\text{Hz}$$

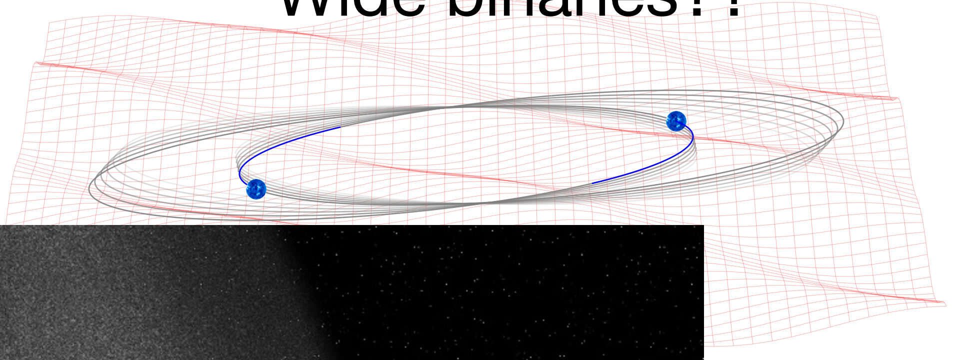
Possible backgrounds

Further ideas in the Solar system

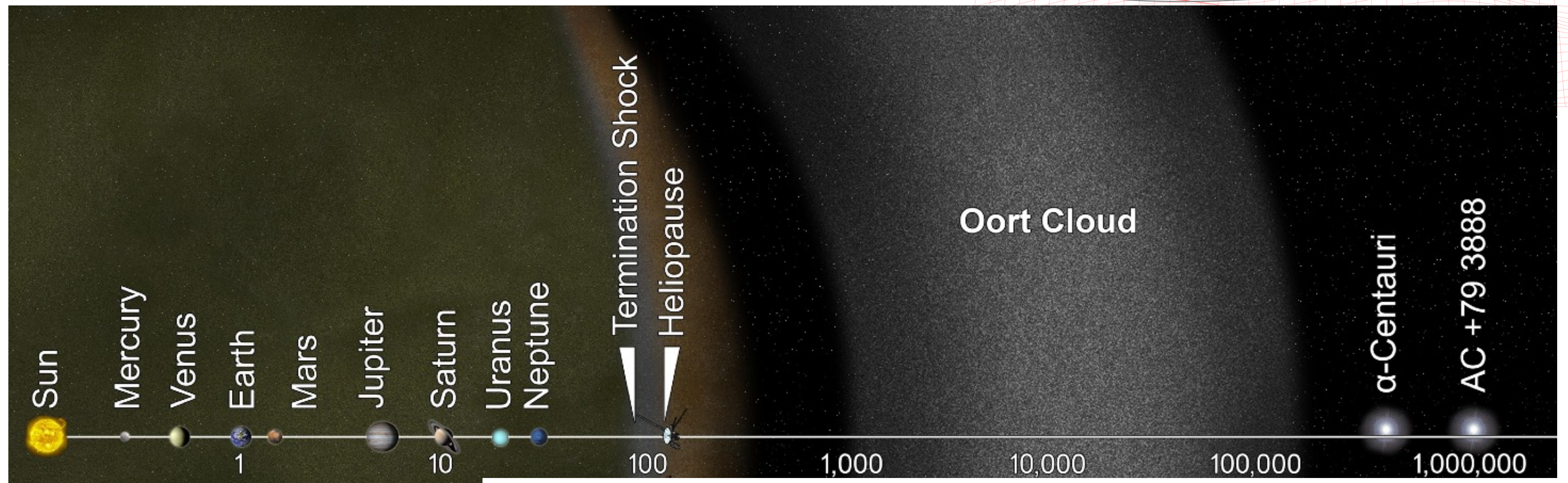
Optimised satellite: Blas, Jenkins, Turyshev (Proposal to NASA FunPAG)

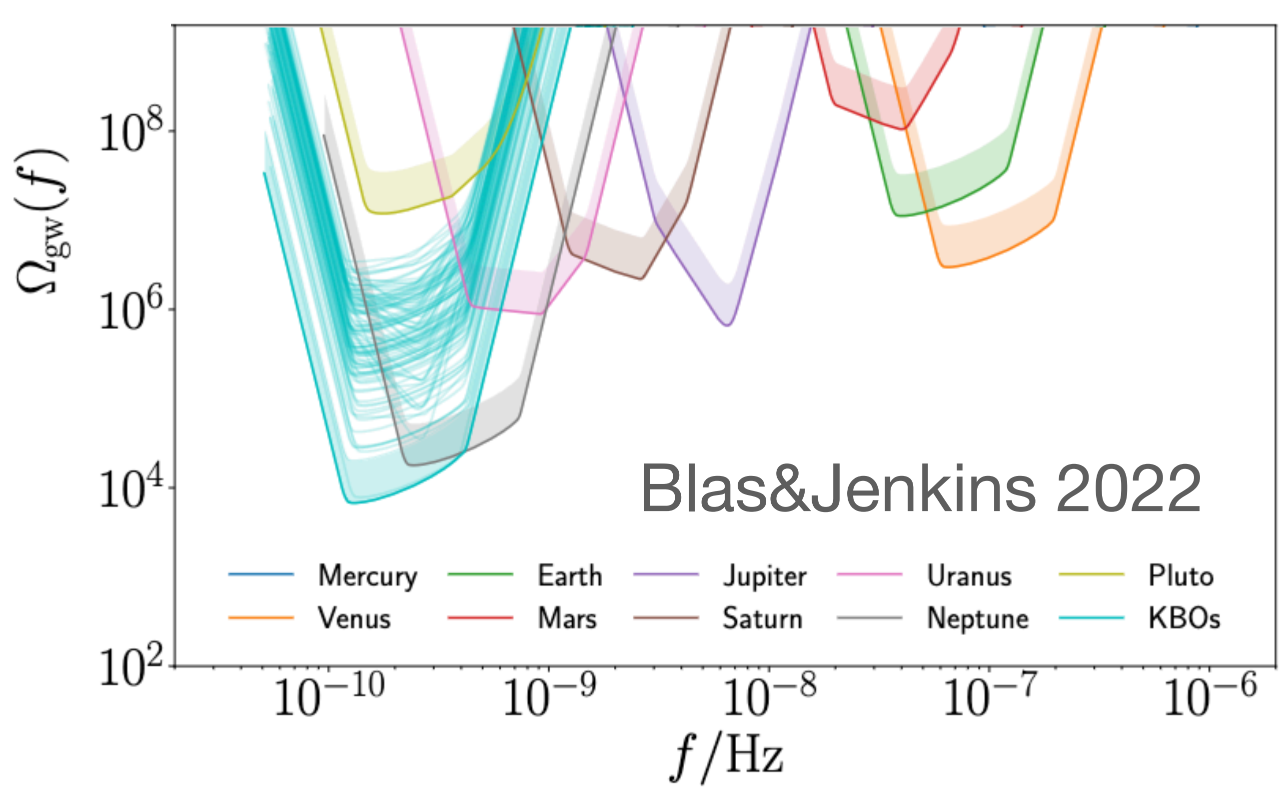


Wide binaries??

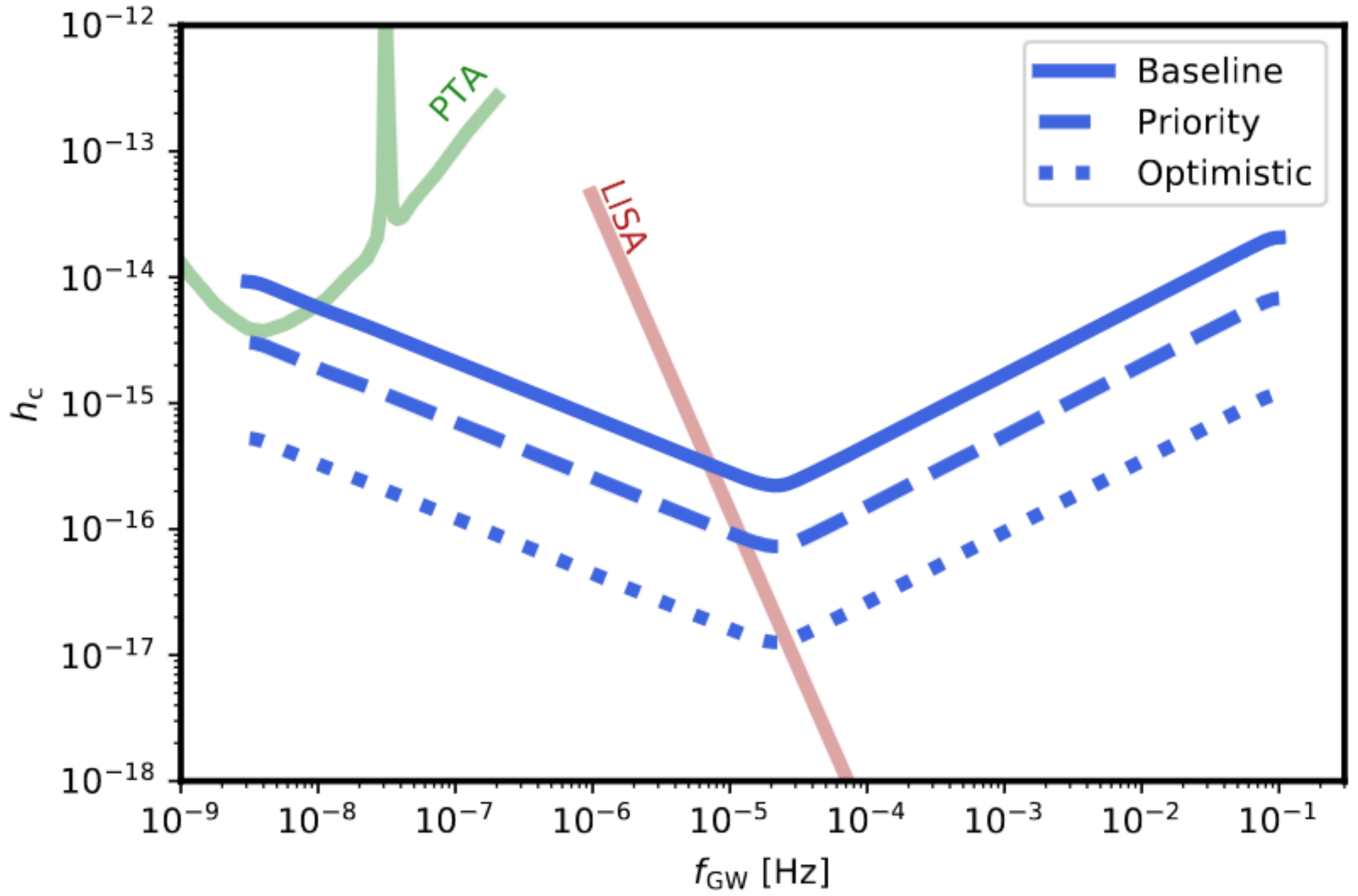


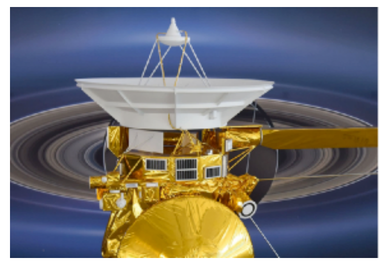
Stability of bounded systems:





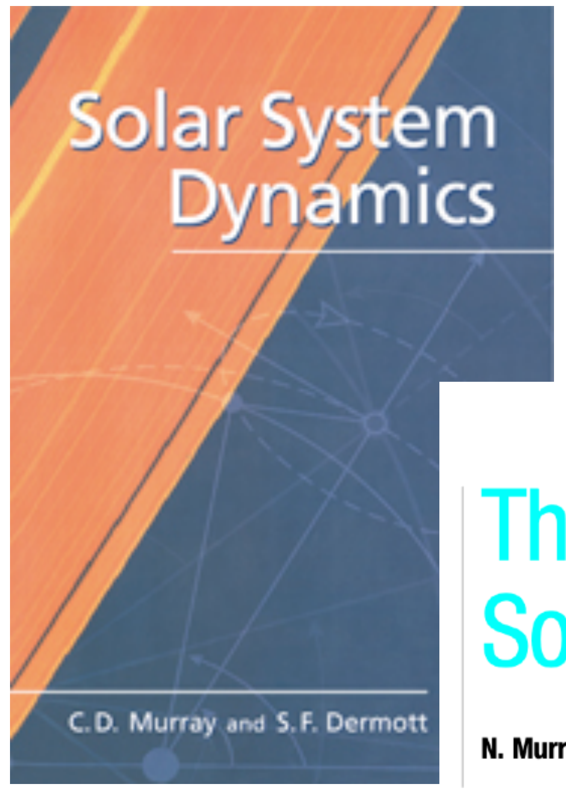
Doppler Tracking of Spacecrafts: Armstrong, Iess, Tortora and Bertotti 03

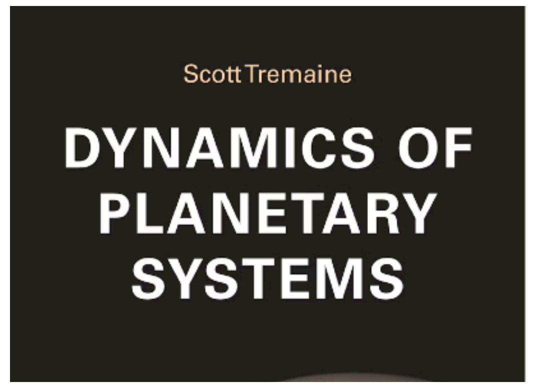


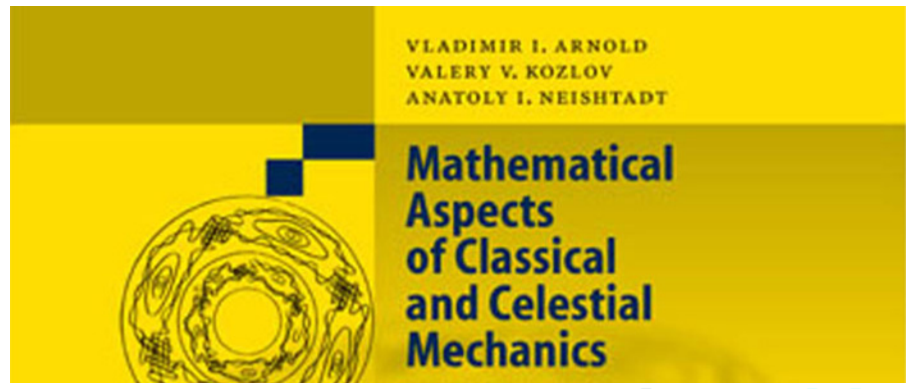


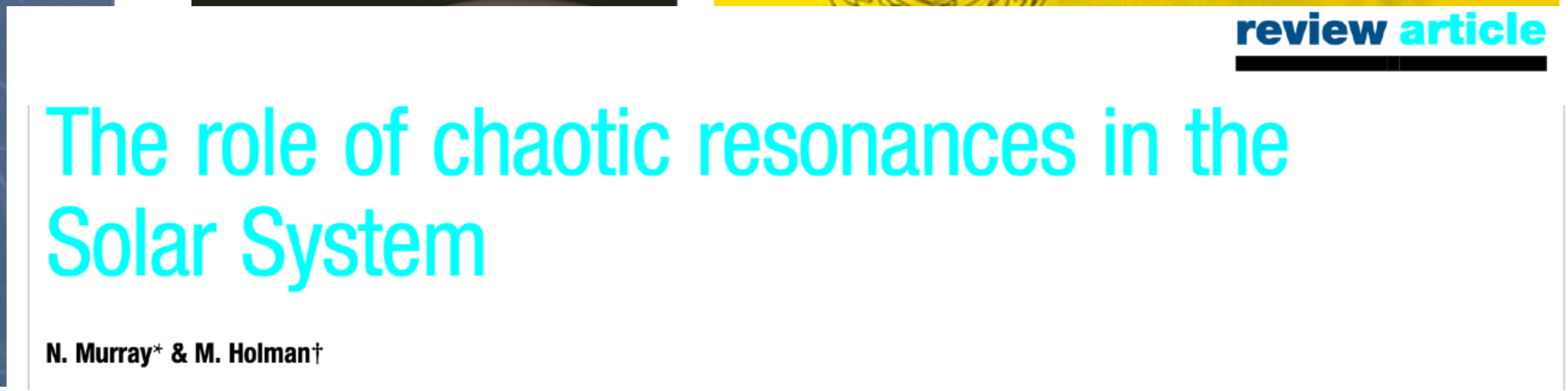
Future missions?
Zwick, Souyer, O'Neill, Derdzinski, Saha, D'Orazio, Blas, Jenkins, Kelley (2406.02306)

Exploit other resonances of the Solar System (also rotation)







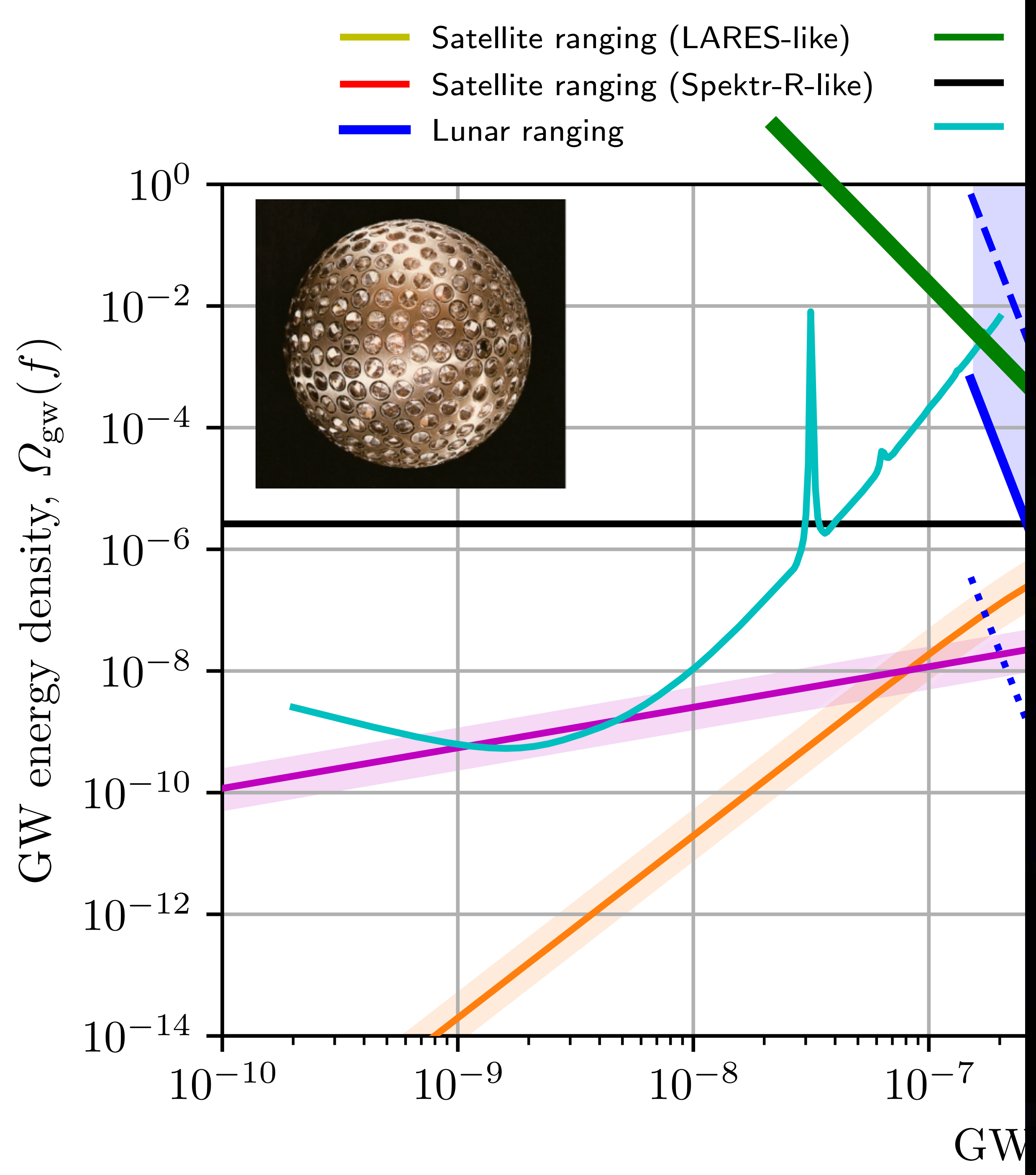

review article

The role of chaotic resonances in the Solar System

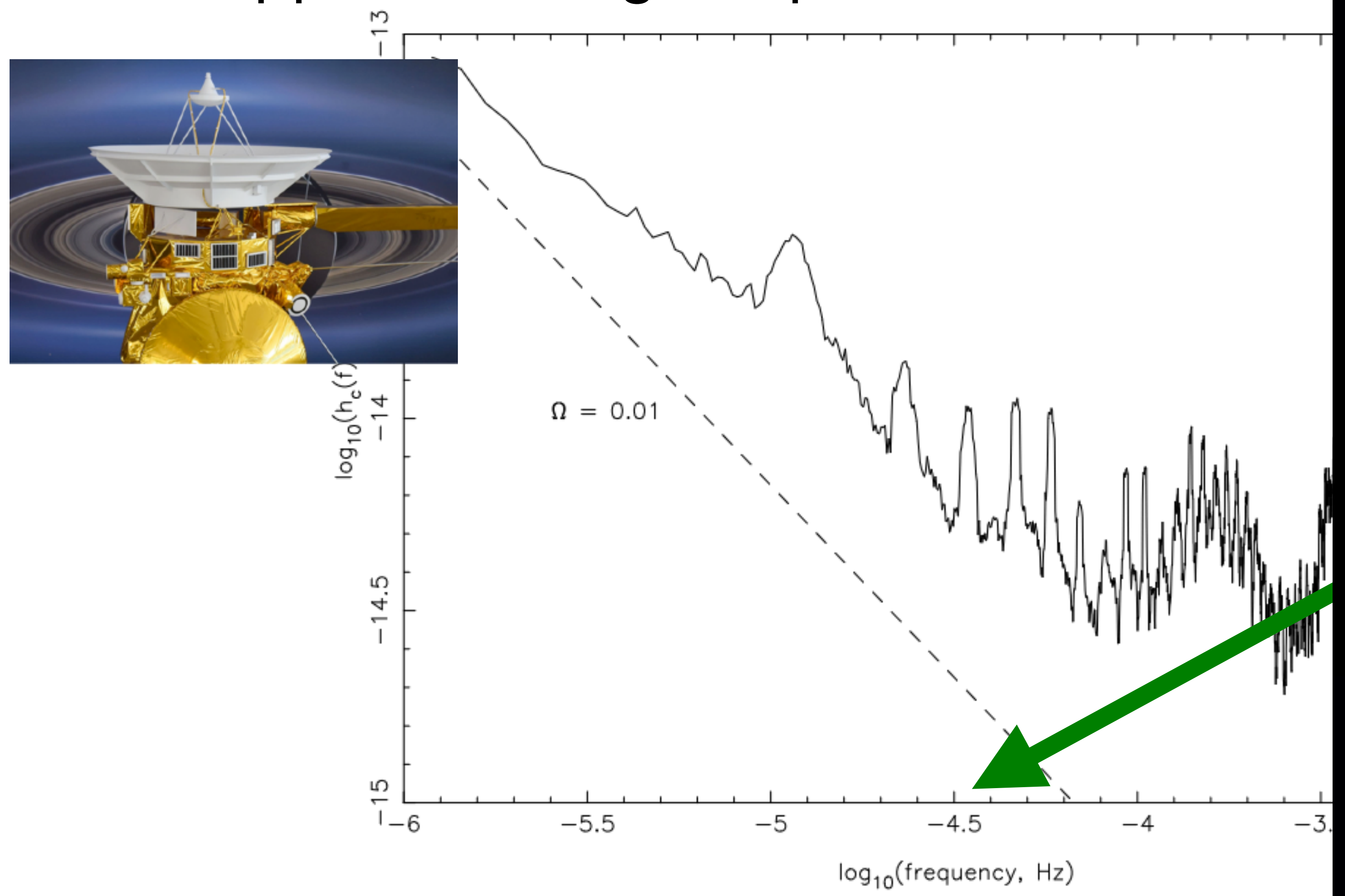
N. Murray* & M. Holman†

Furt… …tem

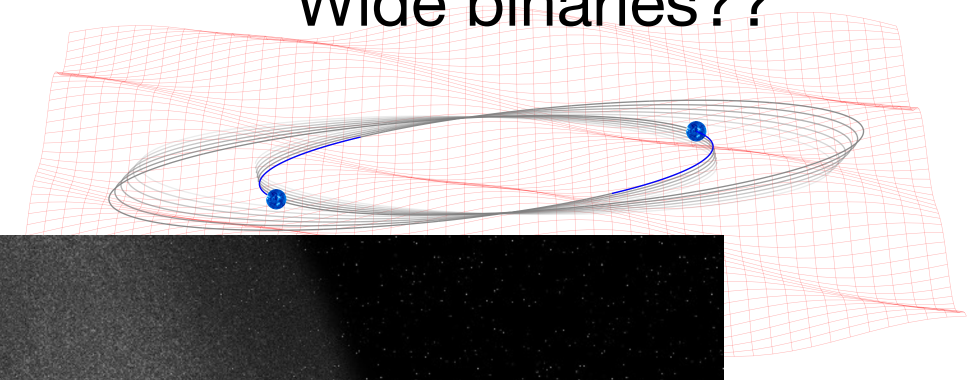
Optimised satellite: Blas, Jenkins, Turysh…



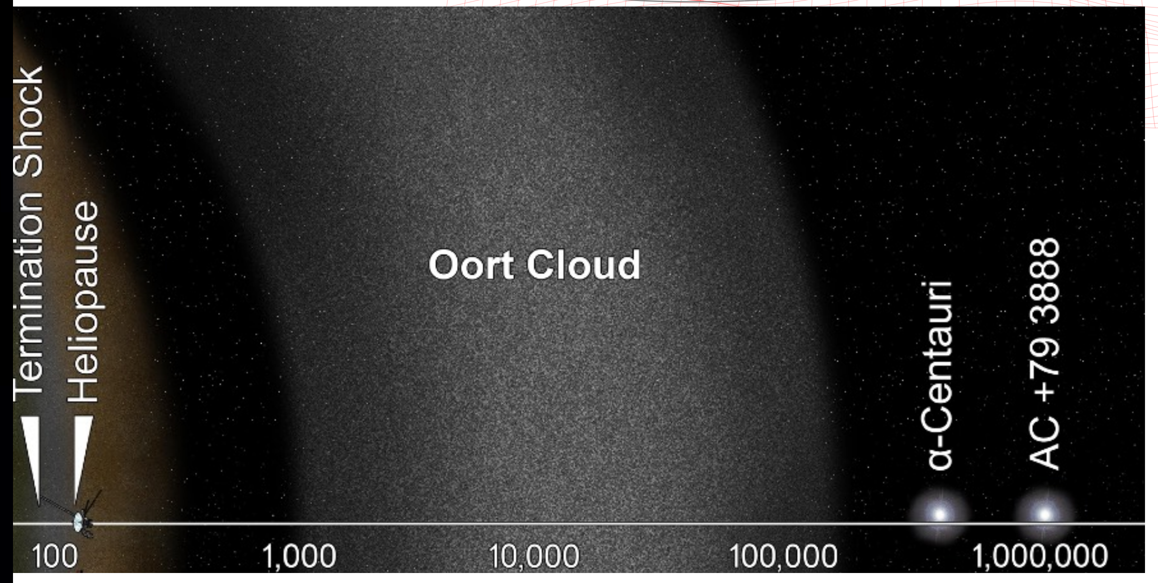
Doppler Tracking of Spacecrafts: Armstr…

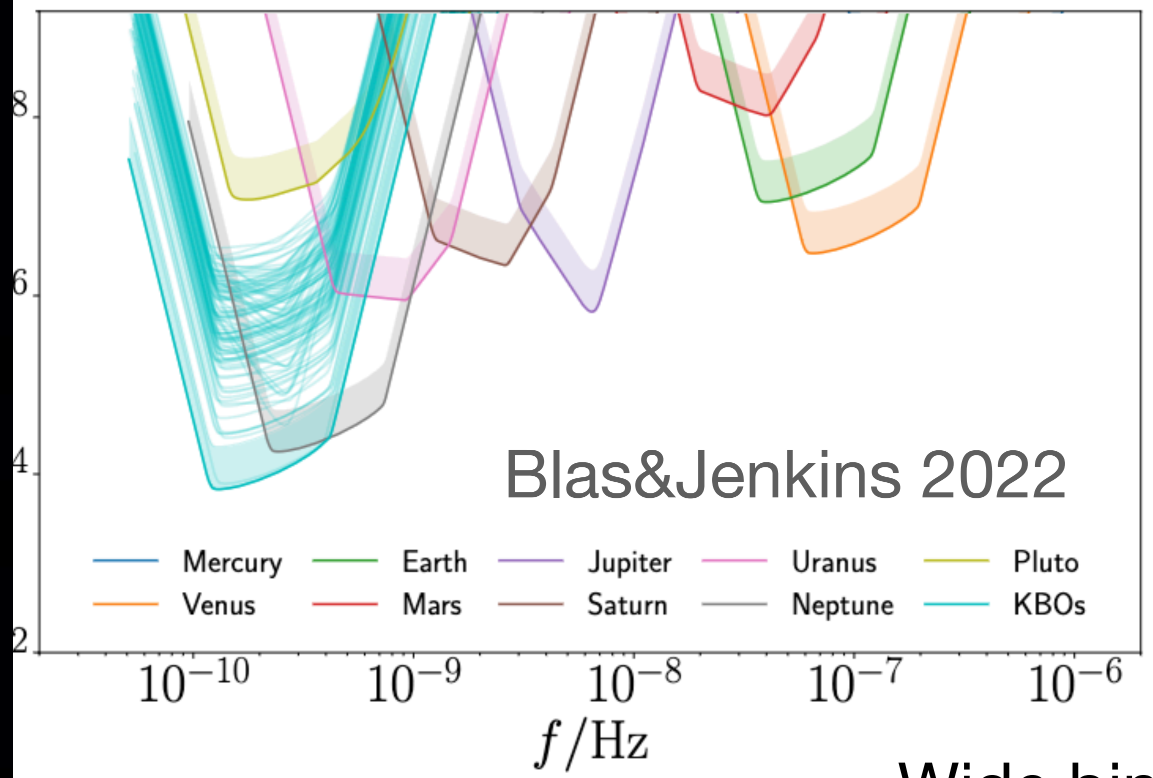


Wide binaries??



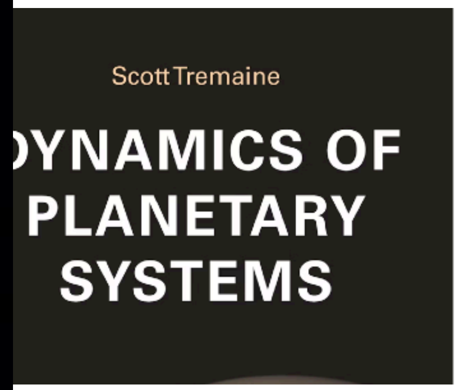
…tems:

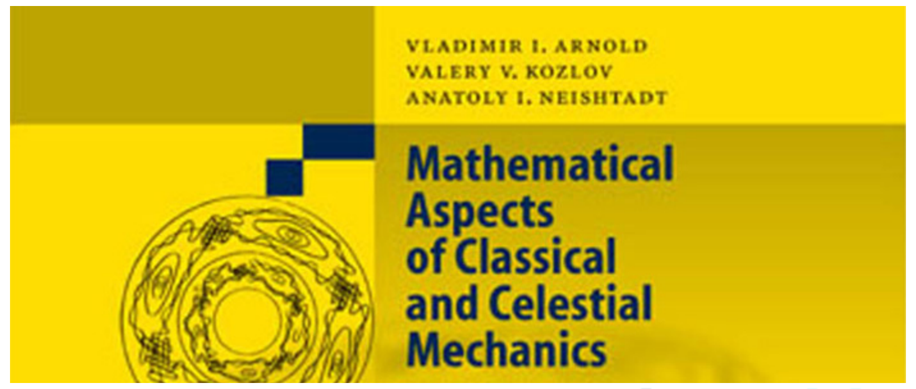




Wide binaries??

…s of the Solar System (also rotation)





review article

…le of chaotic resonances in the …System

…man†

GWs with orbits in the Solar System

- The resonant **absorption of GWs by binaries** (LLR/SLR) gives a new handle to detect gravitational waves at very competitive levels

- The μHz band is very rich for **astrophysical** and **cosmological** sources

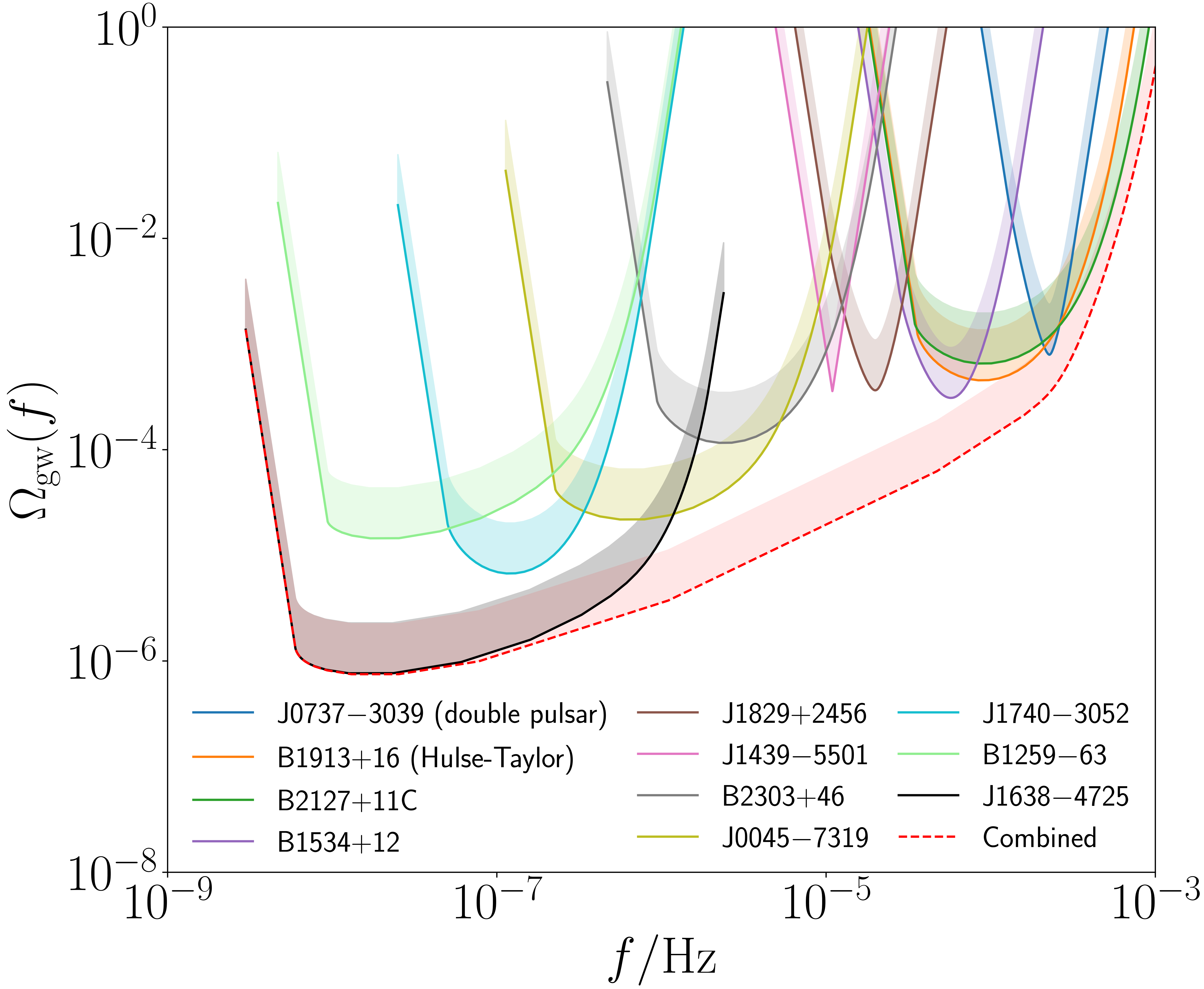
$\Omega_{\rm gw} \geq 4.8 \times 10^{-9}$ $f = 0.85\,\mu{\rm Hz}$

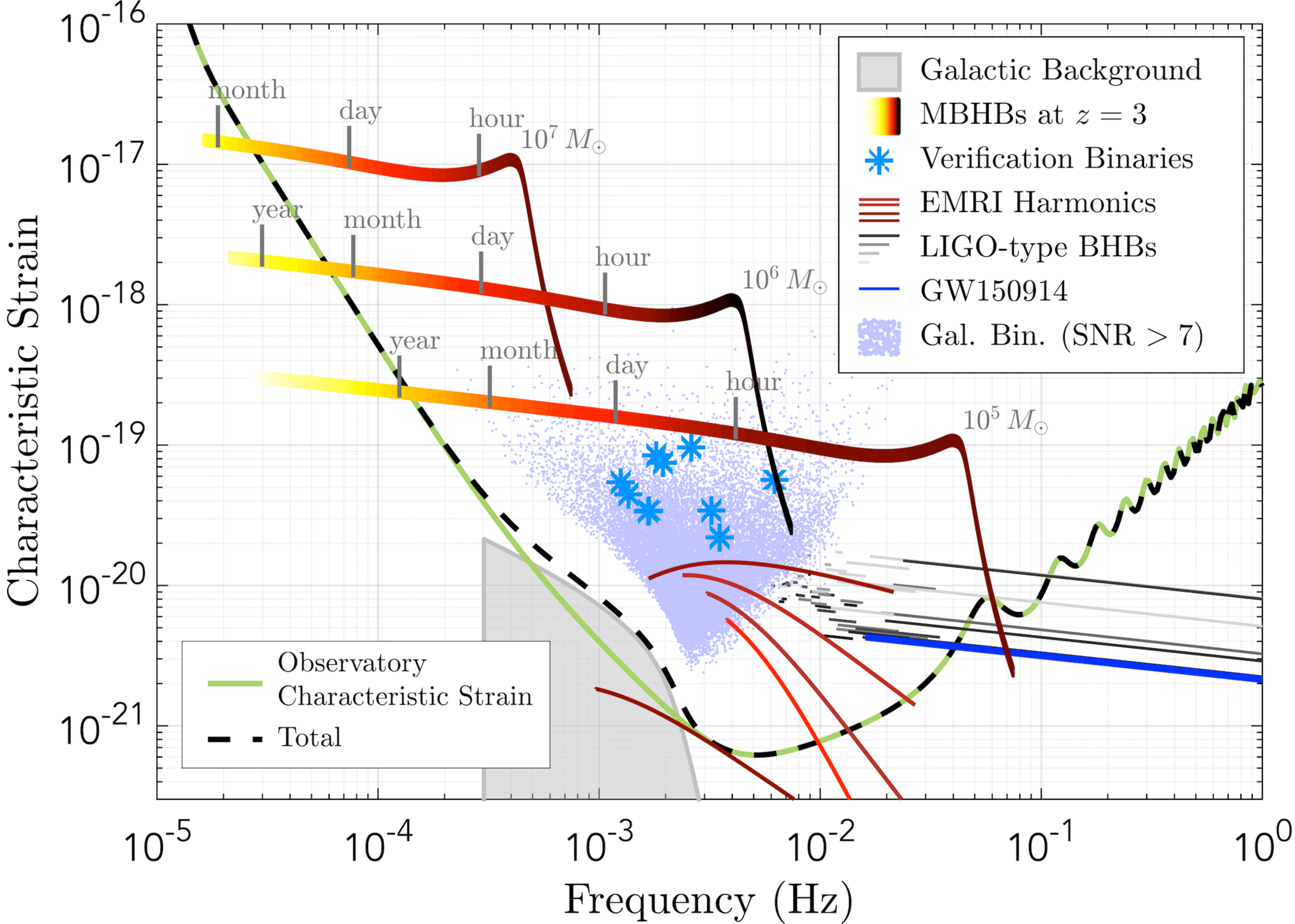
$\Omega_{\rm gw} \geq 8.3 \times 10^{-9}$ $f = 0.15\,{\rm mHz}$

- Detection guaranteed!

- **Future plans:** better analysis. New mirror in the Moon? New optimised satellites?

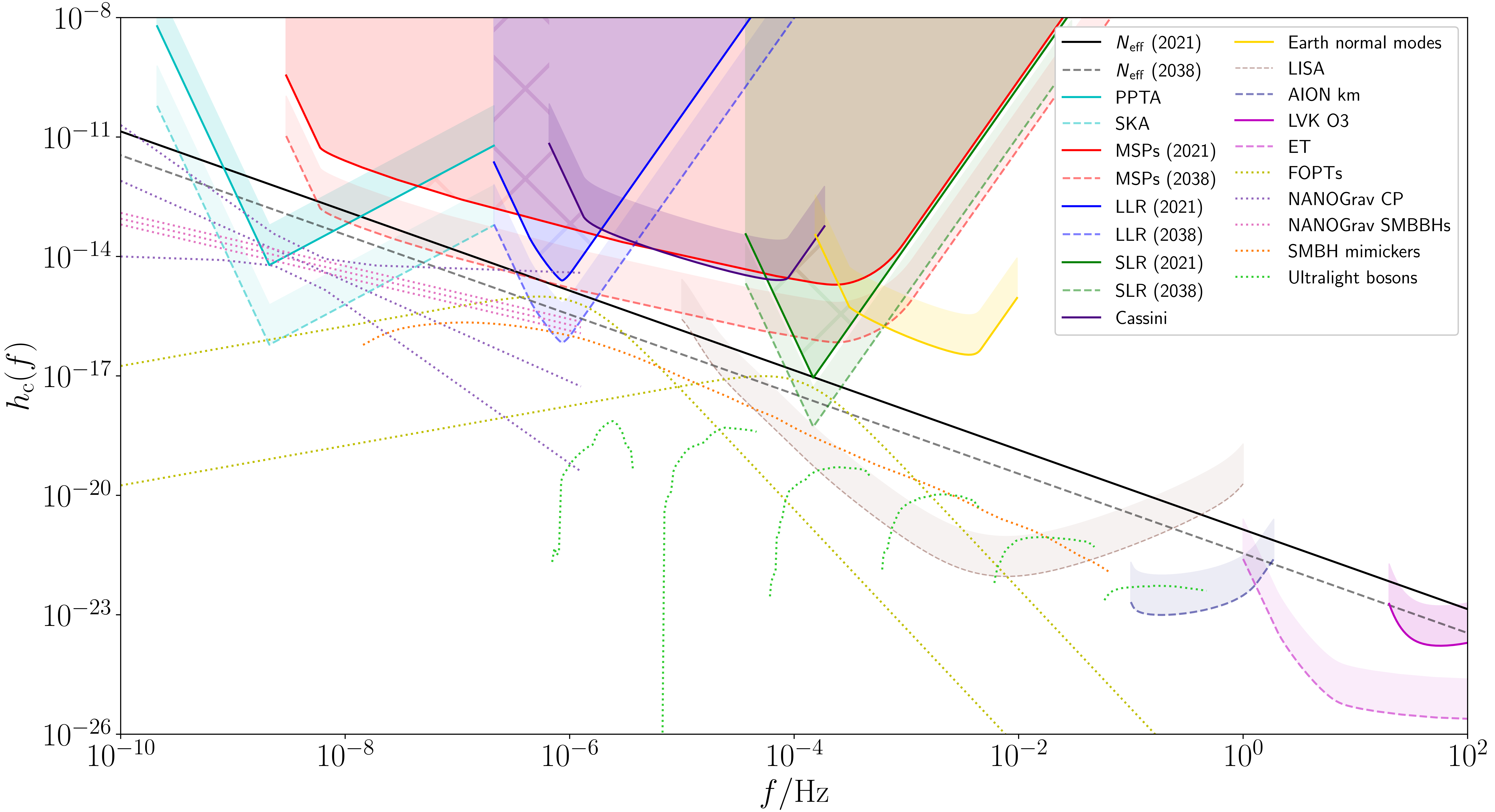
Combining binary pulsar bounds



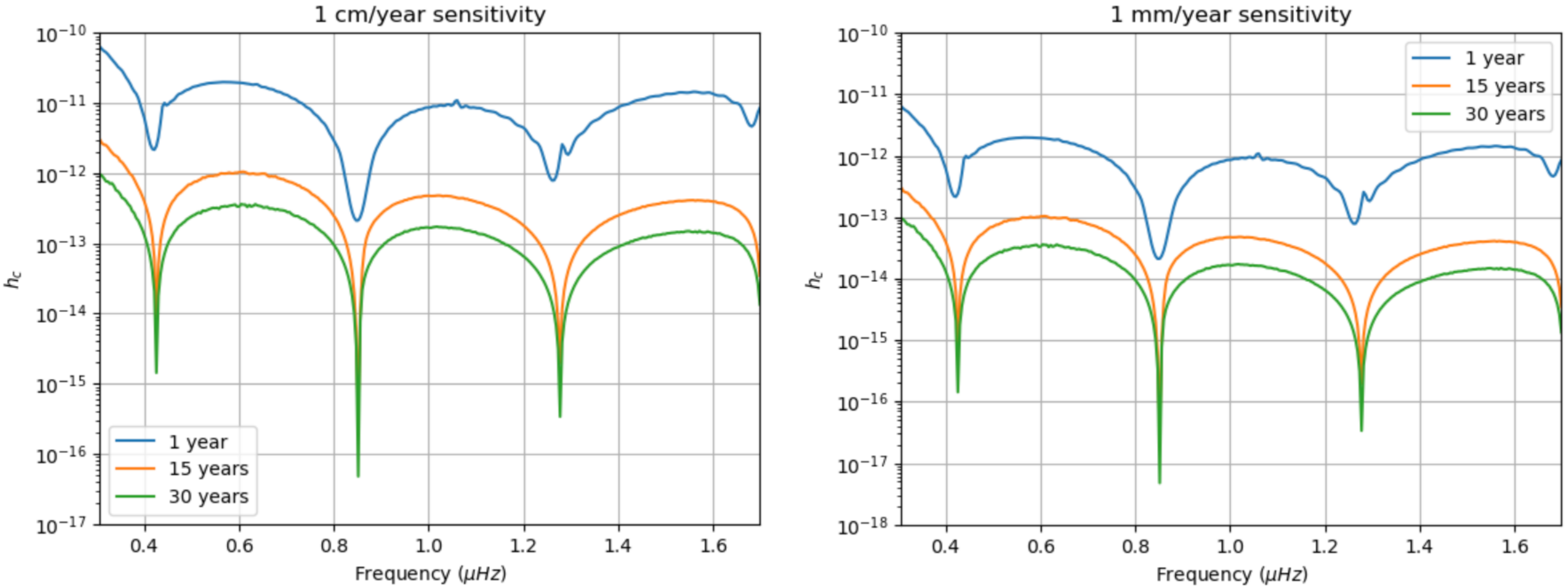


Galactic Background
MBHBs at $z = 3$
Verification Binaries
EMRI Harmonics
LIGO-type BHBs
GW150914
Gal. Bin. (SNR > 7)
Observatory Characteristic Strain
Total
month
day
hour
$10^7 M_\odot$
year
month
day
hour
$10^6 M_\odot$
year
month
day
hour
$10^5 M_\odot$
Characteristic Strain
Frequency (Hz)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

Characteristic strain



Monochromatic sources



Credit:Herrero-Valea
(work in progress:
Blas, Bourguin, Foster, Hees, Herrero, Jenkins, Xue)